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Instituto

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—Bacon.

VOL. XXVI.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.
PARIS: L. WARNIER, LIBRAIRIE DES ASSURANCES, 48 RUE LAFFITTE.
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LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

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OF THE

INSTITUTE OF ACTUARIES.

History of Life Assurance in the United Kingdom. By Cornelius Walford, F.I.A.

(*Continued from* Vol. xxv, p. 437.)

3.—Period of Scientific Exactitude—*(continued).*

A CASE arose about this time—the middle of the eighteenth century—which, as it illustrated some points in practice and an important principle in the law of Insurance, I will briefly review. It is the case of *Cleeve* v. *Gascoigne.* In June 1749, the defendant (Gascoigne) had applied to an Office-keeper, or Broker, to insure £1,600 for one year at 5 per-cent on the life of one Poulton, from whom the defendant had agreed to purchase an estate, whereof Poulton had the reversion in fee, and also an intervening interest for his own life. The Broker answered that the course of the office was to produce the person whose life was to be insured, in order to be inspected by the Insurers or Underwriters; to which the defendant replied that the person lived a great distance in the country; and that it was very inconvenient for him to come to London: and on the Broker asking the defendant whether he was a good life, answered in the affirmative, and added, that he would leave a note of his name and place of abode, which he accordingly did. On this declaration of the defendant, the Broker procured 16 persons to underwrite a policy of assurance for £1,600 on Poulton's life at the rate and for the time before mentioned. About six months afterwards Poulton died. In 1750 the defendant brought an action on the policy in the Court of Common Pleas and obtained a verdict; in consequence of which, the present plaintiff, who was one of the Underwriters, paid him £98, as for his part of the Insurance-money. He then brought his suit in the Court of Chancery to obtain repayment of the money, on the ground that the Insurance had been obtained by fraud and imposition. The defendant admitted in his answer that Poulton was a drunken fellow; upon which the Lord Chancellor directed an action to be brought which was done, and there was a verdict for the present plaintiff; but the jury acquitted the defendant of fraud. The case now came before the Lord Keeper for further directions. He spoke of Sir Crisp Gascoigne, the present defendant as being as worthy a gentleman and in as full credit as any merchant in London; and proceeded: "*I have constantly attended at Guildhall* "*during the last war, and was concerned in various causes of this* "*nature; and by all the foreign books as well as our own, the* "*learning concerning Insurances, as well of Lives as of Ships, was* "*thoroughly settled and established: every party concerned in an* "*Insurance must communicate every circumstance he knows, in* "*order to ascertain the premium, which otherwise will be affected* "*and the policy void.* . . . I should not sit here were I to "suffer a man to receive a sum of money, and keep it without "making any satisfaction for the detention: What would the "merchants of Lombard Street think of such proceedings?"

It will be an instructive, as well as a necessary part of our review of the History of Life Assurance, to take note of the class of cases here referred to by the Lord Keeper, in which he had been "concerned." To this end I quote the following account from David Scott's *Every Man his own Broker,* first published in 1761, where, in reviewing the Gambling Insurance transactions of that period, he says:

> "Another manner of spending the vacation formerly was in insuring the lives of such unfortunate gentlemen as might happen to stand accountable to their country for misconduct. I am not willing to disturb the ashes of the dead, or I could give an account of this cruel pastime, the like of which is not to be met with in any civilized nation. . . . A practice likewise prevailed of insuring the lives of well-known personages, as soon as a paragraph appeared in the newspapers announcing them to be dangerously ill. The insurance rose in proportion as intelligence could be procured from

the servants, or from any of the faculty attending, that the patient was in great danger. This inhuman sport affected the minds of men depressed by long sickness; for when such persons, casting an eye over a newspaper for amusement, saw their lives had been insured in the Alley at 90 per-cent, they despaired of all hopes, and thus their dissolution was hastened. But to the honour of the principal merchants and underwriters, they caused an advertisement some years since to be fixed up at Lloyd's Coffee-house declaring that they would not transact business with any Brokers who should be engaged in such infamous transactions."

That such practices prevailed, does not rest upon the sole testimony of any one writer; but this author had exceptional means of knowledge.

The formation of Assurance Associations had remained in abeyance for more than an entire generation—from 1720 onwards. It is true that various Annuity Institutions had in the interval been founded, but there were not many of these. It ought also to be stated that when the collapse of the South Sea scheme occurred, certain Annuity Societies of earlier date did not succumb. It was now shown by De Moivre and other writers how Life Assurance might be employed as a means of making Annuity dealings much more secure than they had previously been. (*See* 4th edition of De Moivre's *Treatise on Annuities* published 1752; also Simpson's *Select Exercises,* of the same year; James Hardy's *Complete System of Interest and Annuities founded upon New, Easy and Rational Principles,* 1753; and Dodson's *Mathematical Repository,* 1755: in which last, Life Assurance was made the subject of distinct calculation without being deemed necessarily conjoined with Annuity Transactions). The process of constructing Tables of *graduated* premiums for Life Assurance was now made plain.

New Epoch.—The year 1760 constitutes a new and important epoch in the history of Life Assurance, for reasons which will soon appear. This, then, is a fitting point whereat to take a brief review of the situation of the business. In doing this I must bring to bear not only the facts which have been recorded in the preceding summary, but also the result of a vast amount of information (here unrecorded) which such an historical examination to be thorough entails. Volume after volume of contemporaneous events is explored, and the results are, in a direct sense, *nil:* but the very absence of information conveys a significance. Finally then, we will say, almost in the language of Mr. Frederick Hendriks, after a like investigation conducted some thirty years since:

There is no doubt "that the insurance of a sum payable under a "contract designed to extend over the whole duration of life, and "either at a graduated or an equalized premium, was, at the time "to which we are now referring, a contract very rarely entered "upon either by the Commercial or other Classes of the community; and we may take it as established that no plan of Life "Assurance, as we now understand it, had been contemplated by "any Company or Society, or had been considered by any "Legislature in Europe prior to the year 1760" (*Vide Assurance Mag*. iv, pp. 308-9). From this point therefore a fresh start may be made.

The Equitable Society.—The year 1760 saw the commencement of steps taken in view of establishing a plan of Life Assurance upon the principle of exact scientific computation, the premium being a fixed annual charge, determined by the age of the insured, at the time of entering into the contract; and the sum insured being likewise a fixed and determinate sum, subject to no diminution on any ground whatever. The computation of Mortality Tables based upon the data of the London Bills of Mortality, by Smart, Simpson, Hodgson, Stonehouse, Brakenridge and Postlethwayte respectively (leaving entirely out of account Halley's labours), justified such a step; as the calculation of the *Northampton* Tables of Mortality by Dr. Price, a few years later fully confirmed it, and indeed showed that the rates of premium previously charged were largely in excess of the actual risk involved in the business. The moving spirit in the formation of the Society which proposed to undertake the business of Life Assurance upon this much improved system, was Mr. Edward Rowe Mores; and he was supported by many persons of good social position.

It is not necessary here to inquire how the application for a Charter for the establishment and conduct of such a Corporation was refused, on the advice of the Law Officers of the Crown, and how the promoters determined to confederate together and constitute a Society or partnership for carrying on the business; or to recount the difficulties encountered and the final success achieved. It was upon the foundation of the *Equitable* Society, as finally established in 1762, that the entire superstructure of modern Life Assurance has been raised. Neither is it necessary here to suppose that, if the *Equitable* Society had never got beyond the first stages of promotion, the state of knowledge which had been developed was such as to have soon broken down the former defective modes of practice, and placed the business on a more just, equitable and permanent basis; and so, perhaps, to have avoided some of the faults and difficulties which were incident to a mere partnership association in the then state of insurance practice; but we must take the facts as we find them.

What seems desirable, is to estimate the amount of Life Assurance business which may have been in force about the date of the foundation of the *Equitable* Society, in order to gain an accurate idea of its growth in subsequent periods. The materials for such a purpose are in some part directly available. The managers of the *Royal Exchange* Assurance Corporation opposed the grant of a Charter to the *Equitable* Society; and for the purpose of showing what small need there was of any other Life Assurance Association, stated, on affidavit, that from the time of its commencement (in 1721) it had received in Life premiums but £10,915. 2*s.* 2*d.* and had disbursed in losses [claims] on life policies £8,263. 17*s.* 8*d.*—showing only about 25 per-cent surplus for expenses and profits. Keeping in mind the rate of premium charged for the annual contracts entered into, it seems clear from these figures, that the Corporation had only issued policies covering about £5,000 per annum, on an average of the period.

Regarding the *London Assurance* Corporation, no such early details were furnished. There was, indeed, a tradition that its managers, having at an early period found Life Assurance contracts unsatisfactory, confined themselves chiefly to the other branches of their business. How long this resolve may have been adhered to is not known with precision. But some details are available. Up to the end of 1751—that is during a period of 30 years—the *London Assurance* had issued 1,472 life policies, being on an average 49 per annum, or less than one per week. In the last year of the term (1751) there had been issued 86 policies insuring £23,681. None of these contracts extended beyond one year's duration, and in no case was the rate of premium less than £5 per-cent;—being sometimes 5 guineas, and sometimes 6—the premium income being thus about £1,200 per annum. The sums insured on any one life varied from £16 (lowest) to £2,500 (highest). There were only 6 policies of £1,000 and upwards; the sums usually ranging from £100 to £300, and being obviously taken out to protect business transactions: being generally "on the life of another." There were no insurances on Female lives. The business was decidedly on the increase then; and probably by 1760 had reached some £40,000 or £50,000 of yearly contracts.

In the *Amicable* Society the business during the 39 years 1710-49 had resulted in 2,967 claims, 76 per annum, on which had been paid £277,104, "and upwards." In 1757 the accumulated fund of the Society had become reduced to £25,000—about half the amount it stood at in 1720—and I suspect the business had been steadily decreasing from that date until 1757. The founding of the *Equitable* Society tended to restore confidence; and by 1770 the accumulated fund had increased to £33,300. This fund at each date included the amount due on account of Annuities, which had been a branch of the business of the Society since 1713.

As to the amount of Life risks annually written by individual underwriters, it is impossible to discover any data for an estimate. The stamps used on Life policies were the same as those used for Marine or Fire Insurance policies, and no light can be obtained from this source.

The following estimate is ventured upon as being an approximation of the sums insured on lives in Great Britain in 1760:

In the *Amicable* Society	£250,000
„ *London Assurance* Corporation	£50,000
„ *Royal Exchange*	£20,000
By private Insurers	£30,000
Total sums insured on Lives	£350,000

I have assumed in the case of the *Amicable* Society that it had its full 2,000 nominees, and that the amount of each death-share would realize £125.

It will be noted that this estimate relates to a period only one century and a quarter ago. It is necessary to keep this fact in view, in relation to the rapidity of its progress since. It may with equal advantage be noted that Life Assurance has certainly been in practice here for nearly two centuries; probably longer.

We must again revert to the *Equitable* Society. It will be instructive to glance at a few of the reasons which the Law officers of the Crown advanced against the grant of a Charter to the new enterprise; and which reasons had been formulated after hearing all the objections which could be urged by those who supposed they had an interest in opposing the project:

"Upon this petition your Lordships will observe:

"1. The petitioners propose to insure upon cheaper terms, and for a longer time than is practised at present in any offices, to which end they have specified the rates at which the assurance is to be done.

"2. They propose to raise a Capital by investing the premiums, together with a small additional sum of 40*s.* to be deposited by every person insured, to answer all losses, and by way of further security to oblige every person insured to become a member of the Corporation, and to declare or covenant that he will bear his proportion upon any call, if the premiums and deposits should prove deficient. . . .

"We are humbly of opinion to advise his Majesty not to comply with the prayer of this petition, for the following reasons:

"1st. Because it appears to us altogether uncertain whether this project will or can succeed in the manner in which it is proposed. . .

"2nd. The success of this scheme must depend upon the truth of certain calculations taken upon Tables of Life and Death whereby the chance of mortality is attempted to be reduced to a certain standard: this is a mere speculation never yet tried in practice, and consequently subject, like all other experiments, to various chances in the execution.

"The tables upon which the calculations are built are the Bills of Mortality of London, and the Breslau Tables, and admitting them to be strictly accurate (of which there is strong reason to believe the contrary), they are compounded of diseased as well as healthy persons, of those who are embarked in dangerous as well as other employments, without pointing out the proportions they bear to each other; and yet, as the petitioners propose to insure only such even of the healthy as are not employed in dangerous occupations, the register of life and death ought to be confined, if possible, for the sake of exactness, to such persons only as are the objects of Insurance; whereas the calculations offered embrace the chance of life in general, the healthy as well as unhealthy parts thereof, which, together with the nature of such persons' occupations, are unknown numbers. . . .

"The Crown has very wisely been always cautious of incorporating traders, because such bodies will either grow too great, and by overwhelming individuals, become monopolies; or else by failing will involve thousands in the ruin attendant upon a corporate bankruptcy. . .

"3rd. The Parliament, in erecting the two great Companies already mentioned [*London Assurance* and *Royal Exchange* Assurance Companies], have sufficiently declared their opinion that such Charters ought not to be granted without some benefit accruing to the public, and were not sure when they passed the Act whether they were not erecting a nuisance; to prevent which a power was reserved to the Crown to abolish the Corporations at any time within the term of 31 years, if they should be found upon trial to be mischievous or inconvenient; and we cannot help observing that, except only in the case of the *Amicable* Society of Serjeant's Inn, and which is formed upon a very narrow bottom, the Crown has never of itself, so far as appeared to us, granted such a Charter as the present, in any case whatsoever; and as the two great Companies paid a very large sum to the public for the privilege of their Charter, we cannot advise the Crown to entrench upon their rights on the bare request of any set of men, without a clearer and more certain prospect of the public good."

For these reasons (given in greater detail) they advised against the Charter, and it was not granted. The closing words clearly indicate that a considerable payment was expected. It is a matter of history that the two Insurance Corporations of 1720 had promised to pay the enormous sum of £300,000 each for their Charters, and did actually pay £150,000 respectively.

The rates of premium charged by the *Equitable* Society under its original scheme, and as increased two years later, were as follows, at the ages stated:

Ages	Original Rates, 1762		Increased in 1764	
	One Year	Whole of Life	One Year	Whole of Life
8	£1 10 6	£2 4 10	£1 15 6	£2 6 4
14	1 11 9	2 7 7	1 16 9	2 9 1
20	1 15 6	2 15 4	2 0 6	2 16 10
30	2 4 6	3 12 3	2 9 6	3 13 9
40	3 2 0	4 12 2	3 7 0	4 13 8
50	4 8 3	5 18 4	4 13 8	5 19 10
60	6 4 10	8 5 2	6 9 10	8 6 8
67	7 18 1	11 18 8	8 3 1	12 0 2

These rates only applied to *healthy males* for all ages, as *girls*, *women* under 50, and males of hazardous occupations, were to be charged extra "adequate to the hazard." Joint Lives and Survivorship Assurances were granted "upon terms equally just and reasonable." There was an "Entrance-fee" of 15*s*. in respect of each £100 insured. In 1765 the rates of premium were increased in the cases under the age of 30; also on all Military persons, and persons not appearing before the board for "inspection." An extra premium was charged for gout. At a later date extras were charged for hernia; also for beer retailers.

The form of *proposal* to be filled up by persons desirous of entering the Society was as follows:

> "(1) Name and profession of the life to be assured........... (2) Place and date of birth........... (3) Present residence........... (4) Age........... (5) Sum........... (6) Term........... (7) By whom made........... (8) To give a reference to 2 persons of good repute (one if possible of the medical profession), to ascertain the present and general state of health of the life to be assured........... (9) If had the smallpox?........... (10) If had the cowpox?........... (11) If had the gout?........... (12) If ever ruptured?...........
>
> "NOTE.—Persons who do not appear before the court of directors, or who cannot refer to a person of the medical profession, are required to give a reference to 3 persons for an account of the present and general state of their health."

And in addition to this, the Deed of Settlement required a "Declaration" to be made that the preceding statements were true. There was as yet no formal medical examination of the persons proposed for insurance; although it was one step in advance that reference to a medical man was required.

The conditions of Assurance were not incorporated in the *Policy;* which document indeed was little more than a certificate that the person named therein was insured for a given sum, at a given rate of premium. The "General Regulations" of the Society were supposed to bind all the Members, and in a Mutual Society no doubt did so. Amongst these were the following.

"4. Every person desiring to make an assurance with the Society must sign a declaration, setting forth the age, state of health, profession, occupation, and other circumstances of the persons whose lives are proposed to be assured. This declaration is the basis of the contract between the Society and the person desiring to make such assurance; and if any artful, false, or fraudulent representation shall be used therein, all claim on account of any policy, *so fraudulently obtained, shall cease, determine, and be void.*

"5. Every person making assurance with the Society becomes a member of the Society, and enters into a covenant, that he will so continue during the term of his assurance; and that he will conform to, observe, and keep the bye-laws, rules, orders, and ordinances of the Society.

"6. No person can make assurance upon the life, unless he make it appear to the court of directors of the Society that he hath an interest in the life of such other person at least equal to the sum which he proposes to assure.

"19. If any premium remain unpaid 30 days after the time stipulated in the policy for payment thereof, such policy becomes void; but if the defaulter shall, within 3 calendar months after the time so stipulated (the person on whose life the assurance was made being then alive and in good health), pay the said premium, together with the added sum of 10*s.* for every £100 assured by such policy, then such policy is revived, and continues in force to all intents and purposes whatsoever.

"23. All claimants upon the Society must produce authentic certificates of the time and place of the death of the persons under whom they claim.

"24. The books of the Society are open to the inspection of any member of the Society for life or years at any Court or Meeting of the Society without fee or reward."

And there was an "arbitration" clause.

I suppose it is known to every one that the high rates of premium charged to its members, combined with fortunate investments in the National Funds (when prices were very much depressed), led to accumulation of a large surplus fund—and hence to large divisions of profits—then termed "Dividends", and more recently designated "Bonuses." The original Deed of the Society had provided for a periodical investigation into the sufficiency of the "Stock"—which we now term the "Life Assurance Fund"—to meet all the liabilities under policies. The first annual investigation of this character took place in 1776; and this constitutes an historical incident. *It was the first Valuation of the affairs of a Life Office* (always properly regarded in all offices as an eventful period) *ever made!*

The "Bonus System" of Life Assurance arose out of the circumstance that even when reasonably reliable data had been obtained for the conduct of the business, yet Life Assurance on a fixed basis was deemed an experiment; and there being no outside capital—beyond the premium contributions paid by the members—an overcharge was essential to safety. And, equitably, this overcharge was to be returned to the members to whom of right it belonged. It was for a long period dealt with much on the footing of a Lottery: some getting more, some getting less, than their fair share. But this feature contributed largely to the expansion of the business.

I cannot pretend to trace in detail the growth of the business of this Society; even although that growth does admittedly constitute a very important feature in the history of Life Assurance—particularly for the first half-century of the Society's existence. Perhaps all that is necessary for present purposes may be accomplished by means of the following Table, taken bodily from the *Insurance Cyclopædia*, with the addition of the figures at the period of the last bonus division in 1879:

Progress of the Equitable Society.

Year ending 31 Dec.	Total Policies in force	Total Insurances in force	Life Assurance Fund	Surplus distributed
1776	913	...	£60,000	£11,000
1786	2,100	£977,330	...	54,000
1792	4,640	2,976,476	500,000	99,000
1799	5,124	3,900,000	1,000,000	258,000
1809	7,320	8,024,000	4,330,000	975,224
1819	9,650	...	6,235,000	2,000,000
1829	8,867	12,417,630	10,411,540	3,408,552
1839	7,481	14,080,247	10,689,932	2,969,502
1849	6,044	8,305,495	8,858,047	2,113,372
1859	4,679	6,262,000	6,564,671	1,514,066
1869	3,785	4,911,496	4,609,736	1,155,339
1879	3,967	4,886,835	4,360,661	1,893,704

The total amount paid in claims and bonus additions up to the end of 1882 had been over £34,000,000.

The falling-off in the business of the Society after 1829 was an almost necessary consequence of the competition which had then arisen; and it is perhaps hardly necessary to state here that the soundness of the Society was in no way necessarily impaired by the reduction in the number of its members. This falling-off can have resulted only in part from the altered plan of allocating the profits to the oldest 5,000 policies, adopted in 1816.

In 1769 there was founded in the City of Philadelphia (in the Commonwealth of Pennsylvania, then an English Plantation) an Association for the "Relief of the Widows and Children of Clergymen." It was in the main an Annuity Association, and hence calls for no detailed mention here. It still exists.

It has been stated by various writers, upon the assumed authority of Mr. Francis Baily, that soon after the establishment of the *Equitable* Society a great number of Assurance Societies sprang up, &c. What Mr. Baily says (*Doctrine of Life Annuities and Assurances* 1813, p. 491) is "Soon after the establishment of "the *Equitable*. . . . a number of Societies sprung up (all about "the same time) which assumed the false and alarming title of "Institutions for the benefit of old age! The Institutions were, "for the most part, gross impositions on the public; proceeding "from ignorance or knavery, and encouraged by credulity and "folly." The institutions so referred to were in fact, without any exception, Annuity Societies. In recklessness of statement they much resembled the Life Assurance projects of 1699-1720. Yet the good which, indirectly, came out of them was considerable. They attracted the attention of two powerful writers—Price and Dale: the latter the more able, the former the more popular. Their works claim passing attention.

In 1769 Dr. Price published his work: *Observations on Reversionary Payments; on Schemes for providing Annuities for Widows and for Persons in Old Age; on the Method of Calculating the Values of Assurances on Lives, &c.* The earliest edition usually met with is that for 1771; it passed through many later editions, and became very famous. He examined the methods of Halley, De Moivre and Simpson, and gave various illustrations of problems in Life Assurance.

The Northampton Table.—The chief feature of his work was that it presented two new Tables of mortality, based upon the

Bills of mortality of the Town of *Northampton* and the City of *Norwich* respectively; to the results of which he found De Moivre's hypothesis very closely approximated. I believe there are grounds for stating that Dr. Price himself considered the *Norwich* Table the more accurate of the two; but it came to pass that the *Equitable* Society selected the *Northampton* Table for the purpose of deducing its new scale of premiums; and the prominence thus gained gave to the Table a fame which we now know it never to have merited; but which it maintained until far into the present century. A detailed record of the part which the *Northampton* Table played in the assurance and financial transactions of this Country, would furnish a history of some importance.

In 1772 there appeared the other works referred to: *Calculations deduced from First Principles, in the most familiar manner, by Plain Arithmetic: for the Use of Societies instituted for the Benefit of Old Age; intended as an Introduction to the Study of the Doctrine of Annuities.* "By a Member of one of the Societies." The author was William Dale, a man occupying a somewhat humble position, and whose exposure of the fallacies of the various Annuity Associations already referred to was productive of much good. A supplement to this work was afterwards published, bearing the initials "W. D." It is rarely met with.

It should be recorded here that in 1770 Isaac Steell, "Teacher of the Mathematics", published in Dublin: *Tables for Computing the Value of Leases,* &c.; Wherein was included "a section containing the methods of estimating the Value of Annuities upon Lives." This is one of the few works on Life Contingencies which have been issued in Ireland.

With the advent of the *Northampton* Table, and the coincident work of Dr. Price, it may be said, in a general sense, that the efforts towards Scientific exactitude in Life Assurance culminated. As I have already said we now know the inherent defects of that Table, and even the causes of them. But for a full generation it was the recognized standard of Life-measurement; and Life Assurance having thus attained a recognized status, I propose to adopt other land-marks by which to note its later progress in the United Kingdom. The subsequent divisions will mainly represent legislative epochs; but will embrace the leading incidents affecting Life Assurance generally, and the Life Assurance Associations specifically falling within each.

4.—LIFE ASSURANCE 1774 TO 1824.

At this period it becomes desirable to review the State of Legislation regarding Assurance Associations, in order to understand their legal Status—their privileges, obligations and powers generally. To this end we must retrace, and supplement to a brief extent, our previous chronological survey.

In 1610 James I issued a Proclamation against Grants and Monopolies generally. In 1622 the same king organized a Commission of enquiry concerning the decay of the Commerce of England, and one of the Subjects of such enquiry was: How far Joint Stock Companies are beneficial or otherwise? This was followed (1623) by the Statute against Monopolies (21 James I, c. 3), whereby persons were forbidden to form Speculative Associations, or to raise *transferable Stocks,* except in the case of Letters Patent for new manufacturers, and Charters granted to Corporations, Municipal or Trading.

On the occasion, in 1687, of two Fire Insurance Offices contending for a Charter before James II in Council, it was declared and recorded, that no such undertaking as the said persons were concerned in establishing ought to be carried on without Royal Warrant and authority.

The Bubble Act of 1720 (6 George I, c. 18) we have already in part reviewed. It contained the declaration: "And whereas the "sole right and prerogative of granting (Charters of Incorporation), "not being such as are repugnant to any laws or statutes of this "realm, doth belong *to your Majesty.*" One of the chief points aimed at by this Act was to prevent the raising of a Capital Stock by means of transferable shares: for by section 21 brokers were made liable to a penalty of £500 for buying or selling shares in such unauthorized undertakings.

But in this Act there was no restriction as to the operations of individual insurers—underwriters; and section 25 provided that the Act should not be construed to extend to prohibit or restrain the carrying on of any home or foreign trade in *partnership,* "in "such manner as hath been hitherto usually and may be legally "done according to the Law of this realm now in force, excepting "only as to the Insuring of Ships and Goods or Merchandize at "Sea, or going to Sea." Hence partnerships could carry on all Insurance business except marine; and as to individuals there was no limitations.

In consequence of this exemption of individuals it came to be determined by the Law Courts that a dozen persons might insure or guarantee each other; and if 12 then any other number; but, whatever the number, they could only be partners at common law, subject to all the rights and liabilities of partners, under which state of things each partner was individually liable for all the debts and engagements of the partnership, and might be personally sued therefor. This was exemplified in the case of *Vansandan* v. *Moore,* relating to the affairs of the *British Annuity* Society, as recently as 1826.

How difficult it became to obtain a Charter of Incorporation, we have already seen in the case of the *Equitable* Society in 1761. There cannot be much doubt that the events of 1720, with regard to the two Corporations created that year, had led the law officers of the Crown to regard the privilege of a Charter as a thing to be paid for; and to be paid for at a very exorbitant rate.

Far greater wisdom characterized the legislation of the sister-isle. By the *Irish* Statutes, 21 and 22 George III, Chapter 46, any number of persons might join in partnership by Deed for 14 years or less, with a joint Stock of not less than £10,000, or more than £50,000. Acting partners might be appointed who should be liable for engagements of the Company; the others, called the anonymous partners, not to be liable beyond their Shares, this being on the model of the French law of Joint-Stock Companies.

The law in England remained in the unsatisfactory state above indicated down to the year 1825, when a useful modification was obtained. I must now resume the chronological narrative.

In 1774 a very useful measure was enacted, namely, 14 George III, Chapter 48, which recited, "Whereas it hath been "found by experience that the making Insurances on Lives, or "other events, *wherein the assured shall have no interest,* hath "introduced a mischievous kind of gambling"; wherefore it was enacted that no insurance should be made on the lives of persons who had no interest in the events insured against; the names of the persons interested to be inserted in every policy issued; no greater sum should be recovered than the amount of the interest insured against. The Act was not to extend to Marine Insurance, where, "interest or no interest", policies were still indulged in—leading to many kinds of frauds. This Act, which is still in force, has been of frequent service in defeating frauds, while some difficulties in practice have arisen under it, to which reference will be made later on.

In 1777 the premiums in the *Equitable* Society were reduced 10 per-cent on the advice of Dr. Price. In 1772 Females were insured at the same rate as Males; while in 1770 the Entrance-fee had been reduced to 5*s*. in respect of each £100 assured.

In this same year there was projected or founded the *New Laudable* Life Office, concerning which we have met with no details.

Insurance Patent.—In 1778 John Knox presented to His Majesty George III a petition wherein he stated that with great assiduity, art and pains, and at considerable expense, he had invented a plan, different to any before that time discovered, for assurance on lives from ten to eighty years of age, upon a two-fold beneficial principle, making a certain provision as well for Subscribers or their representatives upon the death of the person on whose life assurance should be made within the term assured for, as also by means of a reserved capital for the Surviving Subscribers whose nominees should be living at the expiration of such term; and that the said scheme or institution was adapted to the different ranks, professions and circumstances of mankind, wherein Subscribers might insure on the lives of themselves or of others for their own benefit or that of their assignees, and would, if carried into execution, prove of great advantage to His Majesty's subjects, and likewise contribute to the increase of the public revenues by a very considerable consumption of Stamps. A Patent was accordingly granted for a term of 14 years (No. 1197), the only patent so far as the writer is aware ever granted to the promoters of an Assurance Association, although some later schemes have sought protection under the laws of Copyright.

The Scheme was simply one of Mutual Contribution, the members being ranged in 6 classes; and, instead of advancing the business of Life Assurance, was proposing to carry it back to a state of things existing at the commencement of the century, as already fully described. The only new feature was "a separate " class to include the Army and Navy without any distinction of " age." The office was at No. 76, near the Adelphi, in the Strand.

During the same year there was founded the *Universal Institution for Assurance of Lives*, "calculated on a new principle, " whereby the benefits to Subscribers are already defined and " established upon a double security." The prospectus contained three schemes of Assurance—one for the public at large; the next for persons serving in the Army; the third for those serving in the Royal Navy. Each scheme to consist of ten divisions, each division to contain three distinct grades of age, and each class to consist of 500 members, and to extend over a term of ten years. In the first scheme persons of both sexes between 10 and 80 years of age were admitted to insure either on their own lives or the lives of nominees. The first class of age ranged from 10 to 40 years; the second from 40 to 60; the third from 60 to 80. There was a graduated scale of payments for each division. The scheme in truth was a most complicated one; it was a sort of inverted Tontine, the benefits going to the nominees of those who died within the ten years; but the claims of those who died thus early were subjected to a deduction ranging from 29 per-cent downwards, according to the year of death; and these accumulated deductions were to be divided amongst the survivors at the end of the ten years, when each class was to be renewed. Each policy-holder was to pay *7s. 6d.* for expense of policy. The ultimate fate of the project is unrecorded.

In the 6th Edition "corrected" of Cunningham's *Law of Bills of Exchange and Insurance* (1778), section X, which treats of "Insurance on Lives", opens thus:

> "Men's lives with good reasons are and may be insured, to secure to a creditor the reimbursement of a sum advanced to his debtor, for purchasing a post or place, out of the income of which he may have a sufficiency besides his maintenance and expenses and interest and premium, to pay off yearly part of the Capital. However, the lender ought not to insure the life of the borrower without his consent. In some places Insurances are not permitted on the lives of persons at the head of the Government; but in *London* people take the liberty to make Insurances on any one's life without exception; and the Insurers seldom enquire much if there are good or bad reasons for such an Insurance, but only what the person's age is and whether he be of a good constitution. The common premium on a good life from 20 to 50 years of age is 5 per-cent, and from 50 to 60 years old 6 per-cent per annum: *But these premiums are higher than any computation founded on observations concerning the probability of human life will warrant.* People ought by all means to be prevented from getting insurances done with sinister views, especially that inhuman one of committing murder to gain the sum insured; an instance of which villainy happened a few years since in a London Apothecary who, having got his wife's life insured, soon after killed her . . . . . ."

The first edition of this work was published in 1759.

Mr. W. Morgan, long the Actuary of the *Equitable* Society, and nephew of Dr. Price already spoken of, published in 1821 a second edition of *Principles and Doctrines of Assurance, Annuities* *on Lives, &c.* (first edition 1779); and in the preface thereto, referring to certain of the contents of that first edition, said:

> "In the year 1779, when those observations were written, the business of Assurances on Lives was but little understood and but little practised. Excepting the Society in Sergeant's Inn (the *Amicable*), which assured lives at all ages under 45 at the same annual premium, and never exceeded £300 on the same life, and the *Royal Exchange* office, which made a few assurances for a single year at the general premium, I believe, of £5 per-cent, the *Equitable* Society had no competitors, and was the only Society which varied its premiums according to the age of the person assured. The assurances for the benefit of surviving families at this period were but few in comparison with those which were made on the lives of those improvident persons who, in the disposal of their property, seemed to have as little consideration for their families as for themselves; and as the price of an annuity on a life, however young, very rarely exceeded seven years' purchase, the Assurances were seldom made for a longer time: so a very small proportion was made on the whole continuance of life, or with any other view than to secure a purchaser from the risk of losing the price of his annuity . . . . . ."

I have taken some pains in the course of this history to consult and record the views of contemporary writers. Such records ought to possess great value. Mr. Morgan was contemporary with the events of which he here narrates; but his omissions of all reference to the *London Assurance,* which as I have shown was, a few years previously to the date to which he is referring, transacting, apparently, a larger Life business than the *Royal Exchange* (*see* p. 6), throws a doubt over the entire passage. Here was a writer recording incidents with which he must have been quite familiar in his youth, and which in the main, I am disposed to think, he has correctly rendered; but there is the defect indicated. In other cases I have found his statements either general, or so partial, that I shall not again introduce him as an authority in this narrative, if other testimony can be made available.

National Assurance project.—Mr. Francis ("*Annals, &c.*") refers briefly to a proposal set on foot this year (1779) for a scheme of *National Assurance*—"Universal" he calls it—by means of a tax to be levied by the Government. By this all want was to be abolished, and various Utopian benefits were to be received. He adds "as, "however, the scheme was never carried out, it is only worthy of "notice, as indicative of a growing spirit of enquiry. Such "projects still continue even in the present day"—(*see* 1807).

Health Warranty.—In 1780 there came before our Courts the case of *Willis* v. *Poole,* the result of which impressed upon the Life Offices the importance of seeking more precise information regarding the existence of constitutional diseases in persons proposed for assurance. The Insurance had been granted on the life of Sir Simeon Stuart, Bart., against the life of Eliza Edgly Ewer, from the 1st April 1779 to 1st April 1780. The Policy contained a warranty that Sir Simeon was about 57 years of age, *and in good health,* on the 11th May 1779; and that Mrs. Ewer was about 78 years of age. Sir Simeon died within the year. On action brought, the defendant pleaded the warranty of health. It appeared in evidence that although Sir Simeon was troubled with spasms and cramp from violent fits of gout, he was in as good health when the Policy was underwritten as he had been for a long time before. It was also proved by the Broker who effected the Policy that the underwriters were told that Sir Simeon was subject to gout. Dr. Heberden and other gentlemen of the faculty were examined, and proved that spasms and convulsions were symptoms incident to gout. Lord Mansfield, before whom the case was heard, said in his judgment: "The imperfection of language is such that we " have not words for every different idea; and the real intention of " parties must be found out by the subject-matter. By the present " policy the life is warranted to some of the underwriters, 'in health', " to others in 'good health'; and yet there was no difference " intended in point of fact. *Such a warranty can never mean that " a man has not the seeds of disorder.* We are all born with the " seeds of mortality in us. A man subject to the gout is a life " capable of being insured, if he has no sickness at the time to " make it an unequal contract." Verdict was given for the plaintiff. The policy it will be observed was issued by underwriters.

Life Assurance in Glasgow.—Hitherto, with the single exception of a provincial Assurance office (The *Berkshire and Counties*), founded 1709, all Life Assurance business here recorded had been transacted in London. But it may be well assumed that wherever the business of Marine underwriting was carried on in the United Kingdom, the business of underwriting lives also prevailed. We have now to record an instance of a policy effected in *Glasgow* in 1785. Here is the contract, which tells its own tale, and tells it well and briefly:

"For and in consideration of Three pounds three shillings sterling for one hundred pounds sterling, and so in proportion for any greater or lesser sums by us severally hereunto subscribed and to us respectively paid by Robert Walkinshaw, Sheriff Clerk of Renfrewshire, the receipt and payment whereof We do ourselves severally and respectively, every one for himself, his heirs, executors, and administrators, and not jointly nor one for the other of us, by these presents undertake and promise to pay or cause to be paid unto y^{e} Executors, Administrators or Assigns of the said Robert Walkinshaw, The full sum or sums of money hereunto by Us subscribed as aforesaid. In case the said Robert Walkinshaw shall die or cease this Life by any ways or means whatsoever, Suicide and the hands of Justice always excepted, between the seventh day of September current and the seventh day of September next, both days included.

"In witness whereof we the Assurers have subscribed our names and sums at Glasgow this eighth day of September 1785 years, before these Witnesses, John Boyle, Merchant in Glasgow, and Charles Finlay, his Clerk, by whom these presents are written.

"The above-named Robert Walkinshaw, Warranted in good health the seventh current, and not to depart the Kingdom of G$^{t.}$ Britain during the term of this policy.

"MICHAEL ERSKINE,	One hundred pounds.
"JOHN ASTON, Jun.,	One hundred pounds.
"JOHN GORDON,	Two hundred pounds.
"ARCH$^{B.}$ GOVAN,	One hundred pounds.
"GEORGE BOGLE,	One hundred pounds.

"JOHN BOYLE, Witness.
"CHARLES FINLAY, Witness."

This most clear and business-like document, written on a sheet of foolscap paper, bearing impressed stamps 1*s.* 6*d.* and 2*s.* 6*d.*, displays great familiarity with the essential conditions of a Life Assurance contract; and the fact of its being prepared by a merchant indicates familiarity with the business. The insured was 28 at the time of insuring; and the 3-guinea rate was a modification of former practice in Life underwriting—but was also probably due to its being entirely a "home" risk.

In 1787 appeared the first edition of a work which speedily became famous: *A System of the Law of Marine Insurances, with three Chapters on Bottomry; on Insurance on Lives; and on Insurances against Fire.* By James Allan Park, Barrister-at-Law. The single Chapter on Life Insurance dwelt most upon its advantages in connexion with loans on personal security.

Proprietary Insurance Offices.—A revival of the feeling in favour of Joint-Stock (afterwards termed "proprietary") Life Offices, which had begun to manifest itself early in the century, was now apparent. The first practical outcome from this occurred in 1789, when a Bill was introduced into Parliament to incorporate one hundred gentlemen, by the name of "*The Westminster*

Society for granting and purchasing Annuities and Insurances upon Lives and Survivorships", with a Joint-Stock of £300,000, to be divided in Shares of £3,000 each, on which the sum of £1,000 was to be paid within 21 days after the first General Meeting of Proprietors, and the remainder at such times as it should be called for. It was further provided by the Bill that £100,000 should be immediately invested in the 3 per-cent Consolidated Bank Annuities, and the further sum of £10,000 in like manner annually, until the full sum of £200,000 of 3 per-cent Consols should be purchased; that this fund should not be alienated without the authority of Parliament; and that the Society should not divide more than its net profits.

The *Amicable* Society opposed this Bill. The principal argument was that the Capital, professing to be £300,000, would in reality be no more than £100,000, and might therefore prove a fallacy if not properly guarded; since the Subscribers, from insolvency and various other incidental chances in life, though then perfectly solvent, might not be able to advance the second and third £1,000 when called upon.

The Bill passed the Commons, but was thrown out in the Lords, upon the motion of the Chancellor (Lord Thurlow), who among other objections stated that in his opinion the Capital proposed to be raised was not a sufficient security for the public.

The *Westminster* Society thus failed to obtain a Charter, but was established three years later (1792) on the foundation of a Deed of Settlement. Its Capital was £150,000, in shares of £500; and £50,000 was paid up and invested in the public funds in the names of five Trustees chosen from among the proprietors. The prospectus issued by the founders said:

> "Among the many Institutions formed for the Benefit of mankind, those for provision from old age and for the Widow and the Fatherless have been thought most worthy of encouragement; but from the formation or conduct of several, they have frequently failed to produce the beneficial consequences intended and have occasioned indigence and disappointment.
>
> "This Society, apprehending that the offices existing for these purposes are not sufficiently extensive for the accommodation of the public, and considering the good arising from a well-digested plan, suited to every condition of persons, have established a Capital sufficient to answer every demand, and offer in the annexed Table such terms as will, they trust, be considered advantageous to the public.
>
> "The benefits and conveniences of Insurance on Lives, if better understood, would make the practice more general. A person may,

for a small annual premium, secure to his Widow, his Children, or Dependents, a sum to keep them from the distresses and poverty which his death might otherwise occasion.

"A Creditor by insuring the life of his debtor may secure a debt.

"A man in Trade by an assurance on his own life may strengthen a security so as to be enabled to increase his Capital.

"One whose income depends upon the life of another may have such income secured during his own life, after the other's Decease.

"And those who purchase Employments or Annuities depending on the Lives of themselves, or others, may secure the purchase-money paid for the same."

The remainder of the prospectus is devoted to Annuity business, wherein several useful features were introduced. Finally it stated:— "And they purpose carrying their plans into execution on so fair "and equitable a scale as they are confident cannot fail to secure "general approbation."

The following is an abstract of the Table of annual premiums for Life Assurance:

Age	£100 for 1 Year	£100 for 7 Years	£100 for Whole Term of Life
8 to 14	£1 0 1	£1 4 1	£2 2 4
20	1 10 8	1 13 1	2 9 1
25	1 14 5	1 16 3	2 14 3
30	1 17 5	1 19 5	3 0 1
40	2 5 11	2 9 9	3 15 8
50	3 2 0	3 8 5	5 2 2
60	4 8 0	4 18 0	7 3 5
67	5 18 10	7 0 10	9 13 9

Then there were tables of premiums for the first of two lives "within the limits of Europe, but not upon the Seas."

The proposal form was identical with that of the *Equitable* Society. Reference was to be given . . . . "to one or more "persons of repute (one of whom must be a medical man) to "ascertain the identity and present state of health."

"Policies are void if the person whose life is insured shall depart beyond the limits of Europe, or shall die upon the Seas, or enter or engage in any Military or Naval Service, when the policy is on the Insured's own Life, the same is void if he comes to his death by Suicide or Duelling, which is not the case when the Insurance is on the Life of another.

"Persons to be insured not appearing at the office, or to one of the Society's Agents, must pay a fine of 10*s.* per-cent for an Insurance for one year; 15*s.* per-cent for seven years; and 20*s.* per-cent for the whole continuance of life; in the first payment only.

"Persons in the Army or Navy must pay an additional £1 per-cent on the sums insured; and those who have not had the small-pox must pay an additional premium of £10 per-cent, to be computed on the premium.

"Admission 2*s*. 6*d*. per-cent on Insurance on lives, and 5*s*. per-cent on Consideration-money paid for Annuities.

"Attendance daily given from ten till three o'clock."

A perusal of these Regulations shows that all the essential conditions of Life Assurance were then well understood; and that the projectors knew perfectly well the nature of the business they were undertaking. The rate of premiums were in every sense most moderate for this period; and must have greatly interfered with the business of the Life underwriters.

Agency System.—The really novel feature in the Scheme is the appointment of Agents. This was a means of carrying the facilities of Life Assurance into the Provincial Towns of the Kingdom, which had not before existed. It became a feature of most of the subsequent proprietary Life offices, and of some of the mutual ones.

A last special feature remains to be noted: "Every person "must sign an agreement which will be the basis of the Contract "between him and the Society, prior to the granting any Insur- "ance or Annuity." The meaning of which was that the assured agreed to accept the security of the Joint-Stock, and not resort to the individual Stockholders—a point of much consequence, upon which some observations will be offered later (*see* 1825).

The office transacted a moderate business only; and when (in 1799) the promoters of the *Globe* Insurance Company applied for a Charter, it opposed the same, on the ground that, if granted, that Company would obtain "various privileges and advantages over your petitioners."

This being the first of a series of Proprietary Companies organized for carrying on the business of Life Assurance—unconnected with any other branch of Insurance—for profit, has seemed to demand especial mention.

About 1795 there was floating in the parliamentary atmosphere a proposal to tax the premiums of Life Offices. In Hutton's *Mathematical Dictionary* (1795 Edition, volume ii. p. 727) there is the following reference to it: "The present rates of Assurances on "lives are in the Table below. *And though a duty on these "Assurances should take place on the plan lately proposed by the "House of Commons* there is no great probability that these prices "will be increased." I have nowhere met with the details of this proposal.

In 1797 the *Pelican* Life Office was founded as a proprietary company. It is intimated by Eden, in his valuable little work on *Insurance Charters* (1806) that the promoters of this office had been engaged for nearly ten years previously in investigating the then rates of mortality, and in collecting other Statistics and information relative to the business of Life Assurance. It is to be hoped that the information so acquired may some day be made available for the common good. It was probably upon the basis of such information that office adopted a Scale of premiums much lower than had previously prevailed; of which the following are examples:

Age	One Year	Seven Years	Whole of Life
14	£0 17 6	£1 0 6	£1 15 9
20	1 6 0	1 9 3	2 1 6
25	1 10 2	1 13 4	2 6 4
30	1 12 10	1 16 0	2 11 6
40	2 2 3	2 5 9	3 6 3
50	2 15 6	3 1 4	4 10 0
60	4 1 0	4 10 4	6 9 0
70	5 9 6	6 12 8	8 19 0

In 1798 some slight reductions were made on the later ages in life. In opposing the grant of a Charter to the *Globe* Insurance Company a few years later, the Manager of this office asserted that the public was *supplied with insurance to any amount upon the lowest terms on which it could be afforded.* This office originally accepted £3,000 on one Life; in 1799 this was increased to £5,000.

It was announced "this office does not buy or sell Life Annuities." A special feature of the business of this office was Endowments for Children.

Income-Tax.—I have mentioned various attempts, or rumoured attempts, to impose fiscal burdens upon Insurance Offices, and am glad therefore to be able to record a movement in another direction—an exemption in their favour, made by that great Statesman, William Pitt, in proposing his Income-Tax scheme in 1798. He spoke of those who had "recourse to that easy, certain and "advantageous mode of providing for their families by assuring "their lives"—and proposed to exempt from the tax that portion of their income so applied. This step was highly beneficial to Life Assurance at an important juncture.

In the Income-Tax Act of 1852 (16 & 17 Vict. c. 34) a like exemption was made, but such deduction was not to extend to premiums paid beyond one-sixth of the amount of the income of the person claiming such exemption.

A return of the amount of Income-Tax paid by the various Insurance Offices upon their profits, then and now, would be of much interest. Mr. Morgan said in his Address of 1809, that during the preceding four years the *Equitable* Society had paid upwards of £35,000 on account of such tax.

(To be continued.)

The South Australian Life Assurance Companies' Act, 1882.

WE now give in full the text of the South Australian Life Assurance Companies' Act, 1882, to which important statute Mr. R. Teece made detailed reference in his paper on "State Supervision in Insurance" (*J.I.A.* xxv, 350). For the convenience of our readers, we may briefly mention the provisions which differ most materially from those of the Life Assurance Companies' Act of 1870.

Sections 4 *and* 6:

A deposit of £5,000, to be increased each year by one-fourth of the excess of the receipts over the disbursements until the total reaches £20,000, to be made with the Public Trustee by every company transacting life assurance business in South Australia.

Sections 10 *and* 11:

Securities forming the above deposit to be primarily charged with the payment of all the liabilities of the company in South Australia.

Section 18:

Every foreign company—that is, every company having its head office outside the Colony of South Australia—to prepare a statement of all its policies in force at the end of each year, in addition to the usual accounts.

Section 25:

Every foreign company to appoint an agent in the province, upon whom all lawful processes may be served.

Section 33:

Policies protected against the claims of creditors to the extent of £200 after two years from their inception, £500 after five years, and £1,000 after seven years (*see J.I.A.* xxv, 363-4).

Section 47:

Every company to declare the surrender-value at which it becomes bound to accept its policies, and no policy to lapse for non-payment of premium so long as there is a margin left in the surrender-value.

Section 62:

Probate or administration not absolutely indispensable in the case of policies for less than £200.

In the Schedules appended to the Act there are several noteworthy stipulations.

First Schedule (*Revenue Account*):

The new and renewal premiums have to be stated separately, also the commission as chargeable to new and renewal premiums respectively.

Second Schedule (*Balance-Sheet*):

The assets in South Australia have to be given in detail, a separate statement containing similar particulars of the remainder of the assets.

Third Schedule (*Revenue Accounts applicable to Companies transacting Fire and Life business*):

In the Life Assurance Account the new and renewal premiums are to be distinguished.

Fourth Schedule (*Balance-Sheet, complementary to Third Schedule*):

South Australian and remaining assets are to be separately specified, as in the Second Schedule.

Fifth Schedule:

Particulars of the new policies issued by the company and the policies discontinued in the year, the total policies existing at the date of the return, and a statement showing the entire number of policies issued and discontinued since its establishment, are required to be furnished.

There is nothing analogous to this Schedule in or attached to the Life Assurance Companies' Act, 1870.

Sixth Schedule (*Statement respecting the Valuation*):

Very full particulars of the principles adopted in valuing the policy liabilities are required. Net premiums have to be stated in the Summary of Valuation, instead of it being left to the option of the actuary whether they should be computed or not, as is the case under the Life Assurance Companies' Act, 1870.

Eighth Schedule (*Insolvent Companies*):

Annuities to be valued according to the tables used at the time of granting the same; or, failing such tables, by the Government Annuities Experience Table, with 4 per-cent interest.

Policies to be valued by the Institute of Actuaries' Life Tables, with interest at 4 per-cent, net premiums only to be included as an asset.

45 & 46 Vict. No. 277. 1882.

An Act to amend the Law relating to Life Assurance Companies. [Assented to, 17 November 1882.]

Preamble.

WHEREAS it is expedient to amend the laws relating to Life Assurance Companies, with a view to encouraging persons to insure and to protecting persons assured—Be it therefore Enacted by the Governor of the Province of South Australia, by and with the advice and consent of the Legislative Council and House of Assembly of the said province, in Parliament assembled, as follows, that is to say:

Division.

1. This Act is divided into three parts, as follows:

Part I.—Preliminary.
Part II.—Provisions for Security of Assured.
Part III.—Application of Companies' Act, Procedure, and Miscellaneous.

Part I.

PART I.

Preliminary.

Short title.

2. This Act may be cited as "Life Assurance Companies' Act, 1882."

Interpretation.

3. In this Act the following terms have the following meanings, unless the context requires a different construction—

"Company" means any persons, corporate or unincorporate, who grant assurances, endowments, or annuities upon human life within South Australia:

"Chairman" means the person for the time being presiding over the board of directors, committee of management, or other managing body in South Australia of the company:

"Policy" means any contract for assurance, endowment, or annuity on human life:

"Life assurance business" means the granting of policies:

"Financial year" means each period of twelve months at the end of which the balance of the accounts of the company is struck, or if no such balance is struck, then each period of twelve months ending with the thirty-first day of December:

"Court" means the Supreme Court of South Australia:

"Registrar" means the Registrar of Joint-Stock Companies under "The Companies' Act, 1864":

"Local Company" means a company having its head office in South Australia:

"Foreign Company" means a company not having its head office in South Australia.

PART II.

PROVISIONS FOR SECURITY OF ASSURED.

Companies to deposit securities with Treasurer.

4. Every company which shall carry on the business of life assurance within the province shall deposit with the Public Trustee, securities to the value of Five Thousand Pounds, being mortgages of freehold real estate in South Australia, on which the money advanced does not exceed two-thirds of the value of the estate mortgaged, or title deeds or certificates of real estate, or bonds, debentures, or other securities issued by the Government, or by any Municipal Corporation in the province, duly authorised in that behalf: Provided always that any local company shall not be required to deposit more than fifty per centum on the amounts of the premiums actually received until the deposit shall amount to Five Thousand Pounds.

Time for making deposits.

5. As regards any company which now carries on the business of life assurance in the province, the deposit may be made at any time not later than one year after this Act shall come into operation; and as regards any company formed after this Act, the deposit may be made at any time within twelve months of its incorporation or registration; and no company shall be deemed to carry on the business of life assurance by reason only of receiving premiums in respect of policies issued before this Act shall come into operation.

Further deposits to be made out of receipts.

6. Every company which shall carry on the business of life assurance within the province shall send in annually to the Public Trustee, on or before the first day of June in each year, returns, verified by the certificate of the agent or principal officer of the Company in South Australia, of the amounts received and paid by the company during the year ending the thirty-first day of December then last, or on such other date as the financial year of each company shall determine, on account of policies issued in South Australia by the company, whether before or after the commencement of this Act, and shall, until the total amount deposited by the company under section 4 and this section shall amount to the sum of Twenty Thousand Pounds, deposit with the Public Trustee securities similar to those mentioned in section 4 to the value of twenty-five per centum of the excess of the receipts over the disbursements appearing from such returns.

Provision in case of loss of securities.

7. If any securities deposited under this Act are, whilst so deposited, lost, stolen, destroyed, or damaged, the injury occasioned to all persons interested shall be made good out of moneys to be appropriated for the purpose by Parliament.

Further deposits may be made.

8. Any company may from time to time deposit with the Public Trustee any securities of any kind, and to any amount, besides and beyond the securities hereby required to be deposited.

Income and withdrawal of deposits.

9. The company depositing any securities under this Act shall be entitled to receive the income therefrom, and securities deposited may be withdrawn on timely notice, and, where the deposit is compulsory, on the substitution of similar securities of equal value, and the decision of the Public Trustee shall be conclusive in all matters relating to the value of securities under this Act.

Securities deposited to be charged with liabilities in South Australia.

10. All securities deposited with the Public Trustee under this Act shall be primarily charged with the payment and satisfaction of all the liabilities of the company in South Australia, whether arising in respect of policies issued before or after the commencement of this Act or otherwise howsoever, and no part of such securities shall be applied in payment of any liabilities other than those so charged as aforesaid until the whole of such last-mentioned liabilities shall be paid in full.

Discharge of South Australian liabilities of foreign companies.

11. Every foreign company shall keep a separate account of all the life assurance business transacted in South Australia, and of the entire assets of the company in South Australia; and in the event of the company becoming bankrupt, or being ordered to be wound up, the entire assets of the company in South Australia shall be applied, so far as the same will extend, in or towards satisfaction of the liabilities of the company in South Australia, and no part of such assets shall be applied in payment of any liabilities of the company incurred elsewhere than in South Australia until the whole of the liabilities incurred in South Australia shall have been paid in full.

Mode of distribution of assets of insolvent foreign company.

12. If any foreign company is adjudged bankrupt, or ordered to be wound up, elsewhere than in South Australia, such company, so far only as regards its assets and liabilities in South Australia, may, upon application of any policyholder or shareholder, be ordered to be wound up in South Australia in like manner as if such company were registered under "The Companies' Act, 1864", and proof of such company having become adjudged bankrupt, or ordered to be wound up, shall be conclusive evidence that it is unable to pay its debts.

Penalty for infringement of Act.

13. Any director, agent, officer, or servant of any company wilfully committing or assisting in the commission of any breach of section 11 shall be deemed guilty of a breach of trust, and be held liable to replace the amount applied contrary to the said section, and shall also be deemed guilty of a misdemeanour punishable on conviction, at the discretion of the Court, by imprisonment for any term not exceeding three years or by a fine not exceeding Five Hundred Pounds.

Separation of life assurance and other business.

14. Every company transacting other business besides life assurance business shall keep a separate account of all receipts after the passing of this Act in respect of the life assurance business of the company, and the said receipts

shall be carried to and form a separate fund, to be called the "Life Assurance Fund" of the company, and such fund, however invested, shall be as absolutely the security of the life assurance policyholders as though it belonged to a company carrying on no other than life assurance business, and shall not be liable for any contracts of the company for which it would not have been liable had the business of the company been only life assurance business.

Application of preceding section to existing companies.

15. In respect to all existing companies the exemption of the life assurance fund from liability for other obligations than to its life assurance policyholders shall have reference only to the contracts entered into after the passing of this Act, unless by the constitution of the company such exemption already exists; but this and the preceding section shall not apply to any contracts made by any existing company by the terms of whose deed of settlement or articles of association the whole of the profits of all the business are paid exclusively to the life and endowment policy or annuity holders, and on the face of which contracts the liability of the insurers distinctly appears: Provided always, that this Act shall not diminish the liability of the life assurance fund for any contracts of the company entered into before the passing of this Act.

Accounts to be rendered by life assurance companies.

16. Every company transacting life assurance business only shall, at the expiration of each financial year or half-year of such company, prepare a statement of its revenue account for such year or half-year, and of its balance-sheet at the close of such year or half-year, in the forms respectively contained in the First and Second Schedules to this Act.

Accounts to be rendered by companies carrying on life assurance and other business.

17. Every company which, concurrently with the transaction of life assurance business, transacts any other kind of assurance or other business, shall, at the expiration of each financial year of such company, prepare a statement of its revenue account for such year, and of its balance-sheet at the close of such year, in the forms respectively contained in the Third and Fourth Schedules to this Act.

Accounts to be rendered by foreign companies.

18. Every foreign company shall, at the expiration of each financial year of such company, prepare, in addition to all other statements required by this Act, a statement of all its policies in force at the close of such year, in the form contained in the Fifth Schedule to this Act.

Actuarial report and abstract.

19. Every company shall, once in five years, or at such shorter intervals as may be prescribed by the instrument constituting the company, or by its articles of association, regulations, or by-laws, cause an investigation to be made into its financial condition by an actuary approved by the Public Trustee, and shall cause an abstract of the report of such actuary to be made in the form to be prescribed in the Sixth Schedule to this Act.

PART II.

Statement of life annuity business.

20. Every company shall, on or before the thirty-first day of December, one thousand eight hundred and eighty-three, and thereafter within nine months after the date of each such investigation as aforesaid into its financial condition, prepare a statement of its life assurance and annuity business, in the form contained in the Seventh Schedule to this Act, each of such statements to be made up as at the date of the last investigation, whether such investigation be made previously or subsequently to the passing of this Act: Provided as follows—

I. If the next financial investigation, after the passing of this Act, of any company fall during the year one thousand eight hundred and eighty-four, the said statement of such company shall be prepared within nine months after the date of such investigation instead of on or before the thirty-first day of December, one thousand eight hundred and eighty-three:

II. If such investigation be made annually by any company, such company may prepare such statement at any time, so that it be made at least once in every three years.

The expression "date of each such investigation," in this section, shall mean the date to which the accounts of each company are made up for the purposes of each such investigation.

Forms authorised by Imperial Statute, "Life Assurance Companies' Act, 1870," may be used in certain cases.

21. Any company whose head office or principal place of business is in the United Kingdom may, in lieu of the statement and abstract mentioned in the preceding two sections, deposit at the office of the Registrar a copy, certified in accordance with the 17th section of the Act of the Imperial Parliament known as the Life Assurance Companies' Act, 1870, of the last preceding statement or abstract deposited with the Board of Trade in conformity with the provisions of the 10th section of the said Act.

Forms may be altered.

22. The Governor may alter the forms contained in the Schedules to this Act, for the purpose of adapting them to the circumstances of any company, or of better carrying into effect the objects of this Act.

Statements, &c., to be signed and printed and deposited with Registrar.

23. Every statement or abstract hereinbefore required to be made shall be signed by the chairman and two of the directors or committee of management, or by the agent of the company in South Australia, and by the principal officer or agent managing the life assurance business of the company in South Australia, and if the company has a managing director in South Australia, by such managing director, and shall be printed; and the original so signed as aforesaid, together with three printed copies thereof, shall be deposited at the office of the Registrar within nine months of the dates respectively hereinbefore prescribed as the dates at which the same are to be prepared; and every annual statement so deposited after the first investigation, after the passing of this Act, shall be accompanied by a printed copy of the abstract required to be made as aforesaid.

Copies to be furnished.

24. A printed copy of the last deposited statement, abstract, or other document, by this Act required to be printed, shall be forwarded by the company, by post or otherwise, to every shareholder, member, and policyholder of the company in South Australia.

Agent to be appointed for foreign companies.

25. Every foreign company shall, within six calendar months after the coming into operation of this section, or before doing business in this province, in writing, appoint a person resident therein as general agent, upon whom all lawful processes against the company may be served with like effect as if the company existed in this province; and the said writing or power of attorney shall stipulate and agree, on the part of the company making the same, that any lawful process against the said company which is served on the said general agent shall be of the same legal force and validity as if served on the said company.

Copy of writing to be filed in office of Registrar.

26. A copy of the writing, duly certified and authenticated, shall be filed in the office of the Registrar, and copies certified by him shall be sufficient evidence.

Agency to continue as long as any liability outstanding in the province.

27. The said agency shall continue while any liability remains outstanding against the company in this province, and the power shall not be revoked until the same power is given to another and a like copy filed as aforesaid.

Service of process, &c., on agent sufficient.

28. Service of any process, notice, or otherwise upon the said agent shall be deemed sufficient service upon the principal.

Penalty for non-compliance.

29. No person shall act either as general or other agent of a foreign company until he has complied with all the requirements of this Act; and every person so acting without such compliance, or who knowingly procures payment, or any obligation for the payment, of any premium for insurance or endowment, or for sale of an annuity by fraudulent representations, shall be liable to a penalty not exceeding Two Hundred and Fifty Pounds for each offence.

When contracts valid.

30. Every contract for life assurance made by any foreign company without complying with the provisions of this Act contained in the sections numbered 25 to 29, both inclusive, shall be valid and binding on the company; but the agent making the contract shall be liable to the penalty provided in section 31 of this Act; and any such company which neglects to appoint and keep appointed a general agent agreeably to the provisions of this Act shall not recover any premium or other payment on any contract of life assurance with a person resident in South Australia.

PART II.

Companies may be prohibited from transacting business in certain cases.

31. Every company which makes default in complying with the provisions of this Act shall not only be liable to the penalties set forth in the 44th section, but may also, if it be made to appear to the Governor that such default has continued for a period of three months, be prohibited by the Governor from transacting business within the province, either absolutely or for a time, as the Governor may think fit.

Prohibition to be published in *Government Gazette.*

32. Such absolute or temporary prohibition shall be published in the *Government Gazette*; and if any such company, or any person as agent for such company or otherwise for or on behalf of any such company, shall, after such absolute prohibition, or during any such temporary prohibition, receive any applications for any life assurance, or accept any premium for any life assurance, or otherwise carry on the business of life assurance within South Australia, such company and person shall respectively be liable to a penalty of Two Hundred and Fifty Pounds.

Interest of assured not to pass to creditors under certain circumstances.

33. The property and interest of every person assured, or of his personal representatives, in respect of any policy on the life of such person made *bonâ fide* by such person, or in the moneys payable under or in respect of such policy (including every sum payable by way of bonus or profit), shall be exempt from liability to any law now or hereafter in force relating to bankruptcy or insolvency, or to be seized or levied upon by the process of any court whatever, nor shall such property or interest be affected by any assignment for the benefit of creditors made under the insolvency laws in force for the time being: Provided that no such policy nor contributions shall be so protected until the policy shall have endured for at least two years, but that after an endurance of two years such protection shall be afforded to the extent of Two Hundred Pounds of assurance and contributions, and after an endurance of five years to the extent of Five Hundred Pounds, and after an endurance of seven years to the extent of One Thousand Pounds.

PART III.

PART III.

APPLICATION OF COMPANIES' ACT. PROCEDURE AND MISCELLANEOUS.

Application of Companies' Act to life assurance companies.

34. The provisions of "The Companies' Act, 1864," and of the Acts amending the same, except as hereby expressly modified, shall apply to life assurance companies, but no life assurance company shall be required to hold more than one general meeting during the year, or to make or publish any statement in accordance with the 42nd section of "The Companies' Act, 1864."

List of shareholders.

35. Every proprietary company shall provide a book to be called "The Shareholders' Address Book", in which the company shall cause to be entered from time to time, in alphabetical order, the corporate names and places of business of the several shareholders of the company being corporations, and the surnames of the several other shareholders, with their respective christian names, places of abode, and descriptions, so far as the same shall be known to the company; and every policyholder or shareholder, or if such shareholder or policyholder be a corporation, the clerk or agent of such corporations, may at all convenient times peruse such book *gratis*, and the company shall furnish on application, to every shareholder and policyholder of the company, a copy of such book, or of any part thereof, on payment of a sum not exceeding Sixpence for every hundred words to be copied for such purpose.

Deed of settlement to be printed.

36. Every company which is not registered under "The Companies' Act, 1864", shall cause a sufficient number of copies of its deed of settlement, Act, or charter of incorporation, or other instrument regulating the constitution of the company, to be printed, and shall furnish on application to every shareholder and policyholder of the company a copy thereof, on payment of a sum not exceeding Two Shillings and Sixpence.

Amalgamation or transfer.

37. Where it is intended to amalgamate two or more companies, or to transfer the life assurance business of one company to another, the directors of any one or more of such companies may apply to the Court by petition to sanction the proposed arrangement, fourteen days' previous notice of such application being published in the *Gazette*, and the Court, after hearing the directors and other persons whom it considers entitled to be heard upon the petition, may confirm the same if it is satisfied that no sufficient objection to the arrangement has been established.

Procedure.

38. Before any such application is made to the Court, notice of such application, together with a statement of the nature of the amalgamation or transfer, as the case may be, and an abstract containing the material facts embodied in the agreement or deed under which such amalgamation or transfer is proposed to be effected, and copies of the actuaries' or other reports upon which such agreement or deed is founded, shall be forwarded to each policyholder of both companies in case of amalgamation, or to each policyholder of the transferred company in case of transfer, by the same being transmitted through the post, directed according to the registered or other known address of such policyholder, within such period as to admit of its being delivered in the due course of delivery fourteen days at least before the day named for the hearing of such application; and in proving such service it shall be sufficient to prove that such notice was properly addressed and put into the post office; and the agreement or deed under which such amalgamation or transfer is effected shall be open for the inspection of the policyholders and shareholders at the office or offices of the company or companies for a period of fifteen days after the issuing of the abstract herein provided.

Conditions.

39. The Court shall not sanction any amalgamation or transfer in any case in which it appears to the Court that policyholders representing one-fifth or more of the total amount assured in any company which it is proposed to amalgamate, or in any company the business of which it is proposed to transfer, dissent from such amalgamation or transfer.

Confirmation.

40. No company shall amalgamate with another, or transfer its business to another, unless such amalgamation or transfer is confirmed by the Court in accordance with this section: Provided always, that this section shall not apply in any case in which the business of any company which is sought to be amalgamated or transferred does not comprise life assurance business.

Statements in case of amalgamation or transfer.

41. When an amalgamation takes place between any companies, or when the business of one company is transferred to another company, the combined company or the purchasing company, as the case may be, shall, within ten days from the date of the completion of the amalgamation or transfer, deposit at the office of the Registrar certified copies of statements of the assets and liabilities of the companies concerned in such amalgamation or transfer, together with a statement of the nature and terms of the amalgamation or transfer, and a certified copy of the agreement or deed under which such amalgamation or transfer is effected, and certified copies of the actuarial or other reports upon which such agreement or deed is founded, and the statement and agreement or deed of amalgamation or transfer shall be accompanied by a declaration under the hand of the chairman of each company, and the principal managing officers of each company, that, to the best of their belief, every payment made, or to be made, to any person whatsoever on account of the said amalgamation or transfer, is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money, policies, bonds, valuable securities, or other property, by or with the knowledge of any parties to the said amalgamation or transfer.

Regulation as to novations by policyholders.

42. Where a company, either before or after the passing of this Act, has transferred its business to, or been amalgamated with another company, no policyholder in the first-mentioned company who shall pay to the other company the premiums accruing due in respect of his policy shall, by reason of any such payment made after the passing of this Act, or by reason of any other act done after the passing of this Act, be deemed to have abandoned any claim which he would have had against the first-mentioned company on due payment of premiums to such company, or to have accepted, in lieu thereof, the liability of the other company, unless such abandonment and acceptance have been signified by some writing signed by him, or by his agent lawfully authorized.

Inspection of deposited documents.

43. Any person may, on payment of such fees as the Governor may direct, inspect at the office of the Registrar any printed or other document required by this Act to be deposited at such office, and procure copies thereof.

Documents to be received in evidence.

44. Every statement, abstract, or other document deposited with the Registrar under this Act shall be receivable in evidence, and every document purporting to be certified by the Registrar to be such deposited document, and every document purporting to be similarly certified to be a copy of such deposited document, shall, if produced out of the custody of the Registrar, be deemed to be such deposited document as aforesaid, or a copy thereof, and shall be received in evidence as if it were the original document, unless some variation between it and the original document shall be proved.

Penalty for non-compliance with Act.

45. Every company which makes default in complying with the requirements of this Act, and shall continue in such default for seven days after notice by the Registrar, or any person interested in the matter of such default, shall be liable to a penalty not exceeding Fifty Pounds for every day during which the default continues: and in the case of a foreign company the general agent shall be liable to such penalty as well as the company; and in the case of companies registered under "The Companies' Act, 1864", if default continues for a period of three months after notice of default by the Treasurer, which notice shall be published in one or more newspapers, as the Treasurer may direct, the Court may order the winding up of the company, in accordance with the said Act, upon the application of one or more policyholders or shareholders.

Penalty for falsifying statements.

46. If any statement, abstract, or other document required by this Act is false in any particular to the knowledge of any person who signs the same, such person shall be guilty of a misdemeanour, and being convicted thereof shall be liable, at the discretion of the Court, to be imprisoned for any term not exceeding three years, or to a penalty not exceeding Five Hundred Pounds.

Surrender-value to be declared.

47. Every life assurance society shall declare the surrender-value at which the said society becomes bound to accept their policies, and no policy shall lapse to the society for non-payment of premium so long as the premiums and interest in arrear are not in excess of the surrender-value.

PART III.

Recovery and application of penalties.

48. Every penalty imposed by this Act shall be recovered and applied in the same manner as penalties imposed by "The Companies' Act, 1864," are recoverable and applicable.

Winding up of company.

49. The Court may order the winding up of any company in accordance with "The Companies' Act, 1864", on the petition of five or more policyholders or shareholders, upon its being proved to the satisfaction of the Court that the company is insolvent; and in determining whether or not the company is insolvent the Court shall take into account its contingent or prospective liability, under policies and annuity and other existing contracts.

Security for costs.

50. The Court shall not give a hearing to the petition until security for costs, for such amount as a Judge shall think reasonable, shall be given, and until a *primâ facie* case shall also be established to the satisfaction of the Judge.

Proprietary company.

51. In the case of a proprietary company having an uncalled capital of an amount sufficient, with the future premiums receivable by the company, to make up the actual invested assets equal to the amount of the estimated liability, the Court shall suspend further proceedings on the petition for a reasonable time (in the discretion of the Court) to enable the uncalled capital, or a sufficient part thereof, to be called up, and if, at the end of the original or any extended time for which the proceedings have been suspended, such an amount shall not have been realized by means of calls as with the already invested assets shall be equal to the liabilities, an order shall be made on the petition as if the company had been proved insolvent.

Winding up of subsidiary company.

52. Where the business or any part of the business of a company has, either before or after the passing of this Act, been transferred to another company, under an arrangement in pursuance of which such first-mentioned company (in this Act called the subsidiary company), or the creditors thereof, has or have claims against the company to which such transfer was made (in this Act called the principal company), then, if such principal company is being wound up by or under the supervision of the Court, either at or after the passing of this Act, the Court shall (subject as hereinafter mentioned) order the subsidiary company to be wound up in conjunction with the principal company, and may, by the same or any subsequent order, appoint the same person to be liquidator for the two companies, and make provisions for such other matters as may seem to the Court necessary, with a view to such companies being wound up as if they were one company, and the commencement of the winding up of the principal company shall, save as otherwise ordered by the Court, be the commencement of the winding up of the subsidiary company. The Court, nevertheless, shall have regard, in adjusting the rights and liabilities of the members of the several companies between themselves, to the constitution of such companies, and to the arrangements entered into between the said companies, in the same manner as the Court has regard to the rights and liabilities of different classes of contributories in the case of the winding up of a single company, or as near thereto as circumstances admit.

Contracts of insolvent company may be reduced.

53. The Court, in the case of a company which has been proved to be insolvent, may, if it thinks fit, reduce the amount of the contracts of the company or society, upon such terms and subject to such conditions as the Court thinks fit, in place of making a "winding-up order." (Sec. 22 of Act of 1870.)

Provision in case of subsidiary company not being in process of winding up.

54. Where any subsidiary company, or company alleged to be subsidiary, is not in process of being wound up at the same time as the principal company to which it is subsidiary, the Court shall not direct such subsidiary company to be wound up unless, after hearing all objections (if any) that may be urged by or on behalf of such company against its being wound up, the Court is of opinion that such company is subsidiary to the principal company, and that the winding up of such company in conjunction with the principal company is just and equitable.

Application for winding up subsidiary company.

55. An application may be made in relation to the winding up of any subsidiary company in conjunction with a principal company by any creditor or policyholder of, or person interested in, such principal or subsidiary company.

Grouping of companies.

56. Where a company stands in the relation of a principal company to one company, and in the relation of a subsidiary company to some other company, or where there are several companies standing in the relation of subsidiary companies to one principal company, the Court may deal with any number of such companies, together or in separate groups, as it thinks most expedient, upon the principles laid down in this section.

Court may reduce contracts.

57. The Court, in the case of a company which has been proved to be insolvent, may, if it thinks fit, reduce the amount of the contracts of the company, upon such terms and subject to such conditions as the Court thinks just, in place of making a winding-up order.

Valuation of policies.

58. Where a company is being wound up by the Court, or subject to the supervision of the Court, or voluntarily, the value of every life annuity and life policy requiring to be valued shall be estimated in manner provided by the Eighth Schedule.

Rules of Court.

59. The rules in the Eighth and Ninth Schedules shall be of the same force as if they were rules made in pursuance of "The Companies' Act, 1864," and may be altered in manner provided by the said Act; and rules may be made under the said Act for the purpose of carrying into effect the provisions of this Act with respect to the winding up of companies.

Notices to policyholders.

60. Any notice which is by this Act required to be sent to any policyholder may be addressed and sent to the person to whom notices respecting such policy are usually sent; and any notice so addressed and sent shall be deemed and taken to be notice to the holders of such policy.

Statements to be laid before Parliament.

61. The Treasurer shall lay annually before Parliament the statements and abstracts of reports deposited with the Registrar under this Act during the preceding year.

Probate or administration may be dispensed with in certain cases.

62. Upon the death of any holder of a policy upon his own life for a sum not exceeding Two Hundred Pounds, if no probate of his will or letters of administration to his estate be taken out within three months after his death, the company may pay the amount of such policy to his widow, or any adult child of his, and the receipt of such widow or child shall be a valid discharge, both at law and in equity, for the same.

Receipts of executors or administrators valid discharges.

63. The receipt of the executor or administrator of any deceased policyholder shall be a valid discharge, both at law and in equity, for any moneys payable under the policy held by him at the time of his death.

Assignment of policies.

64. Every assignment made after the first day of January, one thousand eight hundred and eighty-three, of a policy, may be made by memorandum of transfer indorsed upon such policy, in the form in the Tenth Schedule, and no such assignment shall be of any validity until registered as hereinafter provided.

Notice and registration.

65. Notice of every such assignment shall be given to the company for the time being liable upon the policy assigned, and such assignment shall be registered in a book to be provided by the company for that purpose, and the date of such registration shall be inserted in the memorandum of transfer, which shall be also signed by the principal officer managing the life assurance business of the company in South Australia, or his deputy, and thereafter the assignee may sue, as well at law as in equity, in his own name, on the policy assigned, and the receipt of such assignee shall be a valid discharge, both at law and in equity, for all moneys payable thereunder. Every such memorandum of transfer signed as last aforesaid shall be conclusive evidence of the registration thereof, and of the date of such registration.

No notice of mortgage or trusts.

66. If any policy is assigned by way of mortgage, or upon any trust, such mortgage or trust shall be effected by way of defeasance or declaration of trust by some separate instrument, and no notice of any such mortgage or trust shall be entered on the memorandum of transfer or indorsed on the policy, and the company shall not be affected either by express, implied, or constructive notice of any such mortgage or trust, nor be bound or concerned

to see to the application of any moneys payable under such policy.

PART III.

Duties of Public Trustee under Act.

67. It shall be the duty of the Public Trustee to secure the due observance by every company of the provisions of this Act; and the Governor may, by regulation to be published in the *Government Gazette*, declare what fees shall be payable by companies in respect of the duties to be performed hereunder by the Public Trustee.

In the name and on behalf of Her Majesty, I hereby assent to this Bill.

WM. F. DRUMMOND JERVOIS,
Governor.

SCHEDULES.

FIRST SCHEDULE.

Revenue Account for the year ending ______________.

	£	s.	d.		£	s.	d.
18 (Date)				18 (Date)			
Amount of funds at beginning of the year				Claims under policies (after deduction of sums re-assured)			
Renewal premiums, after deduction of re-assurance premiums				Surrenders			
New premiums (after deduction of re-assurance premiums) on new policies, assuring £ , and yielding an annual revenue of £ , after deduction of re-assurances				Annuities			
Consideration for annuities granted				Commission on new premiums			
Interest and dividends				" on renewals			
Other receipts (accounts to be specified)				Expenses of management			
				Dividends and bonus to shareholders (if any)			
				Other payments (accounts to be specified)			
				Amount of funds at the end of the year, as per Second or Fourth Schedule			

Note 1.—Companies having separate accounts for annuities, to return particulars of their annuity business in a separate statement.

Note 2.—Items in this and in the accounts in the Third and Fifth Schedules should be the net amounts, after deduction of the amounts paid and received in respect of re-assurances.

Balance Sheet of the__________on the__________18 .

LIABILITIES.	£ s. d.	£ s. d.
Shareholders' capital paid up (if any) ...		
Assurance fund		
Annuity fund (if any)		
Other funds (if any, to be specified) ...		
Total funds, as per Third Schedule		
Claims admitted but not paid *		
Other sums owing to the company (accounts to be specified) *		
	£	

ASSETS.	£ s. d.	£ s. d.
In South Australia—		
South Australian Government securities		
Other Government securities (particulars to be specified)		
Mortgages		
Loans on the company's policies		
Loans upon personal security		
Railway and other debentures, and debenture stocks ...		
Railway shares (preferential and ordinary)		
House property		
Other investments (to be specified)		
Agents' balances		
Outstanding premiums		
Ontstanding interest		
Cash—On deposit		
In hand and on current account		
Other assets (to be specified)		
Total assets in South Australia		
Elsewhere than South Australia—		
British Government securities		
Indian and Colonial Government securities		
Foreign Government securities		
Mortgages (stating where)		
Loans on the company's policies		
Loans upon personal security		
Railway and other debentures, and debenture stocks ...		
Railway shares (preferential and ordinary)		
House property (stating where)		
Other investments (to be specified)		
Agents' balances		
Outstanding premiums		
Outstanding interest		
Cash—On deposit		
In hand and on current account		
Other assets (to be specified)		
Total assets elsewhere than in South Australia ...	...	
Total assets	£	

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* These items are included in the corresponding items in the First Schedule.

THIRD SCHEDULE.

Revenue Accounts of the ______________ for the year ending ______.

(No. 1).—*Life Assurance Account.*

(Date)	£ s. d.	(Date)	£ s. d.
Amount of life assurance fund at the beginning of the year		Claims under life policies, after deduction of sums re-assured	
Renewal premiums, after deduction of re-assurance premiums		Surrenders	
New premiums (after deduction of re-assurance premiums) on new policies assuring £ , and yielding an annual income of £ , after deduction of re-assurances		Annuities	
		Commission	
		Expenses of management	
Consideration for annuities granted		Other payments (accounts to be specified)	
Interest and dividends		Amount of life assurance fund at the end of the year, as per Fourth Schedule	
Other receipts (accounts to be specified)			
£		£	

NOTE.—Companies having separate accounts for annuities to return the particulars of their annuity business in a separate statement.

(No. 2).—*Fire Account.*

(Date)	£ s. d.	(Date)	£ s. d.
Amount of fire insurance fund at the beginning of the year		Losses by fire, after deduction of re-assurance	
Premiums received, after deduction of re-assurances ...		Expenses of management	
Other receipts (to be specified)		Commission	
		Other payments (to be specified)	
		Amount of fire insurance fund at the end of the year, as per Fourth Schedule	
£		£	

NOTE.—When marine or any other branch of business is carried on, the income and expenditure thereof to be in like manner stated in a separate account.

(No. 3).—*Profit and Loss Account.*

(Date)	£ s. d.	(Date)	£ s. d.
Balance of last year's account		Dividends and bonuses to shareholders	
Interest and dividends not carried to other accounts ...		Expenses not charged to other accounts	
Profits realized (accounts to be specified)		Loss realized (accounts to be specified)	
Other receipts		Other payments	
		Balance as per Fourth Schedule	
£		£	

NOTE.—This account is not required if the items have been incorporated in the other accounts of this schedule.

Balance Sheet of the ______________ on the ______________ 18___.

LIABILITIES.	£ s. d.	ASSETS.	£ s. d.	£ s. d.
Shareholders' capital		*In South Australia*—		
General reserve fund (if any)		South Australian Government securities		
Life assurance fund *		Other Government securities (particulars to be specified)		
Annuity fund (if any)*		Mortgages		
Fire fund		Loans on the company's policies		
Marine fund		Loans upon personal security		
Profit and loss (if any)		Railway and other debentures and debenture stocks		
Other funds (if any, to be specified)		Railway shares (preferential and ordinary)		
£		House property		
Claims under life policies admitted but not yet paid *		Other investments (to be specified)		
Outstanding fire losses		Agents' balances		
Outstanding marine losses		Outstanding premiums		
Other sums owing by the company (accounts to be specified)		Outstanding interest		
		Cash—On deposit		
		In hand and on current account		
		Other assets (to be specified)		
		Total assets in South Australia		
		Elsewhere than in South Australia—		
		British Government securities		
		Indian and Colonial Government securities		
		Foreign Government securities		
		Mortgages (stating where)		
		Loans on the companies' policies		
		Loans upon personal security		
		Railway and other debentures and debenture stocks		
		Railway shares (preferential and ordinary)		
		House property (stating where)		
		Other investments (to be specified)		
		Agents' balances		
		Outstanding premiums		
		Outstanding interest		
		Cash—On deposit		
		In hand and on current account		
		Other assets (to be specified)		
		Total assets elsewhere than in South Australia		
£		Total assets £		

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* If the life assurance fund is in accordance with section 20 of this Act, a separate trust fund for the sole security of the life policyholders, or separate balance sheet for the life branch, may be given in the form contained in the Second Schedule. In other respects the company is to observe the above form. See also note to Second Schedule.

FIFTH SCHEDULE.

1. New policies issued by the________during the year ending______.

	No. of Policies	Sum Assured	Single Premiums	Annual Premiums
		£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
Assurances				
Endowment		(per annum)		
Annuity				
Total				

2. Policies discontinued in the________during the year ending______.

How Discontinued	Assurance Policies			Endowment Policies			Annuity Policies		
	No.	Sum Assured	Annual Premiums	No.	Sum Assured	Annual Premiums	No.	Annuity per Annum	Annual Premiums
		£ *s.* *d.*	£ *s.* *d.*		£ *s.* *d.*	£ *s.* *d.*		£ *s.* *d.*	£ *s.* *d.*
By death or maturity									
By surrender . . .									
By forfeiture . . .									
Total . . .									

3. Policies of the________existing on the______.

	In South Australia		Elsewhere	
	No. of Policies	Sum Assured	No. of Policies	Sum Assured
		£ *s.* *d.*		£ *s.* *d.*
Assurance				
Endowment		(per annum)		(per annum)
Annuity.				
Total.				

4. Progress of the life assurance business of the ____________ since its establishment to ______________.

	No. of Policies	Sums Assured	Annuity per Annum.	Annual Premiums
		£ s. d.	£ s. d.	£ s. d.
Total issued				
Total discontinued . .				
Existing				
Annual Income from Interest				
Total Annual Income				

SIXTH SCHEDULE.

Statement respecting the Valuation of the Liabilities under Life Policies and Annuities of the ______________ to be made to the Actuary.

[The answers should be numbered to accord with the numbers of the corresponding questions.]

1. The date up to which the valuation was made?
2. The principles upon which the valuation was made, and whether these principles were determined by the instrument constituting the company or by its regulations, or by by-laws or otherwise?
   (*a*) Were the policies valued individually or in classes?
   (*b*) If in classes, how was the valuation age determined?
   (*c*) What portion (if any) of a year's premium was assumed to be due?
   (*d*) Were lives assured at increased rates assumed to be of the age at entry corresponding to the premium charged?
3. The table or tables of mortality used in the valuation?
4. The rate or rates of interest assumed in the calculations?
5. By what table of mortality, and according to what rate of interest, have the net premiums valued been computed? Give specimens of such premiums for a whole-life policy of £100, for ages at entry, 20, 25, 30, 35, 40, 45, 50, 55, 60.
6. The proportion of the annual premium income (if any) reserved as a provision for future expenses and profits? [*If none, state how this provision is made.*]
7. The consolidated revenue account since the last valuation, or, in case of a company which has made no valuation, since the

commencement of the business? [*This return must be made in the form annexed.*]

8. The liabilities of the company under life policies and annuities at the date of the valuation, showing the number of policies, the amount assured, and the amount of premiums payable annually under each class of policies, both with and without participation in profits, and also the net liabilities and assets of the company, with the amount of surplus or deficiency? [*These returns must be made in the forms annexed.*]
9. The principles upon which the distributions of profit are made?
10. The time during which a policy must be in force to entitle it to share in the profits.
11. The results of the valuation, showing:
    - (*a*) The total amount of profit made by the company.
    - (*b*) The amount of profit divided among the policy-holders, and the number and amount of policies which participated.
    - (*c*) Specimens of bonuses allotted to policies for £100, effected at the respective ages of 20, 30, 40, 50, and having been respectively in force for five years, ten years, and upwards, at intervals of five years, together with the amount appropriated under the various modes in which the bonus might be received.

[Form referred to under heading No. 7 in Sixth Schedule.]

Consolidated Revenue Account of the__________for__________years, commencing________________and ending________________

	£ s. d.		£ s. d.
Amount of funds on 18 , the beginning of		Claims under policies, after deduction of sums re-assured	
Premiums, after deduction of reassurance premiums		Surrenders	
Consideration for annuities granted		Annuities	
Interest and dividends ...		Commission	
Other receipts (accounts to be specified)		Expenses of management	
		Dividends and bonuses to shareholders (if any) ...	
		Other payments (accounts to be specified)... ...	
		Amount of funds on 18 , the end of the period, as per First (or Third Schedule) ...	
£		£	

[Form referred to under heading No. 8 in Sixth Schedule.]

Summary and Valuation of the Policies of the________as at______

Description of Transactions	Particulars of the Policies for Valuation						Valuation					
							Value by the table. Interest per cent.					
	No. of Policies	Sum Assured	Bonus Addition	Office Yearly Premiums	Net Yearly Premiums	Loading	Sum Assured	Bonus Addition	Office Yearly Premium	Net Yearly Premium	Loading	Net Liability
Assurances—												
1. With participation in profits:												
For whole term of life . .												
Other classes (to be specified)												
Extra premiums payable .												
Total assurances with profits .												
2. Without participation in profits:												
For whole term of life . .												
Other classes (to be specified)												
Extra premiums payable. .												
Total assurances without profits												
Total assurances												
Deduct re-assurances												
Net amount of assurances . .												
Adjustments (if any)												
Annuities—												
Immediate.												
Other classes (to be specified) .												
Total of the results												

The item "extra premium" in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country, at rates of premium deduced from tables other than the European mortality tables adopted by the company, separate schedules, similar in form to the above, must be furnished.

[Form referred to under heading No. 8 in Sixth Schedule.]

Valuation balance sheet of__________as at__________18____.

Dr.	£	*s.*	*d.*	Cr.	£	*s.*	*d.*
To net liability under assurance and annuity transactions (as per summary statement provided in Seventh Schedule) .				By life assurance and annuity funds (as per balance sheet under Third or Fifth Schedule)			
To surplus (if any) . . .				By deficiency (if any) . .			
£				£			

SEVENTH SCHEDULE.

Statement of the Life Assurance and Annuity business of the______ on the_________.

[The answers should be numbered to accord with the numbers of the corresponding questions. Statements of re-assurances, corresponding to the statements in respect of assurances under headings 2, 3, 4, 5, 6 and 7, are to be given.]

1. The published table or tables of premiums for assurances for the whole term of life which were in use at the date abovementioned?
2. The total number of policies and amounts assured on lives for the whole term of life which were in existence at the date abovementioned, distinguishing the portions assured with and without profits, stating separately the total reversionary bonuses, and specifying the sums assured for each year of life, from the youngest to the oldest ages?
3. The amount of premiums receivable annually for each year of life after deducting the abatements made by the application of bonuses in respect of the respective assurances mentioned under heading No. 2, distinguished ordinary from extra premiums?
4. The total amount of premiums which have been received from the commencement upon all policies mentioned under heading No. 2, which were in force at the above-mentioned date?
5. The total number of policies and amounts assured under classes of assurance business other than for the whole term of life, distinguishing the number of policies and the sums assured under each class, and stating separately the amount assured, with and without profits, and the total amount of reversionary bonuses?
6. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under heading No. 5, distinguishing ordinary from extra premiums?
7. The total amount of premiums which have been received from the commencement upon all policies under each special class mentioned under heading No. 5, which were in force at the date above-mentioned?
8. The total amount of immediate annuities on lives, distinguishing the amount for each year of life?
9. The amount of all annuities other than those specified under heading No. 8, distinguishing the amount payable under each class, the amount of premiums annually receivable, and the amount of consideration money received in respect of each such class, and the total amount of premiums received from the commencement upon all deferred annuities?

10. The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation, together with a statement of the manner in which such average has been computed?

11. A table of minimum values (if any) allowed for the surrender of policies for the whole term of life, and for endowments, and endowment assurances, or a statement of the method pursued in calculating such surrender values, with instances of its application to policies of different standing, and taken out at various interval ages, from the youngest to the oldest?

12. Separate statements to be furnished for business at other than European rates, together with a statement of the manner in which policies on unhealthy lives are dealt with?

EIGHTH SCHEDULE.

Rule for Valuing an Annuity.

An annuity shall be valued according to the tables used by the company which granted such annuity at the time of granting the same; and where such tables cannot be ascertained or adopted to the satisfaction of the Court, then according to the table known in England as the Government Annuities Experience Table, interest being reckoned at the rate of Four Pounds per centum per annum.

Rule for Valuing a Policy.

The value of the policy is to be the difference between the present value of the reversion in the sum assured on the decease of the life, including any bonus or addition thereto made before the commencement of the winding-up, and the present value of the future annual premiums.

In calculating such present values the rate of interest is to be assumed as being Four Pounds per centum per annum, and the rate of mortality as that of the tables known in England as the Institute of Actuaries' Life Tables.

The premium to be calculated is to be such a premium as, according to the said rate of interest and rate of mortality, is sufficient to provide for the risk incurred by the office in issuing the policy, exclusive of any addition thereto for office expenses and other charges.

NINTH SCHEDULE.

When an assurance company is being wound up by the Court, or subject to the supervision of the Court, the official liquidator, in case of all persons appearing by the books of the company to be entitled to or interested in policies granted by such company for life assurance endowment, annuity, or other payment, is to ascertain the values of such policies, and give notice of such value to such persons; and any person to whom notice is so given shall be bound by the value so ascertained, unless he gives notice of his intention to dispute such value in manner and within a time to be prescribed by a rule or order of the Court.

TENTH SCHEDULE.

Memorandum of Transfer.

Date of Transfer, and state whether Transfer absolute or partial, and if partial to what amount transferred	Signature of Transferor	Witness	Transferee			Signature of Transferee	Witness	Date of Registration of Transfer	Signature of principal Officer of Company
			Name in full	Address	Occupation				

ACTUARIAL NOTE.

On Mr. Woolhouse's Formula for the Adjustment of Tables. By Charles Daniel Higham, *Assistant Actuary of The Metropolitan Life Assurance Society.*

Though between the graphic method on the one hand, and Mr. Higham's on the other, the formula used by Mr. Woolhouse in graduating the Institute Tables seems to have lost something of its pre-eminence, it may be convenient to have on record a general expression for it, correct to third differences, derived as on pages 341 and 350 of the twenty-third volume of the *Journal.*

Let the numbers living (or dying) at each age be represented by u_0, u_1, u_2, &c., the central term of those immediately under observation being u_c, n denoting distance from the central term, and the average of x curves being required; and let $c-n=a$: then, by the well-known formula of Finite Differences,

$$u_{a-x}=u_0+(a-x)d_1u_0+\frac{(a-x)(a-x-1)}{1.2}d_2u_0+\frac{(a-x)(a-x-1)(a-x-2)}{1.2.3}d_3u_0$$

$$u_a=u_0+a\,d_1u_0+\frac{a(a-1)}{1.2}d_2u_0+\frac{a(a-1)(a-2)}{1.2.3}d_3u_0$$

and $$u_{a-x}-u_a=-x\,d_1u_0+\frac{x(x-2a+1)}{1.2}d_2u_0+\frac{(a-x)(a-x-1)(a-x-2)-a(a-1)(a-2)}{1.2.3}d_3u_0\quad .\quad .\quad (A)$$

Similarly,

$$u_{a+x}-u_a=x\,d_1u_0+\frac{x(x+2a-1)}{1.2}d_2u_0+\frac{(a+x)(a+x-1)(a+x-2)-a(a-1)(a-2)}{1.2.3}d_3u_0\quad .\quad .\quad (B)$$

and adding $$u_{a-x}-2u_a+u_{a+x}=x^2d_2u_0+x^2(a-1)d_3u_0,$$

whence $$d_2u_0=\frac{u_{a-x}}{x^2}-\frac{2u_a}{x^2}+\frac{u_{a+x}}{x^2}-(a-1)d_3u_0;$$

or, putting $c-n$ for a, $$d_2u_0=\frac{u_{c-n-x}}{x^2}-\frac{2u_{c-n}}{x^2}+\frac{u_{c-n+x}}{x^2}-(c-n-1)d_3u_0.$$

Multiplying A by $(x+2a-1)$ and B by $(x-2a+1)$, and subtracting the latter product from the former,

$$(x+2a-1)u_{a-x}-2(2a-1)u_a-(x-2a+1)u_{a+x}=-2x^2d_1u_0+\frac{2x^2\{3a(a-1)-(x^2-1)\}}{6}d_3u_0$$

whence $$d_1u_0=-\frac{x+2a-1}{2x^2}u_{a-x}+\frac{2a-1}{x^2}u_a+\frac{x-2a+1}{2x^2}u_{a+x}+\frac{3a(a-1)-(x^2-1)}{6}d_3u_0;$$

or, putting $c-n$ for a,

$$d_1u_0 = -\frac{x+2(c-n)-1}{2x^2}u_{c-n-x} + \frac{2(c-n)-1}{x^2}u_{c-n} + \frac{x-2(c-n)+1}{2x^2}u_{c-n+x} + \frac{3(c-n)(c-n-1)-(x^2-1)}{6}d_3u_0.$$

Again, $$u_c = u_0 + cd_1u_0 + \frac{c(c-1)}{1.2}d_2u_0 + \frac{c(c-1)(c-2)}{1.2.3}d_3u_0$$

and $$u_{c-n} = u_0 + (c-n)d_1u_0 + \frac{(c-n)(c-n-1)}{1.2}d_2u_0 + \frac{(c-n)(c-n-1)(c-n-2)}{1.2.3}d_3u_0$$

whence $$u_c = u_{c-n} + nd_1u_0 + \frac{n(2c-n-1)}{2}d_2u_0 + \frac{c(c-1)(c-2)-(c-n)(c-n-1)(c-n-2)}{6}d_3u_0.$$

Substituting in the last expression the values for d_1u_0 and d_2u_0 found above,

$$\begin{aligned} u_c = {} & u_{c-n} \\ & -\frac{n\{x+2(c-n)-1\}}{2x^2}u_{c-(x+n)} + \frac{n\{2(c-n)-1\}}{x^2}u_{c-n} + \frac{n\{x-2(c-n)+1\}}{2x^2}u_{c+(x-n)} + \frac{n\{3(c-n)(c-n-1)-(x^2-1)\}}{6}d_3u_0 \\ & + \frac{n(2c-n-1)}{2x^2}u_{c-(x+n)} - \frac{n(2c-n-1)}{x^2}u_{c-n} + \frac{n(2c-n-1)}{2x^2}u_{c+(x-n)} - \frac{n(2c-n-1)(c-n-1)}{2}d_3u_0 \\ & + \frac{c(c-1)(c-2)-(c-n)(c-n-1)(c-n-2)}{6}d_3u_0 \end{aligned}$$

and summing $$u_c = -\frac{n(x-n)}{2x^2}u_{c-(x+n)} + \frac{x^2-n^2}{x^2}u_{c-n} + \frac{n(x+n)}{2x^2}u_{c+(x-n)} - \frac{n(x^2-n^2)}{6}d_3u_0.$$

This being a general term, let $n=-n$,

then
$$u_c=\frac{n(x+n)}{2x^2}u_{c-(x-n)}+\frac{x^2-n^2}{x^2}u_{c+n}-\frac{n(x-n)}{2x^2}u_{c+(x+n)}+\frac{n(x^2-n^2)}{6}d_3u_c,$$

and adding these two values of u_c, and dividing by 2,

$$u_c=\frac{x^2-n^2}{2x^2}(u_{c-n}+u_{c+n})+\frac{n(x+n)}{4x^2}(u_{c-(x-n)}+u_{c+(x-n)})-\frac{n(x-n)}{4x^2}(u_{c-(x+n)}+u_{c+(x+n)}),$$

the third differences vanishing.

Then making $n=0, 1, 2,$ &c., $\frac{x-1}{2}$ (x being always an odd number, or intermediate terms in the series u_0, u_1, &c., will be introduced), and doubling all the expressions except the first,

$$u_c=u_c$$

$$2u_c=\frac{x^2-1^2}{x^2}(u_{c-1}+u_{c+1})+\frac{1(x+1)}{2x^2}(u_{c-(x-1)}+u_{c+(x-1)})-\frac{1(x-1)}{2x^2}(u_{c-(x+1)}+u_{c+(x+1)})$$

$$2u_c=\frac{x^2-2^2}{x^2}(u_{c-2}+u_{c+2})+\frac{2(x+2)}{2x^2}(u_{c-(x-2)}+u_{c+(x-2)})-\frac{2(x-2)}{2x^2}(u_{c-(x+2)}+u_{c+(x+2)})$$

&c. &c. &c.

$$2u_c=\frac{x^2-\left(\frac{x-1}{2}\right)^2}{x^2}\left[u_{c-\frac{x-1}{2}}+u_{c+\frac{x-1}{2}}\right]+\frac{\frac{x-1}{2}\left(x+\frac{x-1}{2}\right)}{2x^2}\left[u_{c-\left(x-\frac{x-1}{2}\right)}+u_{c+\left(x-\frac{x-1}{2}\right)}\right]-\frac{\frac{x-1}{2}\left(x-\frac{x-1}{2}\right)}{2x^2}\left[u_{c-\left(x+\frac{x-1}{2}\right)}+u_{c+\left(x+\frac{x-1}{2}\right)}\right]$$

Summing these values, and dividing by x,

$$u_c=\frac{u_c}{x}+\frac{x^2-1^2}{x^3}(u_{c-1}+u_{c+1})+\frac{x^2-2^2}{x^3}(u_{c-2}+u_{c+2})+\&c.+\frac{x^2-\left(\frac{x-1}{2}\right)^2}{x^3}\left(u_{c-\frac{x-1}{2}}+u_{c+\frac{x-1}{2}}\right)$$

$$+\frac{\left(\frac{x-1}{2}\right)\left(x+\frac{x-1}{2}\right)}{2x^3}\left[u_{c-\left(x-\frac{x-1}{2}\right)}+u_{c+\left(x-\frac{x-1}{2}\right)}\right]+\&c.+\frac{2(x+2)}{2x^3}(u_{c-(x-2)}+u_{c+(x-2)})+\frac{1(x+1)}{2x^3}(u_{c-(x-1)}+u_{c+(x-1)})$$

$$-\frac{1(x-1)}{2x^3}(u_{c-(x+1)}+u_{c+(x+1)})-\frac{2(x-2)}{2x^3}(u_{c-(x+2)}+u_{c+(x+2)})-\&c.-\frac{\frac{x-1}{2}\left(x-\frac{x-1}{2}\right)}{2x^3}\left[u_{c-\left(x+\frac{x-1}{2}\right)}+u_{c+\left(x+\frac{x-1}{2}\right)}\right]$$

where $x=$ the number of curves of which the average is desired.

This formula is the general expression of Mr. Woolhouse's method; if, therefore, $x=5$, then $\frac{x-1}{2}=2$, and

$$u_c=\cdot200u_c+\cdot192(u_{c-1}+u_{c+1})+\cdot168(u_{c-2}+u_{c+2})$$
$$+\cdot056(u_{c-3}+u_{c+3})+\cdot024(u_{c-4}+u_{c+4})$$
$$-\cdot016(u_{c-6}+u_{c+6})-\cdot024(u_{c-7}+u_{c+7}),$$ as in *Journal* xv, 392.

It will be observed in the above general expression for the value of u_c that for each integral 2 in the value of x, six terms of u will be added to $\frac{u_c}{x}$ to form the expression, the sum of the coefficients of such six terms being

$$2\left\{\frac{x^2-\left(\frac{x-1}{2}\right)^2}{x^3}+\frac{\frac{x-1}{2}\left(x+\frac{x-1}{2}\right)}{2x^3}-\frac{\frac{x-1}{2}\left(x-\frac{x-1}{2}\right)}{2x^3}\right\}=\frac{2x^2}{x^3};$$

and since there are $\frac{x-1}{2}$ integral twos in x, the sum of all the coefficients in the expression for the value of $u_c = \frac{1}{x} + \frac{x-1}{2} \cdot \frac{2x^2}{x^3}$ $= 1$, as it should be.

CORRESPONDENCE.

ON LOGARITHMS CORRECT TO TEN PLACES OF DECIMALS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having had occasion to require the logarithms of certain numbers to 10 places of decimals, my attention was directed to the Tables of Mr. S. Pineto of St. Petersburg, and I have found these so simple and convenient in working, that I feel sure the readers of the *Journal* will be interested in having their method and use explained.

The method referred to depends on the following simple proposition. If A be the number whose logarithm is sought, and M be any other number whatever, then

$$A = AM \times \frac{1}{M}; \text{ and } \operatorname{Log} A = \operatorname{Log} AM + \operatorname{Log} \frac{1}{M}.$$

The tables of Mr. Pineto consist of two parts;

1st. A subsidiary table of the logarithms to 10 places of the reciprocals of certain multipliers, M, which multipliers, when applied to any number whatever, suffice to bring the first seven figures of the product within the limits 1,000,000, and 1,011,000.

2nd. A table of the logarithms to 10 places of all numbers between the aforesaid limits, with tables of proportional parts, which enable the logarithms of any number of not more than 11 digits to be found with the greatest ease.

The tables are comprised in a small volume of 56 pages. The following is an example from the introductory chapter illustrating their use.

Let it be required to find to 10 places the logarithm of the number $\pi = 3 \cdot 14159\,26536$. From the auxiliary table we find that the requisite multiplier is 32 and that $\operatorname{Log} \frac{1}{32} = \bar{2} \cdot 49485\,00216 \cdot 80$

Then

$$\begin{array}{rr} \pi = & 3{\cdot}14159\;26536 \\ & 32 \\ \hline M\pi = & 100{\cdot}53096\;49152 \end{array}$$

$$\begin{array}{lr}
\text{Log}\,\frac{1}{32} & =\bar{2}{\cdot}49485\;00216{\cdot}80\\
\text{Log}\,100\;5309 & =2{\cdot}00229\;95705{\cdot}75\\
\text{Additions by proportional parts}\left\{\begin{array}{l}64\\ \quad 91\\ \qquad 52\end{array}\right. & \begin{array}{r}-\qquad 2764{\cdot}80\\ 39{\cdot}31\\ {\cdot}22\end{array}\\
\hline
\text{Log}\,\pi & =0{\cdot}49714\;98726{\cdot}88
\end{array}$$

It is hardly necessary to say that by a converse process the number can be found corresponding to any logarithm to 10 places.

Unfortunately, Mr. Pineto's tables are out of print; but another set of tables by Mons. A. Namur of Thuin-sur-Sambre, Belgium, based on the same principle, can be readily obtained.*

In the latter, the results are tabulated to 12 places of decimals. The numbers to which the logarithms are given lie between the limits 433,300 and 434,300, and Mons. Namur makes use of the features of the logarithmic differences at this part of the table, to find the logarithm of any number between 433,300,000,000 and 434,300,000,000 without the aid of the usual tables of proportional parts. Hence, the very small compass of 11 pages within which his tables are comprised. I must refer the reader for a further explanation to the introduction to the tables, written by Mons. P. Mansion, Professor at the University of Ghent.

It must be added, however, that, in respect of facility in use, Mr. Pineto's tables are superior to those of Mons. Namur. In the latter it is necessary to apply *two* consecutive multipliers in the great majority of cases to the number whose logarithm is sought to bring the first six figures of the product within the limits given by the tables, and the mode of finding the addition to the tabulated logarithm corresponding to the last six figures of the product requires the exercise of great care in fixing the place of the decimal point.

I send for publication in the *Journal* the annexed short table, compiled by an acquaintance of mine on the principle of Pineto's tables, which will enable the logarithms of numbers to be found to eight places of decimals, by means of the extended portion lying between the numbers 100,000 and 108,000 in any good logarithmic tables, such as those of Hutton, Babbage, Callet, Schrön, &c.

The logarithms of $\frac{1}{M}$ have been obtained from the table of logarithms to 20 places given by Hutton.

I am, Sir,
Your obedient servant,
JAS. CHISHOLM.

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* The Tables were published in 1877 at Brussels and Paris by the Royal cademy of Belgium, and the price is 1*s*.

Table for computing to Eight Decimals the Logarithm of any Number by means of the Eight-figure Logarithms of the Numbers from 100,000 *to* 108,000, *given in the Tables of Hutton, Babbage, Schrön and others.*

N	M	Log $\frac{1}{M}$	Log
108	96	$\bar{2}$·01772,87670	·987
112	9	$\bar{1}$·04575,74906	·955
120	88	$\bar{2}$·05551,73279	·945
121	84	$\bar{2}$·07572,07139	·936
125	8	$\bar{1}$·09691,00130	·904
134	77	$\bar{2}$·11350,92748	·887
140	72	$\bar{2}$·14266,75036	·877
143	7	$\bar{1}$·15490,19600	·846
153	66	$\bar{2}$·18045,60645	·820
162	63	$\bar{2}$·20065,94505	·817
164	61	$\bar{2}$·21467,01650	·810
167	6	$\bar{1}$·22184,87496	·779
179	58	$\bar{2}$·23657,20064	·772
182	55	$\bar{2}$·25963,73105	·741
195	54	$\bar{2}$·26760,62402	·739
197	51	$\bar{2}$·29242,98239	·731
200	5	$\bar{1}$·30102,99957	·699
216	48	$\bar{2}$·31875,87626	·686
224	45	$\bar{2}$·34678,74862	·676
228	44	$\bar{2}$·35654,73235	·645
244	41	$\bar{2}$·38721,61433	·634
250	4	$\bar{1}$·39794,00087	·603
269	39	$\bar{2}$·40893,53930	·592
276	38	$\bar{2}$·42021,64034	·588
277	36	$\bar{2}$·44369,74992	·557
300	34	$\bar{2}$·46852,10830	·550
303	33	$\bar{2}$·48148,60601	·524
323	31	$\bar{2}$·50863,83062	·510
334	3	$\bar{1}$·52287,87453	·478
359	29	$\bar{2}$·53760,20021	·464
371	27	$\bar{2}$·56863,62358	·431
400	25	$\bar{2}$·60205,99913	·413
430	24	$\bar{2}$·61978,87583	·376
455	22	$\bar{2}$·65757,73192	·355
477	21	$\bar{2}$·67778,07053	·334
500	2	$\bar{1}$·69897,00043	·302
539	19	$\bar{2}$·72124,63990	·288
556	18	$\bar{2}$·74472,74949	·256
599	17	$\bar{2}$·76955,10786	·237
625	16	$\bar{2}$·79588,00173	·209
667	15	$\bar{2}$·82390,87409	·179
715	14	$\bar{2}$·85387,19643	·147
771	13	$\bar{2}$·88605,66477	·114
829	121	$\bar{3}$·91721,46297	·112
834	12	$\bar{2}$·92081,87540	·080
900	112	$\bar{3}$·95078,19773	·074
910	11	$\bar{2}$·95860,73148	·042
981	102	$\bar{3}$·99139,98282	·037
991	101	$\bar{3}$·99567,86262	·033

DIRECTIONS FOR USE OF THE TABLE.

The column M contains multipliers, one of which, properly selected, will always bring the natural number to be dealt with within the limits 10,000,000 and 10,800,000. The column Log $\frac{1}{\text{M}}$ contains the log., to 10 places, of the reciprocals of those multipliers.

To find the log. to eight places of any number over 10,800,000. Take the multiplier corresponding to the first three figures of the number in the column N., or to the next *lower* if those figures are not found there. Multiply by it the number to be dealt with. Find the log. of the result from one of the tables referred to. Add to that log. the Log $\frac{1}{\text{M}}$ from this table. The sum, properly reduced to eight places, gives the log. required.

To find the natural number to eight places of any log. over ·033 . . . Add thereto the Log $\frac{1}{\text{M}}$ corresponding to the first three places of the log. in the above table, or, if not found there, to the next *lower* places there found. The resulting log. will be between ·00000000 and ·03300000. Find its natural number, which is then to be multiplied by the corresponding M. The product is the natural number sought.

It will make the result in all cases rather more accurate if 25 in the ninth and tenth places be *added* to the tabular log. where the last figure has *not* been increased, and *subtracted* where it *has* been increased in those tables which give information on that point.

PREMIUMS FOR CONTINGENT ASSURANCES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have read with interest Mr. Chatham's letter which appears in the last number of the *Journal* (*see J.I.A.*, xxv, 439). For some time I have thought it would be desirable to tabulate corrections, such as those which he gives on pp 441 and 442, by means of which the premium for assuring x against y and for t years longer might be easily deduced from the premium for assuring x against y.

It appears to me, however, not unlikely that the H^{M} Table, which by some of the best authorities is held inapplicable to the calculation of premiums generally, may, on examination, prove specially inappropriate for quoting rates for contingent assurances.

Contingent assurances, as we know, are generally effected to cover loans advanced upon reversions in cases where the reversioner must survive the life-tenant in order to acquire a vested interest. The persons on whose lives these assurances are effected are probably somewhat under the average in point of vitality, being, many of them impecunious and some of them dissipated; the life-tenants of the trust estates on the other hand are generally in comfortable circumstances, and I believe considerably more than half of them are women, and their vitality will therefore probably exceed that of the H^M Table. It is of great importance not to under-estimate the risk attaching to these assurances, since they are effected in great numbers and often for very large amounts.

The exact calculation of a table of contingent insurance premiums, in which the lives insured follow one table of mortality and the lives insured against another, and in which also the effect of selection in each case is duly taken account of, is one of some difficulty. On this point I have been making some experiments, in order to frame a set of premiums for the use of my own office, and give you in the annexed table some of the results which I have obtained, reserving for a future letter an explanation of the processes employed. The mortality tables employed in the calculation of these rates are—(1) for the lives assured, Mr. Sprague's Select Mortality Tables, given pp 398, 399, of Vol. xxii of the *Journal*, with this modification, practically unimportant, that the column $l_{[x-4]+4}$ has been omitted and its place supplied by the column l_x; (2) for the lives assured against, Mr. Finlaison's Female Government Annuitants' Analyzed Table of 1884.

It will be noticed that the premiums of my table, as a rule, exceed those given by Mr. Chatham on pp 442 and 443, based on the H^M Table, the difference in some cases being very considerable. Take, for instance, the case of 20 against 60. The single premium given below is 45 per-cent higher than the H^M 4 per-cent single premium, and the annual premium is over 31 per-cent higher than the H^M 4 per-cent annual premium.

I am, Sir,
Your obedient servant,
A. W. SUNDERLAND.

National Life Assurance Society,
2, King William Street, London,
30 *June* 1886.

Table of Single and Annual Premiums for Insuring x against y, the Mortality Table for x being Sprague's Select Mortality Tables (modified), for y Finlaison's Female Government Annuitants' Analyzed Table, 1884. *Interest* 4 *per-cent.*

Table of $A^{\;1}_{[x][y]}$.

x =	$y=40$	$y=50$	$y=60$	$y=70$	$y=80$
20	·15322	·12687	·09902	·06962	·04486
30	·18574	·14689	·10704	·06961	·04168
40	...	·20997	·14969	·09286	·05286
50	...	...	·23843	·14906	·08332
60	...	...	...	·26462	·15656

Table of $\varpi^{\;1}_{[x][y]}$.

x =	$y=40$	$y=50$	$y=60$	$y=70$	$y=80$
20	·010244	·009537	·009016	·008591	·008066
30	·012663	·011145	·009765	·008573	·007473
40	...	·016685	·014050	·011609	·009543
50	...	...	·023947	·019380	·015344
60	...	...	...	·037767	·030368

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, 1886.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE.
(PART I.)

Examiners.—H. ANSELL, Esq.; W. SUTTON, Esq., M.A.; F. W. WHITE, Esq.

First Paper.

1. Define a vulgar fraction and a decimal fraction. Simplify

$$\frac{\dfrac{1}{2\frac{1}{3}} + 4\frac{1}{4} + 6\frac{1}{6}}{3\frac{1}{3} + 4\frac{1}{4} + 5\frac{1}{6}}$$

Express 14 cwt. 3 qrs. 17 lbs. 5 oz. as a decimal fraction of a ton.

2. At what times are the hands of a clock exactly one over the other?

3. State and prove the rule for finding the value of a mixed circulating decimal. What is the value of $\cdot 2\dot{9}9754\dot{2}$ of £1. 13*s*. 11*d*.?

4. Give an algebraical proof of the rule for finding the greatest common measure of two algebraical expressions.

Reduce to its lowest terms the fraction $\frac{3x^3+5x^2-x+2}{4x^4+9x^3+2x^2-2x-4}$.

5. If α, β, be the roots of the equation $ax^2+bx+c=0$, prove that the roots of the quadratic equation $a^4cx^2+b(b^4-5ab^2c+5a^2c^2)x+ac^4=0$ are $\frac{\alpha^4}{\beta}$ and $\frac{\beta}{\alpha}$.

6. Solve the following equations:

$$(1) \quad \frac{m^2-n}{m-x}+\frac{m+x}{x-\sqrt{n}+1}=2(m+\sqrt{n})$$

$$(2) \quad \frac{5}{x^2-7x+10}+\frac{5}{x^2-13x+40}=x^2-10x+19$$

$$(3) \quad \sqrt{x^2+4x+3}+\sqrt{x^2+2x-3}=x+3$$

7. Define Arithmetic, Geometric, and Harmonic Progression.

(*a*) There are 10 numbers in Arithmetical Progression whose sum is 10, and the sum of whose cubes is $10\frac{10}{11}$. What are the numbers?

(*b*) There are 4 numbers in Geometrical Progression whose sum is 40, and the sum of whose squares is 820. What are the numbers?

8. Define permutations and combinations.

(*a*) How many words of six letters may be formed out of 24 letters of the alphabet, with 2 of the 5 vowels in each word.

(*b*) If ${}_nP_r$ denote the number of permutations of n things taken r together, and $\Sigma({}_nP)$ denote ${}_nP_1+{}_nP_2+\ldots+{}_nP_n$, show that

$$\Sigma({}_{n+1}P)=(n+1)\{\Sigma({}_nP)+1\}.$$

9. Write down the rth term of the expansion of $(a+x)^n$.

Find how many homogeneous products of r dimensions can be formed out of the n symbols a, b, c, . . . &c.

10. Sum the series

(*a*) $1.2x+2.3x^2+3.4x^3+\ldots$ to n terms.

(*b*) $1.3x+4.9x^2+7.27x^3+\ldots$ to infinity where x is <1.

11. Define a *logarithm*.

Enunciate and prove the Exponential Theorem.

Prove that

$$n=1+\log_\epsilon n+\frac{\overline{\log_\epsilon n}|^2}{\underline{|2}}+\frac{\overline{\log_\epsilon n}|^3}{\underline{|3}}+\text{\&c.}$$

12. Describe the various books necessary in ordinary mercantile book-keeping by double entry.

Explain the process described as a *trial balance.*

State exactly the difference between book-keeping by single and by double entry.

It is optional on the part of the Candidate to answer one or more of the following questions; but due weight will be given to answers sent in.

13. Define
A point.
A straight line.
A circle.
Parallel straight lines.

The three angles of a triangle are together equal to two right angles.

14. In any right-angled triangle the square described upon the side subtending the right angle is equal to the squares described upon the sides which contain the right angle.

15. Distinguish between the angle in a segment of a circle, and the angle of a segment of a circle.

Angles in the same segment of a circle are equal to one another.

Second Paper.

1. What is meant by the modulus of a system of logarithms? What relation exists between the moduli of systems whose bases are in geometrical progression?

$$\text{Given } \log_{10}2 = \cdot 3010300$$
$$\log_{10}3 = \cdot 4771213$$

Find $\log_5 1458$, $\log_8 19440$, $\log_9 230400$ and $\log_{12} 38\cdot 88$.

2. Show that for all real values of x the expression ax^2+bx+c has the same sign as a, except when the roots of the corresponding equation $ax^2+bx+c=0$ are real and different, and x lies between them.

For what values of x is $12x^2 < 11bx + b^2$?

3. Solve the equations:

$$(1)\quad \left.\begin{aligned}(x^2+a^2)(y^2+b^2)&=m(xy+ab)^2\\ (x^2-a^2)(y^2-b^2)&=n(bx-ay)^2\end{aligned}\right\}$$

$$(2)\quad \left.\begin{aligned}x^3+y^3+z^3&=3xyz\\ x-a=y-b&=z-c\end{aligned}\right\}.$$

4. Find all the roots of the equation

$$x^4-15x^3+70x^2-120x+64=0,$$

the roots being in geometrical progression.

5. Form the equations whose roots are

$$(1)\quad \pm 4\sqrt{3} \text{ and } 5 \pm 2\sqrt{-1}$$
$$(2)\quad \pm\sqrt{2} \text{ and } \pm\sqrt{-3}.$$

6. A sum of £700 was divided between 4 persons whose shares were in geometrical progression, and the difference between the greatest and least was to the difference between the means as $3{\cdot}08\dot{3}$ to 1. What were the respective shares?

7. If $a+b+c=0$, show that $(a^2+b^2+c^2)^2=2(a^4+b^4+c^4)$.

8. One of two events must happen. The odds against one event are 7 : 5. What is its chance of happening?

A and B are two exclusive events, p is the chance of A, and q of B, happening on any one of a succession of occasions when either A or B must happen. Find the chance that in any given sequence of n occasions A will happen r times and B $n-r$ times. What is the chance that in any sequence of n occasions, A will occur at least r times?

9. A certain sum is to be won by the first person who throws head with a penny.

If there be m throwers, find the chance of the rth person.

10. Given

$$\begin{aligned} u_0 &= 100{,}000 \\ u_4 &= 98{,}391 \\ u_5 &= 98{,}011 \\ u_6 &= 97{,}615 \end{aligned}$$

construct the series from u_{10} to u_{15}, assuming that third differences are constant.

11. Given u_0, u_1, u_2, u_3, u_4, and u_5, and assuming fifth differences to be constant, show that

$$u_{2\frac{1}{2}} = \frac{c}{2} + \frac{25(c-b)+3(a-c)}{256}$$

where

$$\begin{aligned} a &= u_0+u_5 \\ b &= u_1+u_4 \\ c &= u_2+u_3 \end{aligned}$$

It is optional on the part of the Candidate to answer one or more of the following questions; but due weight will be given to answers sent in.

12. Explain the term *locus* as used in geometry, and show that the locus represented by the equation $\mathrm{A}x+\mathrm{B}y+\mathrm{C}=0$ is a straight line.

13. At what point will the straight lines respectively represented by the equations

$$\begin{aligned} \mathrm{A}x + \mathrm{B}y + \mathrm{C} &= 0 \\ \mathrm{A}'x + \mathrm{B}'y + \mathrm{C}' &= 0 \end{aligned}$$

intersect one another?

Hence deduce the condition for the line to be parallel.

14. Find the conditions necessary in order that when the axes of co-ordinates are rectangular the general equation of the second degree

$$\mathrm{A}x^2+\mathrm{B}xy+\mathrm{C}y^2+\mathrm{D}x+\mathrm{E}y+\mathrm{F}=0$$

may represent a circle. What will be the length of the radius?

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW.
(PART II.)

Examiners.—G. F. HARDY, Esq.; G. KING, Esq.; W. VAUGHAN, Esq..

First Paper.

1. Six persons, A, B, C, D, E, F, are of the same age; the rate of mortality at that age being 1 per-cent. What is the probability (to five decimal places):

(α) That they will die in an assigned order?

(β) That A will die first and F last, the remaining order not being fixed?

(γ) That A will die in the first year and be the first to die?

2. The value of an annuity of £1 per annum for n years certain, when interest is convertible p times per annum and the annuity is payable m times per annum, is

$$\frac{1-\left(1+\frac{i}{p}\right)^{-np}}{m\left[\left(1+\frac{i}{p}\right)^{\frac{p}{m}}-1\right]}.$$

Prove this by general reasoning; and show what the formula becomes when m is indefinitely great.

3. Having given the value of $a_{\overline{20}|}$ at $4\frac{1}{2}$ per-cent$=13{\cdot}0079$ and at 5 per-cent$=12{\cdot}4622$; find approximately the rate of interest yielded by an annuity for 20 years, in which the payments are successively 20, 19, 18, &c., when purchased for £150.

4. If P represents the present value of £1 to be received at the end of the year in which a certain status may fail, and Q the present value of £1 per annum to be received during its continuance; prove that $P=\frac{1-iQ}{1+i}$.

Hence show that $(1+i)M_x+iN_x=D_x$.

5. A shareholder in a life company holds £2,000 of its paid-up capital, the dividends on which are increased 10 per-cent every quinquennial valuation. Supposing a valuation to have just taken place and the dividends for the next 5 years to be fixed at £100 per annum, what is the value of his interest in the undertaking upon a 5 per-cent basis?

6. If a life company experiences a mortality in exact accordance with the table adopted in its calculations, and realizes the precise rate of interest it has anticipated, prove that the net premium for any age, x, amounts in n years to $\frac{a_x-a_{x+n}}{1+a_x}$, the claims of each year being duly provided for.

7. Give Mr. Woolhouse's formula for the value of $a_x^{(m)}$. What does this represent when m is fractional and $=\frac{1}{n}$; and how may the formula in this shape be employed to approximate to the values of annuities involving any number of lives?

8. A body of men, kept of uniform strength by annual entrants engaged at age 20, and superannuated at age 60, are grouped into 3 classes, the numbers in the classes being as 3 in the junior, 2 in the intermediate, and 1 in the senior. Assuming promotion to go by seniority, how can a mortality table be used for finding the average age of promotion from one class to another? What causes are in operation to vitiate the results?

9. In two mortality tables, (A) and (B), the probability of living a year by the latter table is throughout less in a constant ratio than by the former. What relation will exist between the policy-values by the two tables?

10. Given a table of the expectation of life at each age, how would you obtain the corresponding mortality table; and what means would you employ for testing the correctness of the work? Prove that the expectations in the case of a stationary population may be determined solely from a return of the ages of the inhabitants of the place.

11. Describe briefly the object of the $\mathrm{H}^{\mathrm{M}(5)}$ Mortality Table, the materials upon which it is based, and the mode of using it for the purposes of a life office valuation.

12. Show how successive cash bonuses may be applied to limit the term for which the premiums are payable under a whole-term assurance.

13. A life company divides its business into four classes, with a view to appropriating the profits of each class exclusively to the policyholder in that division. What form of Cash Book would you suggest should be used under these circumstances?

The following question is optional on the part of the Candidate.

14. State Taylor's Theorem, and show that the Binomial Theorem is a special case of it.

Second Paper.

15. Show that the sinking fund to redeem a debt of κ in n years will in t years $(t<n)$ accumulate to

$$\kappa\cdot\frac{a_{\overline{n}|}-a_{\overline{n-t}|}}{a_{\overline{n}|}}.$$

16. By paying a certain rate per annum, i, of interest in quarterly instalments, that rate is increased by ·017 of itself. Explain in what way you would obtain a series of approximations to the value of i, performing the first of them.

17. Show how to obtain the probability that x will die in the lifetime of y, Q^1_{xy}; and give Mr. Woolhouse's formula for this probability. Explain why $\mathrm{Q}^1_{xy}\times\mathrm{Q}^1_{xz}$ does not equal Q^1_{xyz}.

18. The number of deaths at the various ages in one year in a large district unaffected by migration are compared with a suitable mortality table; and it is found that the ratio of the actual deaths at each age to those of the table for the same age constantly diminishes as the age increases. In what way is this to be accounted for; and how far does the same cause of disturbance explain the defects of Dr. Price's Northampton Table?

19. Show how the value of an annuity payable so long as any one of 3 lives x, y, z, may be living can be approximately found from the "Institute Life Tables."

20. Find the value of a temporary annuity payable for m years if (x) survive, or for n years if (y) survive, (x) having died within the n years. Assume $n<m$.

21. The yearly rents of an estate belong in equal shares to four sisters, being equally divided at the end of each year amongst all of the sisters who survive that year, the last survivor taking the whole for the rest of her life. Find in terms of the single and joint-life annuities an expression for the present value of the interest of any one of them. Write down the general formula for the same thing, when the number of persons is n.

22. How would you approximate to the annual premium for an assurance payable on the death of a person aged 48, in the event of his dying before the survivor of two persons aged respectively 75 and 70, or within 1 year after the death of such survivor?

23. Show that the single premium for an assurance, with return of the premium along with the sum assured, is equal to the value of all the future fines on successive lives, where the lives are all to be nominated of the same age as that at present of the life in possession. Explain verbally why this should be so.

24. Show how by a continuous process to construct commutation columns, M^1_{xy}, for survivorship assurances.

25. A company proposes to calculate its premiums to allow for a reversionary bonus at the rate of 1 per-cent per annum on the sum assured and all previous bonuses. Show how to constuct commutation columns so that the annual premium may be obtained by a single division.

26. There are 44 policies of 1 each all effected at age 40, and having been in force 1, 2, 3, &c., up to 44 years respectively. The sum of their values is 17·52789. Find the values separately of the aggregate sums assured, and of the future net premiums having given $A_{40}=\cdot379434$.

The following questions are optional on the part of the Candidate.

27. Show that when fourth differences are constant, $\dfrac{d^3u_x}{dx^3}=\Delta^3u_{x-\frac{3}{2}}$.

28. Explain in what way integration is equivalent to the summation of an infinite number of infinitely small terms; and integrate

$$x^3; \quad \sqrt{a^2-x^2}; \quad x\log x.$$

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW.
(PART III.)

Examiners—W. HUGHES, Esq.; A. W. SUNDERLAND, Esq., M.A.; T. E. YOUNG, Esq., B.A.

First Paper.

1. Describe Mr. Berridge's method of Graduation applied to the Peerage Mortality Table of Messrs. Bailey and Day, and sketch the process by which Mr. Berridge's equations may be obtained.

2. What mortality table would you use if you were called on to calculate for an ordinary life office a set of tables of premiums for single-life insurances?

What objections are there to the use of the H^M Table for this purpose?

3. What are the chief points to be considered in forming a scheme for distributing the policyholders' share of the profits of a life office?

An office which always makes a net premium valuation by the H^M Table of Mortality divides the profit shown at each valuation among the participating policies in proportion to the premiums paid on them during the valuation period. Under what circumstances is this mode of distribution fairly equitable?

4. How would you proceed to investigate the solvency of a life office?

A life office having been found insolvent, how would you treat the following assets:

(1) Half-credit premiums.
(2) Loans on policies within their surrender-values.

5. State the provisions of the Married Women's Property Act, 1870, in regard to Life Assurance Policies. How are these provisions modified by the Act of 1882?

Can you suggest any improvements in the eleventh section of the Married Women's Property Act of 1882?

6. A policy is conveyed to trustees. On the claim arising, the solicitor to the trustees presents the policy, properly discharged, and a letter to the office signed by the trustees requesting that the sum assured be paid to him. What recent important decision in the Court of Appeal bears upon the case? What course should be adopted by the office?

7. State concisely the state of the law as prescribing the circumstances under which one person may insure the life of another.

How is the insurable interest of a creditor in the life of his debtor affected by the repayment of the debt?

Why is it that, in cases of insurance by one person on the life of another, the insurance office usually makes enquiries as to the object of the insurance?

8. Give a short description of the English National Debt as at present existing. What arrangements for its gradual extinction are in operation?

9. A scarcity of the standard is considered to be one of the chief causes of the present state of low prices. An issue of £1 notes has been suggested as a remedy for this scarcity. State, generally, the advantages of such an issue, and what effect it would be likely to have upon prices. Discuss, from an economic point of view, the expediency of such an issue as a remedy in present circumstances.

10. An absolute reversion, falling in on the death of a person aged 55, to a trust fund invested in British Government Stock, English Railway Debenture and Ordinary Stocks, Foreign Government Stocks, Shares in Foreign Railways, Mortgages of English Freeholds and Leasehold Houses in London, is offered for sale to an Insurance Company. On what principles would you value the various securities for the purpose of estimating the price which should be given for the reversion, and what enquiries concerning the securities should be made by the Company before it enters into a contract to buy?

11. A, aged 45, has a reversionary life interest in a freehold estate expectant on the decease of B, aged 75. A has also, in the event of his predeceasing C, aged 25, the reversion to the fee simple of the estate expectant on his own decease and that of B. A wishes to raise £2,000 on his interest, by way of reversionary charge. State the amount and nature of the reversionary charge, and the insurance required:

(i) If the estate produces £20,000 a year.

(ii) If the estate produces £400 a year.

12. A, born 30th April 1861, has a life interest in £1,000 a year (amply secured and payable free of any deductions) in reversion expectant on the death of B, born in January 1816, provided B leaves no issue male. B has a wife, aged 60, in good health, and has no issue. Find the market value of A's interest assuming the following rates of premium:

£1.	18*s.*	0*d.*	per-cent per annum for ordinary non-profit insurance on A's life.
0	10	0	per-cent single premium for making the above insurance whole world.
4	0	0	per-cent single premium for insurance against issue to B.

NOTE.—Candidates may refer to *Jones on Annuities* for any further data required for the solution of this problem.

It is optional on the part of Candidates to answer the following questions.

13. A sum of money is invested for *n* years at compound interest, at nominal rates fluctuating between $2\frac{1}{2}$ and 4 per-cent per annum, the interest being added to the principal from day to day.

Find an expression for the accumulated amount.

14. $m+n$ persons aged x are taken under observation for a year, within which time n of them die.

Find the probability, as deduced from the above observations, that a person aged x will die in a year's time.

Second Paper.

15. Describe the mode of construction of Mr. Sprague's Select Mortality Tables.

16. The mortality likely to prevail in the future among the lives insured in an office is deduced from its past experience. Criticize the effect of the element "Survivors."

Describe the method of treating this element adopted by certain American Actuaries, and discuss the value of this method.

17. Discuss the merits of that method of dividing profits known as the "uniform reversionary bonus system."

18. Criticize the following methods of valuing the liabilities of an ordinary life office:

(i) A Carlisle 4 per-cent pure-premium valuation.

(ii) A valuation at 3 per-cent in which the H^{M} pure premiums are valued, and the tables of mortality for reversions and annuities are, for policies less than five years old the H^{M}, for policies more than five years old the $H^{M(5)}$.

(iii) A valuation in which the full premiums are valued, the reversions and annuities used in valuing being such as correspond at 3 per-cent interest with the premiums charged by the office.

19. Give an account of the legislation of 1870 and 1872 dealing with the subject of Insolvency in Life Offices.

20. Explain the difference between the interest of a tenant-for-life and a tenant-in-tail in an estate.

What is the method of raising money on his interest usually adopted by a tenant-in-tail in remainder? What risk is run by the lenders in this method of dealing with a tenant-in-tail's interest?

21. Explain the difference between the bank rate of discount and the rate of interest yielded by first-class Stock Exchange securities, mentioning the chief causes which affect these rates.

Why is the average rate of interest on first-class mortgages higher than the rate of interest yielded by first-class debenture stocks?

22. What is the system of English Banks in regard to Cash Reserves? What remuneration does the Bank of England get for its responsibility in the matter?

How is the Reserve affected by the rates of exchange? How does the Bank of England act in order to make the bank rate effective in the money market?

23. What is meant by the depreciation of silver, and what are the chief causes assigned for it?

Explain how the difference between standard and token money is illustrated by the rupee and English florin.

An English company with the bulk of its investments in British Securities issues policies in India in which the premiums and sums assured are payable in rupees. How would you pass the items of this business through the Annual Accounts, and how would you reserve for such policies at the valuations?

24. A, aged 35, if living at the death of B, aged 70, will then become entitled to the balance of a fund of £10,000, after the following charges have been satisfied thereout:

(1) £1,000 to be paid on the death of B to C (now aged 25) if then living.

(2) An annuity of £100 a-year, payable after the death of B, to a lady now aged 25 for the remainder of her life, and after her death to any husband she may leave behind her for the remainder of his life. (The lady has a husband now aged 40).

The £10,000 is a first charge on a fund in Court amply sufficient to secure it.

How would you estimate the market value of A's interest, and what rate of interest would you use in the calculations?

25. On what general principles would you frame a scheme of surrender-values of ordinary policies?

(i) For a Mutual Society.

(ii) For a Proprietary Company.

26. The manager of a Life Insurance Company is instructed by his directors to report to them on the character, profitableness, and progress of the business of the various branches of the Company. Sketch in detail the points which should be considered in his report.

It is optional on the part of Candidates to answer the following questions.

27. If the force of mortality at age x is a function of the form mq^x where m and q are constants, find an expression for l_x, the number living at age x.

28. Show how Σu_x may be developed in a series proceeding by the differential coefficients of u_x, and prove that no differential coefficient of an even order will appear in the expansion.

Find a formula by means of which Bernoulli's numbers may be successively calculated.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1885–6.

First Ordinary Meeting, 30 *November* 1885.

The President (Mr. T. B. SPRAGUE) in the Chair.

The following gentleman was elected a Fellow of the Institute:

Mackenzie, Alexander George.

The following gentleman was elected an Associate of the Institute:

Lidbury, Isaac Stephen.

The President delivered an inaugural address.

Second Ordinary Meeting, 21 *December* 1885.

The President (Mr. T. B. SPRAGUE) in the Chair.

Mr. A. H. Bailey read a paper on "The Income Tax, especially with reference to its Incidence on Assurance Companies."

The following gentlemen took part in the discussion:—Messrs. Ackland, Colquhoun, A. Day, C. D. Higham, G. King, Lancaster, Priestley, and the President.

Third Ordinary Meeting, 25 *January* 1886.

Mr. M. N. ADLER (Vice-President) in the Chair.

Mr. W. T. Gray read a paper on "Average Rates of Mortality as affected by the Grouping of the Numbers exposed to Risk at different Ages."

The following gentlemen took part in the discussion:—Messrs. Crisford, G. King, Manly, Rusher, Ryan, and the Chairman.

Fourth Ordinary Meeting, 22 *February* 1886.

The President (Mr. T. B. SPRAGUE) in the Chair.

Mr. G. S. Crisford read a paper on "Office Premium Loadings, and how they are dealt with in Valuation and Distribution of Profits.—Part II."

The following gentlemen took part in the discussion:—Messrs. Ackland, Adler, J. Chisholm, S. Day, W. T. Gray, Manly, Ryan, and the President.

Fifth Ordinary Meeting, 29 *March* 1886.

Mr. M. N. ADLER (Vice-President) in the Chair.

Mr. Jas. Chisholm read a paper on "The Assessment of Life Risks."

The following gentlemen took part in the discussion:—Drs. Bennett, Hall, Pollock, Thomson, Thornhill, and Tirard; and Messrs. Colquhoun, Humphreys, Ryan, Whittall, and the Chairman.

Sixth Ordinary Meeting, 19 *April* 1886.

The President (Mr. T. B. SPRAGUE) in the Chair.

The admission of the following gentleman as a Fellow of the Institute was proposed:

Tennant, John Bell.

The admission of the following gentlemen as Associates of the Institute was proposed:—

Buckley, Thomas John Westley.
Chatham, James.
Hume, Alexander Sherman.

The President read a paper on "The Graphic Method of Constructing Mortality Tables."

The following gentlemen took part in the discussion:—Messrs. Ackland, Bailey, Burridge, A. J. Finlaison, C. D. Higham, G. King, and Manly.

The Thirty-ninth Annual General Meeting, 5 *June* 1886.

The President (Mr. T. B. SPRAGUE) in the Chair.

Mr. H. W. MANLY (hon. sec.) read the circular convening the meeting, the minutes of the ordinary meeting held in April (which were signed as correct by the President), and the following Report of the Council, and Statement of Accounts:

"On the occasion of meeting the members at this, the Thirty-ninth Annual General Meeting of the Institute since its foundation, and the second since its incorporation by Royal Charter, the Council have much pleasure in reporting that the Bye-Laws, which were prepared by the Council, and adopted by a General Meeting of the members held on 10 October last, were, with a few immaterial alterations, allowed by the Lords of the Privy Council on 26 February last; and that all the formalities required by the Charter having been duly observed, the Institute has now fairly started on its new career of usefulness as an incorporated society.

"Under the provisions of the Charter the new Class of Students has been formed, and 24 gentlemen have up to this time been admitted to it.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
The number of members existing on the list in each class on 31 March 1885 was as follows .	3	153	269	0	9	434
These were diminished by						
Deaths	0	6	1	0	0	23
Transfer by Examination	0	0	4	0	0	
„ Election .	0	0	1	0	0	
Resignations . . .	0	0	4	0	0	
Default in Payment of Subscriptions . .	0	0	7	0	0	
	3	147	252	0	9	411
And increased by						
Election . . .	0	1	1	24	0	30
Transfer by Examination	0	4	0	0	0	
Leaving on the Books at the close of the financial year, 31 March 1886 . .	3	152	253	24	9	441

"The financial position of the Institute continues to be very satisfactory. The Accounts, which have been duly audited, show that the income of the year was £1,148. 14*s*. 9*d*., being £37. 8*s*. 8*d*. more than last year; and that the expenditure was £802. 11*s*. 4*d*.; so that a sum of £346. 3*s*. 5*d*. has been added to the total funds, which now stand at £3,357. 5*s*. 5*d*. It should, however, be mentioned that since these accounts were made up, a sum of £194. 18*s*. 11*d*. has been paid which was incurred in connection with the passing of the Bye-Laws; and that Part II of the *Text-Book* will involve a heavy expenditure, no portion of which has yet been paid; so that the above-mentioned increase in the funds may be considered as a reserve to provide for charges already incurred.

"The Council have decided to divide Part II of the *Text-Book* into two volumes; and they have much pleasure in reporting that the first of these volumes is nearly ready, and will probably be published in the course of the current year.

"The ordinary sessional meetings have been well attended. At the first meeting, held on 30 November 1885, the President (Mr. T. B. Sprague) delivered an opening address; and at the subsequent meetings the following papers were read and discussed, namely:

"21 *December* 1885—'On the Income Tax, especially in reference to its incidence on Assurance Companies'—by Mr. A. H. Bailey.

"25 *January* 1886—'On Average Rates of Mortality as affected by the grouping of the numbers exposed to risk at different ages'—by Mr. W. T. Gray.

"22 *February* 1886—'On Office Premium Loadings, and how they are dealt with in Valuation and Distribution of Profits; Part II'—by Mr. G. S. Crisford.

"29 *March* 1886—'On the Assessment of Life Risks'—by Mr. James Chisholm.

"19 *April* 1886—'On the Graphic Method of constructing Mortality Tables'—by the President (Mr. T. B. Sprague).

"The examinations of the Institute were held on 30 April and 1 May; and from the reports of the examiners it appears that for the First Part thirty-two candidates presented themselves, of whom seven passed, namely:

Beames, F.
Faulks, J. E.
Foot, H.
Griffin, R. W.
Hazell, J. S.
Mullins, B. W.
Newman, P. L.

"For the Second Part, fourteen candidates presented themselves, of whom seven passed, namely:

Bell, F.
Cooper, W. G.
Lemon, W. K.
Longdon, A. E.
Milner, J. W.
Pulley, W. P.
Woods, A. B.

"And for the Third Part, eleven candidates presented themselves, of whom four passed, namely:

Berry, B. A.
Dickinson, A. L.
Schooling, F.
Simon, L. M.

"In all these cases the names of the successful candidates are, in accordance with a recent resolution of the Council, arranged in alphabetical order.

"The Council cannot allow the occasion to pass without expressing their deep sense of the loss which the Institute has sustained by the deaths of Mr. W. B. Hodge and Mr. Cornelius Walford. Mr. Hodge was one of the first members of the Institute, and contributed some valuable papers to the earlier volumes of the *Journal*. He always took a lively interest in the affairs of the Institute, and was for two years (1870-1, 1871-2) its President. Mr. Walford was a voluminous writer on all subjects relating to insurance; and he took an active part in the discussions at the sessional meetings of the Institute. Almost his last work was the 'Essay on the History of Life Assurance', for which the Institute awarded him the 'Samuel Brown' First Prize."

INSTITUTE OF ACTUARIES.

Income and Expenditure for the Year ending 31 *March* 1886.

Dr.

1885-6.	£	s.	d.	£	s.	d.
Amount of Funds, 31 March 1885—						
Messenger Legacy Fund	299	16	9			
Brown Prize Fund	233	8	11			
General Fund	2,477	16	4			
				3,011	2	0
Annual Subscriptions—						
Town Fellows	290	17	0			
Country ,,	87	3	0			
Town Associates	328	13	0			
Country ,,	108	3	0			
Students' Fees	25	4	0			
Students' Subscriptions	25	4	0			
				865	4	0
Dividends				92	18	0
Interest—*Nil*					..	
Overpaid by Bank, as per last Account				0	9	0
Mortality Experience Sales	8	16	3			
Institute of Actuaries' Life Tables Sales	9	15	3			
				18	11	6
Journal Sales				100	11	3
Text-Book, Part I, Sales				14	11	0
Logarithm Card Sales				0	17	0
Examination Fees				55	13	0
				£4,159	16	9

Cr.

1885-6.	£	s.	d.	£	s.	d.
Rent	160	0	0			
Salaries	140	0	0			
Journal Expenses	242	19	7			
Examination Expenses	66	18	0			
Stationery	35	13	6			
Library Expenses	25	0	5			
Ordinary Meeting Expenses	27	16	2			
Bye-Laws Expenses	65	18	0			
House Expenses	15	12	0			
Income Tax on Salaries	2	0	0			
Fire Insurance Premium	0	15	0			
Postage	16	2	6			
Miscellaneous	5	1	2			
				802	11	4
Amount of Funds, 31 March 1886—						
Messenger Legacy Fund	305	19	3			
Brown Prize Fund	241	3	11			
General Fund	2,810	2	3			
				3,357	5	5
				£4,159	16	9

Balance Sheet, 31 *March* 1886.

Dr.

	£	s.	d.	£	s.	d.
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	203	17	8			
Unappropriated Dividends	102	1	7			
				305	19	3
Brown Prize Fund, £200 Metropolitan Railway 4 per-cent Debenture Stock, cost	200	0	0			
Unappropriated Dividends	41	3	11			
				241	3	11
General Fund				2,810	2	3
				£3,357	5	5

Cr.

	£	s.	d.	£	s.	d.
£1,600 Consols, cost				1,498	19	3
£1,200 Metropolitan Railway 4 per-cent Debenture Stock, cost				1,225	17	3
Cash at London and Westminster Bank				598	0	0
Cash in hand				14	9	11
Arrears of Subscription:—						
Town Fellow	3	3	0			
Country Fellows	4	4	0			
Town Associates	6	6	0			
Country Associates	6	6	0			
				19	19	0
				£3,357	5	5

Examined and found correct,

(Signed) Geo. B. Bellamy,
Alfred B. Adlard,
Edward A. Rusher, } *Auditors.*
15 *April* 1886.

The President, after adverting to the stride made by the Institute during the past 12 months, and to its now-recognized position as an Incorporated Body, moved the adoption of the Report and Accounts; and the motion having been seconded by Mr. G. Humphreys, they were unanimously adopted.

At the ballot which followed, the Scrutineers, Messrs. E. Justican and G. H. Ryan, reported the following to be the list of President, Vice-Presidents, Council and Officers, elected to serve during the ensuing year:

President.

Archibald Day.

Vice-Presidents.

Edward Docker, M.A.
Henry William Manly.
Benjamin Newbatt.
William Sutton, M.A.

Council.

*Thomas G. Ackland.
Marcus N. Adler, M.A.
Arthur H. Bailey.
George William Berridge.
*David A. Bumsted.
James Chisholm.
*Thomas H. Cooke.
George Stephen Crisford.
Archibald Day.
*David Deuchar.
*Edward Docker, M.A.
Alexander John Finlaison.
*Richard C. Fisher.
John Ralph Grimes.
Ralph Price Hardy.
*Augustus Hendriks.
*Charles D. Higham.
George Humphreys, M.A.
George King.
Henry William Manly.
*Benjamin Newbatt.
Thomas Bond Sprague, M.A.
*Thomas Y. Strachan.
William Sutton, M.A.
*James Terry.
*Spencer C. Thomson, B.A.
*George Todd, M.A.
John Hill Williams.
*Frank B. Wyatt.
Thomas Emley Young, B.A.

Note.—Those marked * are new members.

Treasurer.

George Humphreys, M.A.

Honorary Secretaries.

George Stephen Crisford.
Thomas Emley Young, B.A.

Mr. W. K. Lemon moved, and Mr. J. Miller seconded, the election of Mr. Adey, Mr. Adlard, and Mr. Rusher, as auditors; and the motion was agreed to.

On the motion of Mr. B. Newbatt, seconded by Mr. E. Justican, a cordial vote of thanks was passed to the President, Vice-Presidents, Council and Officers, for their services during the past two years, and received a brief acknowledgment in each case.

The proceedings then terminated.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Graphic Method of Adjusting Mortality Tables.—A description of its objects, and its advantages as compared with other methods, and an application of it to obtain a Graduated Mortality Table from Mr. A. J. Finlaison's Observations on the Mortality of the Female Government Annuitants, 4 years and upwards after purchase. By T. B. Sprague, M.A., *President of the Institute of Actuaries.*

[Read before the Institute, 19 April 1886.]

I HAVE on various occasions stated that, in my opinion, the graphic method is the best one to employ in graduating a mortality table; and I have more than once had it in contemplation to explain fully the grounds for this opinion, describing the advantages of the graphic method, and the objections I see to the use of some other methods which are commonly employed. In order to do this satisfactorily, it is clearly essential to give a graduation of some mortality observations; and my attention having been again drawn to the subject by reading Mr. Higham's brief, but extremely interesting and suggestive paper, *On the Graduation of Mortality Tables* (*J.I.A.* xxv, 15), in which he speaks unfavourably of the graphic method, I think that an examination of his reasoning and results will furnish me with the most suitable opportunity I could have for stating my views on the whole subject.

Mr. Higham's paper commences with a reference to the report I made in August 1883, on Mr. A. J. Finlaison's new tables, which show in an improved form, the mortality among the Government Life Annuitants: (see the Parliamentary Return entitled *Papers relating to the Government Annuities Act* 1882, ordered by the House of Commons to be printed, 7 February 1884); and I think it will be convenient to my readers if I begin by making a few extracts from the reports of Mr. Finlaison and myself, as given in the Return. Mr. Finlaison says (p. 13): " The most satisfactory system of adjustment of Tables of " Mortality at present known, when a sufficiently large number " of observed facts have to be dealt with, is that devised by Mr. " W. S. B. Woolhouse for the construction of the Table of " Mortality derived from the experience of Life Assurance " Companies by the Institute of Actuaries; but the method " becomes inapplicable unless the numbers involved are so large " as to themselves produce a series which demands no more than " a slight rectification of its progression. When the number of " persons under observation is limited, * * * * * other " means for the adjustment of the death-rates must be employed " to arrive at an orderly progressive series.

" The criterion which should be applied, however, to whatever " system of adjustment may be adopted, consists in the derived " series being found to return an equivalent number of deaths to " the original facts, when the adjusted death-rate is assumed to " be experienced by a body of persons exactly similar to that " upon which the table is founded."

And on page 14: " The Tables for each of the first four years " of the currency of the annuities, have been adjusted, as the " numbers under observation were relatively small, in the following " manner: in the first place, an adjustment was made according " to the formula well known among actuaries by the name of its " author, Mr. Benjamin Gompertz, F.R.S., according to which the " number of persons living at any age of life (x) is to be repre- " sented by the algebraical expression $a . b^{c^x}$, in which a, b, and c, " are constants deduced from the original facts. The death-rate " obtained from such preliminary series, was then applied to the " actual original number of persons exposed to the risk of " mortality at each age; and the number of deaths obtained from " the computation, compared with the deaths that actually occurred, " in groups of short intervals of age. The constants in the " formula were then varied at such intervals of age, in a manner

" which tended to bring the deaths returned by the final series " obtained by this means, into an agreement with the actual " original facts.

" In the case of the Tables for four years and upwards, when " the numbers under observation were of sufficient extent, the " method of Mr. Woolhouse, already referred to, has been used.

" The object in view throughout the whole of the computations, " has been that the results to be obtained from the adjusted series " of death-rates, should correspond, as closely as the nature of the " case would permit, with the deductions primarily made from the " crude facts."

I entirely accept the tests of good graduation here described by Mr. Finlaison. In the first place, the adjusted facts must form an orderly progressive series, that exhibits no breaks or irregularities; and, secondly, the graduated series must in the aggregate yield the same results as the ungraduated; for instance, the same number of deaths when the numbers exposed to risk are identical. But I say with regard to the process which Mr. Finlaison has adopted, firstly, that although Mr. Gompertz's formula is very suitable for terms of years of greater or less duration, it is not applicable, with the same values of constants, throughout the whole of life; and that the adoption of new constants at certain points, causes a break in the regularity of the series at each of those points; and secondly (p. 74), " If we take " Mr. Finlaison's adjusted rate of mortality for females [being the " figures quoted by Mr. Higham on pages 20 and 21 of his paper], " and write down the differences, we shall find that for certain " series of years they run very smoothly, say from age 40 to age " 50; but that for other series of years, say from 50 to 60, they " run more irregularly, increasing and decreasing without any law. " This also clearly appears from the form of curve as given in " diagram* No. 3; and these irregularities are a consequence of " the use of Mr. Woolhouse's method of adjustment, and an " inseparable defect attaching to the use of it. The result of my " own study of the subject of graduation of tables, is, that the " most satisfactory system of adjustment is the graphic method. " This assumes no formula like Mr. Gompertz's method, and no " rigid course of procedure like Mr. Woolhouse's; but the " computer first draws a smooth curve representing approximately " the series of values to be adjusted, and is then at liberty to

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* This is Diagram No. 1 of the present paper.

“ amend it by successive trials until it satisfies sufficiently both of “ the conditions of good graduation.”

Mr. Higham in effect admits the justice of my criticism of Mr. Woolhouse's method, and proposes formulas of his own, which (he says) give results that are on the whole very similar to Mr. Woolhouse's, but proceed with greater regularity.

Mr. Woolhouse's graduation formula may be written in the following form:

$$\begin{aligned} U_x = {\cdot}200u_x &+ {\cdot}192(u_{x-1}+u_{x+1}) + {\cdot}168(u_{x-2}+u_{x+2}) \\ &+ {\cdot}056(u_{x-3}+u_{x+3}) + {\cdot}024(u_{x-4}+u_{x+4}) \\ &- {\cdot}016(u_{x-6}+u_{x+6}) - {\cdot}024(u_{x-7}+u_{x+7}). \end{aligned}$$

We see from this, that if we have a series of unadjusted numbers,

$$u_0,\, u_1,\, u_2,\, \ldots\ldots\, u_{x-1},\, u_x,\, u_{x+1},\, \ldots\ldots\ldots,$$

the adjusted number U_x, corresponding to u_x, is found by means of u_x and the 7 preceding and 7 succeeding numbers, taking the percentages of each indicated by the formula. Mr. C. Ansell, in his *Statistics of Families*, page 21, says that the effect of the formula “ is to cause the adjusted number dying at any given age “ to be composed of the following percentages of the unadjusted “ numbers dying at that and contiguous ages, namely:

	At each age.		Total.	
“ At the given age	20	per-cent	20	per-cent
“ + Ages 1 year older and 1 year younger	19·2	„	38·4	„
“ + „ 2 „ „ 2 „	16·8	„	33·6	„
“ + „ 3 „ „ 3 „	5·6	„	11·2	„
“ + „ 4 „ „ 4 „	2·4	„	4·8	„
			108·0	„
“ − „ 6 „ „ 6 „	1·6	„	3·2	„
“ − „ 7 „ „ 7 „	2·4	„	4·8	„
		Total	100·0	„

“ This formula yields results the progressive differences in “ which are sufficiently regular; and, except at the beginning and “ end of the table to which it may be applied, it has the advantage “ of maintaining exactly the aggregate mortality indicated by the “ unadjusted numbers, only altering its distribution at various “ ages. It has the disadvantage, however, of making the result “ for any age, dependent on the observed mortality of so long a “ period of life as 7 years before and 7 years after that age, or 15 “ years altogether.”

In practice it will cause no misunderstanding if we omit the subscript x's in Mr. Woolhouse's formula, and write it

$$\begin{aligned} U = {\cdot}200u + {\cdot}192(u_{-1}+u_{+1}) + {\cdot}168(u_{-2}+u_{+2}) + {\cdot}056(u_{-3}+u_{+3}) \\ + {\cdot}024(u_{-4}+u_{+4}) - {\cdot}016(u_{-6}+u_{+6}) - {\cdot}024(u_{-7}+u_{+7}). \end{aligned}$$

Mr. Ansell himself employs a formula which may be similarly written

$$U = \cdot 17\dot{3}u + \cdot 16(u_{-1}+u_{+1}) + \cdot 12(u_{-2}+u_{+2}) + \cdot 08(u_{-3}+u_{+3})$$
$$+ \cdot 04(u_{-4}+u_{+4}) + \cdot 01\dot{3}(u_{-5}+u_{+5}).$$

Mr. Higham gives us the choice between two formulas. In the one which he describes as the most easy in working, the graduated central value is

$$U = \frac{2\Sigma - 3S}{125}$$
$$= \cdot 200u + \cdot 192(u_{-1}+u_{+1}) + \cdot 144(u_{-2}+u_{+2}) + \cdot 080(u_{-3}+u_{+3})$$
$$+ \cdot 024(u_{-4}+u_{+4}) - \cdot 016(u_{-6}+u_{+6}) - \cdot 016(u_{-7}+u_{+7})$$
$$- \cdot 008(u_{-8}+u_{+8}).$$

But in the one which he offers as the best in regard to smoothness of graduation,

$$U = \frac{64\Sigma}{10000} - \frac{4S}{300}$$
$$= \cdot 210\dot{6}u + \cdot 192(u_{-1}+u_{+1}) + \cdot 1418\dot{6}(u_{-2}+u_{+2})$$
$$+ \cdot 0794\dot{6}(u_{-3}+u_{+3}) + \cdot 024(u_{-4}+u_{+4}) - \cdot 005\dot{3}(u_{-5}+u_{+5})$$
$$- \cdot 016(u_{-6}+u_{+6}) - \cdot 0144(u_{-7}+u_{+7}) - \cdot 0069\dot{3}(u_{-8}+u_{+8}).$$

For the meanings of the symbols S and Σ (which are not the same in the two formulas), reference must be made to Mr. Higham's paper.

The latter formula gives, he says, a column of which the second differences present a wavy line instead of a zigzag. He has applied it to graduate the rate of mortality among the female Government annuitants, 4 years and upwards after purchase; and he has adopted the very convenient course of placing by the side of his own graduated results, (1) those of a graduation he has made by Mr. Woolhouse's formula, (2) the actual observed probabilities of dying, (3) the adjusted probabilities given by Mr. Finlaison. It is to be noted that Mr. Woolhouse himself applies his formula to graduate the actual numbers-dying, d_x, in the mortality table; but Mr. Higham applies it to graduate the probability of dying in a year, q_x, which is also the function that Mr. Ansell graduates by means of his formula.

The graduated probabilities of Mr. Higham and Mr. Finlaison, cut down to 3 significant figures, are given in the following table, which also contains my own results obtained by the process to be presently explained:

Table A.

Adjusted probability of dying in a year, as calculated by Mr. Finlaison, Mr. Higham, and Mr. Sprague.

Age	Finlaison	Higham according to		Sprague	Age	Finlaison	Higham according to		Sprague
		His new formula	Wool-house's				His new formula	Wool-house's	
27		·0192	·0205	·0135	49	·0130	·0116	·0114	·0121
28		·0175	·0176	·0128	50	·0132	·0126	·0126	·0130
29		·0146	·0137	·0121	51	·0141	·0139	·0141	·0140
30		·0119	·0120	·0116	52	·0151	·0152	·0151	·0155
31		·0103	·0108	·0110	53	·0163	·0163	·0162	·0163
32		·0096	·0091	·0105	54	·0175	·0173	·0174	·0170
33		·0099	·0096	·0103	55	·0181	·0180	·0181	·0177
34		·0107	·0113	·0103	56	·0185	·0187	·0185	·0184
35		·0113	·0112	·0106	57	·0194	·0194	·0194	·0191
36		·0115	·0115	·0109	58	·0204	·0202	·0204	·0199
37		·0115	·0114	·0112	59	·0209	·0210	·0209	·0208
38		·0115	·0118	·0118	60	·0219	·0219	·0219	·0219
39		·0117	·0114	·0120	61	·0233	·0231	·0233	·0232
40	·0108	·0120	·0122	·0122	62	·0248	·0247	·0248	·0249
41	·0110	·0124	·0122	·0121	63	·0266	·0268	·0266	·0270
42	·0113	·0126	·0128	·0119	64	·0292	·0292	·0292	·0294
43	·0115	·0126	·0125	·0116	65	·0320	·0320	·0320	·0321
44	·0117	·0122	·0125	·0114	66	·0348	·0347	·0348	·0350
45	·0120	·0117	·0115	·0113	67	·0375	·0374	·0374	·0380
46	·0122	·0112	·0111	·0113	68	·0403	·0401	·0401	·0410
47	·0125	·0109	·0109	·0114	69	·0434	·0431	·0431	·0440
48	·0127	·0110	·0110	·0115	70	·0467	·0466	·0466	·0470

It will be seen from this table that from the age of 46 to 70 the two graduations given by Mr. Higham, as made by his own and Mr. Woolhouse's formulas, are practically identical. It is also seen that, as was to be expected, Mr. Finlaison's graduated results from the age of 51 to 70, agree still more closely with those given by Mr. Higham, as obtained by the use of the same formula, namely, Mr. Woolhouse's. Mr. Higham gives, on page 20 of his paper, the first and second differences of each of the three graduated sets of results; and by means of these, or by differencing out the values in the above table, we find that, in the graduation by Mr. Higham's formula, there are 9 changes of sign in the series of second differences, against 23 given by Mr. Woolhouse's. This constitutes, in my opinion, a great improvement, and fully justifies Mr. Higham's claim to have produced greater smoothness in the graduated results. It will be noticed that the ungraduated values used in Mr. Higham's formula, extend over 17 years of life, gainst 15 in Mr. Woolhouse's formula; and this circumstance in lf would lead us to expect that it would give smoother results. f we next examine Mr. Finlaison's figures, in the light of his remarks quoted above, it is clear that his graduation from age 40 to 50 has been made by means of Gompertz's formula, and from age 51 to 70 by Mr. Woolhouse's. During the former interval, the probabilities of dying proceed by almost constant first differences, and there are no changes of sign in the series of second differences; but, during the latter interval, there are 6 changes of sign in the second differences, against 3 and 6 in the two graduations given by Mr. Higham according to his own and Mr. Woolhouse's formulas respectively.

The best way of comparing the three graduations and judging whether the graduated results exhibit a satisfactory degree of smoothness, is to represent each series of probabilities by a curve, in which the age is the abscissa, and the probability of dying in a year is the ordinate. This is done for certain ages in the appended diagram (No. 2), which brings before us in a more striking way the greater smoothness of the results given by Mr. Higham's formula as compared with those given by Mr. Woolhouse's. The continuous line represents the graduation by Mr. Higham's formula; the dotted line, the graduation by Mr. Woolhouse's; and the heavy discontinuous line, Mr. Finlaison's graduation; the two latter lines not being continued beyond the ages of 46 and 52 respectively, because, as already mentioned, the graduations practically coincide with Mr. Higham's thereafter.

Although, as above stated, I consider the graduation by Mr. Higham's new formula to be better than that given by Mr. Woolhouse's, the question still remains to be considered, whether it satisfies the conditions of a good graduation. Does it, in the aggregate, represent the same rate of mortality as the original observations? and have all accidental irregularities been got rid of? Mr. Higham answers the first of these questions by showing the departures, + and −, of his graduated probabilities of dying, from the rough probabilities deduced from the original observations; but I do not consider this a satisfactory method of comparison, as it virtually attributes the same weight to the observed probabilities at the ages where the persons under observation are very few, as it does to those at the ages where they are very numerous. It is, I think, greatly preferable to introduce into the calculation the actual numbers of lives at risk; and a simple and satisfactory way of doing this, is to calculate the expected deaths at each age according to the graduated table, and to compare the numbers with the actual deaths. This method of comparison has also the advantage of furnishing us with a means (which Mr. Higham's does not) of making an exact numerical estimate of the degree of accuracy with which the graduated table reproduces, in the aggregate, the original mortality. In Table B the expected deaths at each age from 27 to 70 according to Mr. Higham's graduated table, are compared with the actual deaths; and in Table C the totals for quinquennial groups of ages

Table B.—*Showing for ages 27–70 the Expected Deaths, according to Mr. Higham's Graduation, and comparing them with the Actual Deaths.*

Age	Number of Deaths		Difference (2)−(3)	Age	Number of Deaths		Difference (2)−(3)	Age	Number of Deaths		Difference (2)−(3)
	Expected	Actual			Expected	Actual			Expected	Actual	
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
27	1·7	1	+ ·7	42	10·5	12	−1·5	57	86·9	86	+ ·9
28	1·7	1	+ ·7	43	11·9	9	+2·9	58	98·7	90	+ 8·7
29	1·9	3	−1·1	44	13·4	12	+1·4	59	110·9	123	−12·1
30	1·9	2	− ·1	45	14·7	17	−2·3	60	119·2	138	−18·8
31	2·1	...	+2·1	46	15·9	21	−5·1	61	139·5	116	+23·5
32	2·1	2	+ ·1	47	17·2	13	+4·2	62	158·6	157	+ 1·6
33	2·5	3	− ·5	48	19·4	14	+5·4	63	181·2	182	− ·8
34	3·0	3	0	49	22·8	26	−3·2	64	211·5	209	+ 2·5
35	3·6	5	−1·4	50	27·2	22	+5·2	65	243·2	258	−14·8
36	4·2	5	− ·8	51	33·1	37	−3·9	66	272·9	254	+18·9
37	4·8	3	+1·8	52	41·0	43	−2·0	67	301·4	317	−15·6
38	5·4	6	− ·6	53	47·4	51	−3·6	68	328·7	353	−24·3
39	6·7	2	+4·7	54	57·6	59	−1·4	69	358·0	343	+15·0
40	7·8	13	−5·2	55	66·2	57	+9·2	70	390·1	396	− 5·9
41	9·1	9	+ ·1	56	76·7	83	−6·3				
									3,534·3	3,556	−21·7

Table C.—*Containing the figures in Table B arranged in quinquennial groups.*

Ages	Number of Deaths		Difference (2)−(3)
	Expected	Actual	
(1)	(2)	(3)	(4)
27–30	7·2	7	+ ·2
31–35	13·3	13	+ ·3
36–40	28·9	29	− ·1
41–45	59·6	59	+ ·6
46–50	102·5	96	+ 6·5
51–55	245·3	247	− 1·7
56–60	492·4	520	−27·6
61–65	934·0	922	+12·0
66–70	1,651·1	1,663	−11·9
	3,534·3	3,556	−21·7

are given. My conclusion from an examination of these figures is, that the agreement between the actual and the expected deaths, is not remarkably close, but is probably sufficient.

The second question is whether all accidental irregularities in the progression of the numbers have been got rid of. I submit that a glance at the curve representing Mr. Higham's results in Diagram 2, is sufficient to show that his process has retained irregularities which ought to be got rid of. Is it credible that the normal rate of mortality among annuitants 4 years after the date of purchase, should increase and decrease in the manner shown by the diagram? It will be seen that, according to his graduation, the rate of mortality decreases from the age of 27 to 32, then increases, but somewhat irregularly, to 43, decreasing again to 47, and from that point continually increases. If we were prepared to admit that the decreases in the rate from age 27 to 32, and from 43 to 47, are true to nature, I think there can be no doubt that the undulation between 32 and 43, should not be preserved, but should be smoothed away. I understand that Mr. Higham, in effect, admits this; in fact, that he does not even insist on retaining the decrease of mortality from 43 to 47; for he says that he does not question the soundness of the judgement, with which this feature of the mortality has been ignored by Mr. Finlaison, who, as is shown by the curve, makes the mortality increase very regularly from 40 to 50. It seems to be a fair inference from Mr. Higham's remarks, that he does not, like Mr. Woolhouse, submit his formula as giving a graduated mortality table that may be adopted without further alteration by the actuary, but that he contemplates a subsequent "final adjustment". If this is the correct view, and the curve obtained by Mr. Higham's process is to be submitted to a further smoothing or adjustment, the question arises, whether it is worth while to devote so much labour to obtaining that curve. Is it not better in every way to confine ourselves, in the first instance, to making a rough graduation, by simply grouping the observed facts, in such a way as to lessen the original irregularities, and obtain an indication of the general law which the facts follow?

This is the course I adopt in making a graduation by the graphic method. I do not start, as Mr. Higham supposes, with a preconceived idea of what the curve ought to be, or what I wish it to be; but I begin by carefully studying the observed facts, and combining them so as to make them, if possible, tell their own tale, and indicate the law which they follow. I make various trial groupings of the facts, and select that one which presents the fewest or least irregularities. No positive rule can be given for making these groupings; but the negative one may be laid down, that we have no right to expect that a grouping according to quinquennial ages, or in any other definite way, will be successful. In fact, as the irregularities of the observed facts do not follow any law, or occur at any fixed intervals, it is obvious that they cannot be neutralized and got rid of, by any definite mode of procedure, such as quinquennial grouping. This argument, when fully considered, seems to be conclusive against all such methods of graduation as Mr. Woolhouse's, Mr. Ansell's, and Mr. Higham's; and we learn that, though these may lessen the irregularities, they can never entirely remove them. We must therefore follow a tentative method, and determine by trial which groupings are most suitable.

I will now apply my graphic method to deduce a graduated mortality table from the same observations. I may here remark, in passing, that though I speak of "my" method, I do not put forward any claim to have originated the graphic method. The idea is, I imagine, a very old one; but, starting with that idea, I have elaborated a method of procedure which I think I may fairly claim as my own. The original facts, as given by Mr. Finlaison, namely, the numbers at risk and the deaths at each age, are given in Table D (p. 87), which contains also the unadjusted rate of mortality, calculated to three decimal places; and confining my attention for the present to the ages up to 51, I first group the facts as shown in the first division of Table E (p. 87).

We have thus greatly reduced the irregularities of the original facts, but we have not as yet got any trustworthy indication of a law. The rate of mortality still shows within short intervals of age considerable fluctuations, which seem to me more likely to arise from the insufficiency of the number of persons under observation, than from any special or climateric condition. I therefore next arrange the facts in the larger groups shown in divisions 2 and 3 of Table E. The laws indicated by these two groupings are essentially different, and we have to consider which of them is to be preferred. According to the former of them, the rate of mortality must be considered as approximately constant from age 19 to 49; and if we make one large group of all the facts up to age 49 inclusive, we get, Number at risk = 16037, Deaths = 186; and these figures give us, Rate of Mortality = ·0116. According to the latter, the rate of mortality is considerable at the youngest ages, falls to a minimum between the ages of 30 and 35, and thereafter continually increases. The fundamental question, TABLE D.—*Female Government Annuitants: four years and upwards after purchase. Number exposed to risk, Number who died, and the unadjusted rate of Mortality.* (*P.* 24 *of Return.*)

Age	Number at Risk	Deaths	Rate of Mortality	Age	Number at Risk	Deaths	Rate of Mortality	Age	Number at Risk	Deaths	Rate of Mortality
19	9	...	...	47	1,578	13	·008	75	7,214	592	·082
20	14	...	...	48	1,761	14	·008	76	6,713	567	·084
21	18	...	...	49	1,958	26	·013	77	6,205	548	·088
22	23	...	...	50	2,160	22	·010	78	5,704	565	·099
23	31	...	...	51	2,387	37	·016	79	5,196	562	·108
24	37	...	...	52	2,669	43	·016	80	4,664	549	·118
25	54	2	·037	53	2,909	51	·018	81	4,142	558	·135
26	66	2	·030	54	3,330	59	·018	82	3,576	479	·134
27	85	1	·012	55	3,682	57	·015	83	3,070	481	·157
28	99	1	·010	56	4,104	83	·020	84	2,575	399	·155
29	130	3	·023	57	4,473	86	·019	85	2,146	404	·188
30	163	2	·012	58	4,885	90	·018	86	1,706	342	·200
31	196	...	...	59	5,281	123	·023	87	1,332	299	·224
32	223	2	·009	60	5,644	138	·024	88	1,018	210	·206
33	248	3	·012	61	6,038	116	·019	89	788	208	·264
34	280	3	·011	62	6,422	157	·024	90	569	160	·281
35	319	5	·016	63	6,762	182	·027	91	391	115	·294
36	360	5	·014	64	7,247	209	·029	92	265	72	·272
37	416	3	·007	65	7,599	258	·034	93	189	63	·333
38	473	6	·013	66	7,863	254	·032	94	121	39	·322
39	571	2	·004	67	8,061	317	·039	95	78	31	·397
40	653	13	·020	68	8,197	353	·043	96	47	12	·255
41	733	9	·012	69	8,307	343	·041	97	34	18	·529
42	833	12	·014	70	8,372	396	·047	98	16	9	·563
43	941	9	·010	71	8,292	401	·048	99	7	4	·571
44	1,097	12	·011	72	8,156	432	·053	100	3	...	...
45	1,252	17	·014	73	7,905	518	·066	101	3	2	·667
46	1,416	21	·015	74	7,588	532	·070	102	1	...	...

TABLE E.—*Containing the observations in* TABLE D, *for ages* 19–51, *groupt in various ways.*

Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality
19–26	252	4	·0159	19–33	1,396	16	·0115	19–29	566	9	·0159
27–29	314	5	·0159								
30–33	830	7	·0084					30–35	1,429	15	·0105
34, 5	599	8	·0133	34–37	1,375	16	·0117				
36, 7	776	8	·0103					36–38	1,249	14	·0112
38	473	6	·0127	38–40	1,697	21	·0124				
39, 40	1,224	15	·0123					39–44	4,828	57	·0118
41, 2	1,566	21	·0134	41–44	3,604	42	·0117				
43, 4	2,038	21	·0103								
45–47	4,246	51	·0120	45–49	7,965	91	·0114	45–51	12,512	150	·0120
48, 9	3,719	40	·0108								
50, 1	4,547	59	·0130	50, 1	4,547	59	·0130				
19–51	20,584	245									

therefore, which we have to determine, is whether our table is to begin at the age of 19 with a rate of mortality about equal to ·0115, or with a rate larger than ·0159. I say a rate *larger than* this, because ·0159 is the aggregate rate over the ages 19 to 29, and if the rate is rapidly diminishing to a minimum of ·0105 in the next group of ages 30–35, it must be still higher at the beginning of the group, or at the age of 19.

The graduations given by Mr. Higham according to his own and Mr. Woolhouse's formulas, begin with rates of mortality equal to ·0192 and ·0205 respectively, at the age of 27; but I consider that these are manifestly too large. It is true that, if we take the observations for the 5 ages 25–29, we get the rate of mortality =9÷434=·0207; but this leaves out of account the fact that at the ages 19–24 we have 132 years of life without any deaths. Looking at the matter from another point of view, we may say that Mr. Higham's method of procedure attaches undue weight to the heavy rates of mortality which the original observations give at the ages of 25 and 26, namely, 2÷54=·0370, and 2÷66=·0303; but by throwing the small numbers at these ages into a single group along with all the others up to age 29, we roughly assign to the observation at each age a weight proportional to the observed numbers. If we had no further information beyond what is contained in the above table, it would be difficult to say which of the two groupings should be preferred, and the safer plan would be to take the former and make the rate of mortality approximately constant from 19 up to 49. But Mr. Finlaison's observations furnish us with other facts, namely, those relating to the years 0, 1, 2, 3, after the purchase of the annuity; and these will assist us in coming to a decision. Grouping these facts, as explained above, I get the following results:

Table F.—0 *Years elapst.*

Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality
15–24	79·2	1	·0126	15–30	225·1	3	·0133
25–30	145·9	2	·0137				
31–34	163·3	1	·0061	31–37	383·3	2	·0052
35–37	220	1	·0046				
38, 9	176·6	1	·0057	38–41	433·3	2	·0047
40, 1	256·7	1	·0039				
42, 3	288	2	·0069	42–49	1,501·9	11	·0073
44–46	515·9	4	·0078				
47–49	698	5	·0071				
Total	2,543·6	18	·0071				

TABLE G.—1 *Year elapst.*

Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality
16–27	192	2	·0104	16–33	433	4	·0092
28–33	241	2	·0083				
34–37	361	2	·0055	34–41	923	5	·0054
38–41	562	3	·0053				
42–44	612	5	·0081	42–49	2,055	16	·0078
45–49	1,443	11	·0076				
Total	3,411	25	·0073				

TABLE H.—2 *Years elapst.*

Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality
17–26	113	3	·0265	17–36	563	8	·0142
27–30	142	2	·0141				
31–34	169	2	·0118				
35, 6	139	1	·0079				
37–43	956	9	·0094	37–43	956	9	·0094
44, 5	415	7	·0169	44–49	1,463	22	·0150
46, 7	480	8	·0167				
48, 9	568	7	·0123				
Total	2,982	39	·0131				

TABLE I.—3 *Years elapst.*

Ages	Number at Risk	Deaths	Rate of Mortality	Ages	Number at Risk	Deaths	Rate of Mortality
18–30	220	1	·0046	18–35	415	3	·0072
31–35	195	2	·0103				
36–40	450	3	·0067	36–42	706	6	·0085
41, 2	256	3	·0117				
43, 4	370	4	·0108	43, 4	370	4	·0108
45–47	624	8	·0128	45–49	1,132	13	·0115
48, 9	508	5	·0099				
Total	2,623	26	·0099				

Here we see that three out of the four sets of figures, agree in making the rate of mortality under the age of 30 or 35, greater than that at the following ages. Combining the original figures for the four years, we get the results shown in the following table:

TABLE J.—0–3 *Years elapst.*

Ages	Number at Risk	Deaths	Rate of Mortality
15–20	102·5	2	·0195
20–27	501·6	6	·0120
28–32	675·3	7	·0104
33–37	1,356·0	10	·0074
38–42	2,548·3	19	·0075
43–46	3,179·9	35	·0110
47–49	3,196·0	29	·0091
Total	11,559·6	108	

These results agree with our former ones in indicating a high rate of mortality at the commencing ages, which decreases to a minimum about the age of 35 to 40; and I therefore decide to adopt the second of the two alternative groupings in Table E.

For the ages 52 to 70 the grouping presents no difficulty, and we get

TABLE K.—*Containing the observations in Table D, for ages 52 to 70, arranged in suitable groups.*

Ages	Number at Risk	Deaths	Rate of Mortality
52–55	12,590	210	·0167
56–58	13,462	259	·0192
59–61	16,963	377	·0222
62	6,422	157	·0245
63	6,762	182	·0269
64	7,247	209	·0288
65, 6	15,462	512	·0333
67	8,061	317	·0393
68	8,197	353	·0431
69, 70	16,679	739	·0443
Total	111,845	3,315	

Collecting now our results up to the age of 70, we get the figures shown in the first two columns of Table L. Column (1) contains the various groups of ages which we have adopted; and column (2) the rate of mortality prevailing in each of those groups, as found by dividing the deaths by the number at risk in each group. In columns (3) and (4) are given the first and second differences of the rate of mortality; and in column (5) is inserted the middle age of each group of ages in column (1). The remaining columns will be explained presently. (*See* p. 96.)

TABLE L.—*Showing the unadjusted rate of mortality in certain groups of ages from* 19 *to* 70.

Ages	Rate of Mortality	Differences		Middle Age	Number of Years in Interval	Divided First Differences (3) ÷ (6)	Differences of (7)
		First	Second				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
19–29	·0159			24			
		−·0054			8½	−·00063	
30–35	·0105		+61	32½			+·00079
		+ 7			4½	+ 16	
36–38	·0112		− 1	37			− 3
		6			4½	13	
39–44	·0118		− 4	41½			− 10
		2			6½	3	
45–51	·0120		+45	48			+ 82
		47			5½	85	
52–55	·0167		−22	53½			− 14
		25			3½	71	
56–58	·0192		+ 5	57			+ 29
		30			3	100	
59–61	·0222		− 7	60			+ 15
		23			2	115	
62	·0245		+ 1	62			+ 125
		24			1	240	
63	·0269		− 5	63			− 50
		19			1	190	
64	·0288		+26	64			+ 110
		45			1½	300	
65, 6	·0333		+15	65½			+ 100
		60			1½	400	
67	·0393		−22	67			− 20
		38			1	380	
68	·0431		−26	68			− 300
		12			1½	80	
69, 70	·0443			69½			

The next step is to plot down the values we have obtained on a sheet of diagrammatic paper accurately ruled;* the age being taken as the abscissa, and the rate of mortality as the ordinate. This is done in Diagram (3), where each interval in the direction of the abscissas is taken as a year, and each interval in the direction of the ordinates is taken as representing ·001 in the probability of dying. Consistently with this, the first two significant figures in the rate of mortality, correspond to an exact number of intervals; and the figures in the fourth decimal place have to be estimated as tenths of an interval. In this way we have got 15 points, indicated by the small circles in the figure; and by observing the position of these, we get a better general idea of the course which the mortality curve will follow, than we can obtain from simple inspection of the numerical ratios.

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* Messrs. Layton, the publishers of this *Journal*, have had paper of this kind specially prepared, which I find to be very suitable. Although the ruling is slightly inaccurate in parts, as the lines are not all at precisely the same distance, I do not think this is likely to lead to any inconvenience, as I find the error to be not greater than that which I am myself liable to make in estimating fractions of an interval, and errors arising from both these causes are readily corrected by inspection.

Above the age of 70 the probabilities of dying increase so steadily that very little grouping is required, and we have the figures in the first four columns of the following table. As in Table L, the middle age of each group is given in column (6).

TABLE M.—*Showing the unadjusted rate of mortality in certain groups of ages from* 71 *to* 102.

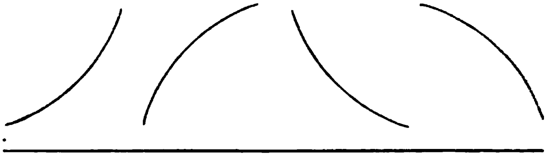
Ages (1)	At Risk (2)	Deaths (3)	Rate of Mortality (4)	Δ(4) (5)	Middle Age (6)	Δ(6) (7)	(5) ÷ (7) (8)	Δ(8) (9)
71	8,292	401	·0484	·0046	71	1	·0046	·0079
72	8,156	432	·0530	125	72	1	125	−79
73	7,905	518	·0655	46	73	1	46	74
74	7,588	532	·0701	120	74	1	120	−96
75	7,214	592	·0821	24	75	1	24	14
76	6,713	567	·0845	38	76	1	38	70
77	6,205	548	·0883	108	77	1	108	−17
78	5,704	565	·0991	91	78	1	91	4
79	5,196	562	·1082	95	79	1	95	16
80	4,664	549	·1177	166	80	1½	111	−3
81, 2	7,718	1,037	·1343	216	81½	2	108	108
83, 4	5,645	880	·1559	324	83½	1½	216	−94
85	2,146	404	·1883	122	85	1	122	118
86	1,706	342	·2005	240	86	1	240	−194
87	1,332	299	·2245	69	87	1½	46	286
88, 9	1,806	418	·2314	498	88½	1½	332	−203
90	569	160	·2812	129	90	1	129	−108
91	391	115	·2941	32	91	1½	21	146
92, 3	454	135	·2973	250	92½	1½	167	140
94	121	39	·3223	613	94	2	307	−16
95–7	159	61	·3836	1164	96	4	291	
98–102	30	15	·5000		100			

For these ages, the scale we have hitherto used is not convenient, and I therefore adopt a considerably smaller scale. Each interval along the base line still represents a year of age, but each interval in the direction of the ordinates will now be taken as representing a probability equal to ·01 instead of ·001. In this way the ratios in Table M give us 22 points; and, joining each adjacent two by straight lines, we get the broken curve shown in Diagram (4).

What we have thus far done really amounts to a partial graduation of the observations, effected by suitably grouping the original facts; and thus far no use has been made of the graphic method. We have now to apply that method in order to substitute for the broken curves, or zigzag lines, given in Diagrams (3) and (4), smooth curves, which on the whole represent the same aggregate mortality, but proceed without any sudden breaks or turns. It will be convenient to begin with the ages above 70, as the irregularities in the figures for those ages are less than at younger ages. Looking now at Diagram (4) we see that, on the whole, the rate of mortality increases very regularly as the age increases; and that, if we leave out of account the slight irregularities which the line exhibits at certain points, the curve will have no undulations, but will have its convexity constantly turned to the base line. In the application of the graphic method, the first point we have to consider is, what deviations from regularity are to be removed, and which of them (if any) are to be retained. Now I have no hesitation in saying that all the irregularities in the curve in Diagram (4) must be considered as accidental, and ought to be removed. If we obtained another large set of observations on a similar body of lives, we should no doubt find that they would exhibit similar irregularities, but not at the same ages; and if we could get observations on a sufficiently large number of lives, these irregularities would either wholly disappear or be reduced to insignificant proportions. Now the object of graduation is to remove all irregularities of this kind, and to obtain the smooth curve that would be yielded by our observations if they were sufficiently numerous. I therefore proceed on the supposition that the graduated curve is to exhibit no undulations from the age of 70 up to the end of life.

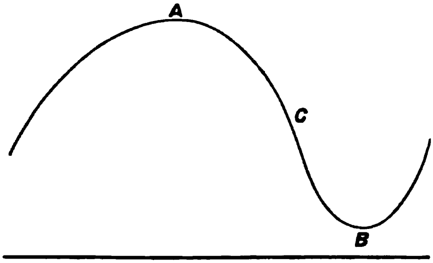
Before doing this it may be useful to mention that, in order to understand and successfully apply the graphic method of graduation, it is necessary to study carefully the relations that exist between the progression of the numbers and their differences, and the form of the curve. If we have any smooth curve, and take the values of the ordinates at equal small intervals along the base line, and then find the first and second differences of these values, we may easily satisfy ourselves that, if the first differences are all positive, the curve continually recedes from the base line; and if they are negative, the curve approaches that line: also that if the second differences are positive, the curve has its convexity turned to the base line; and if they are all negative, it is concave to that line.

First Differences .	*Positive*	*Positive*	*Negative*	*Negative*
Second „ .	*Positive*	*Negative*	*Positive*	*Negative*

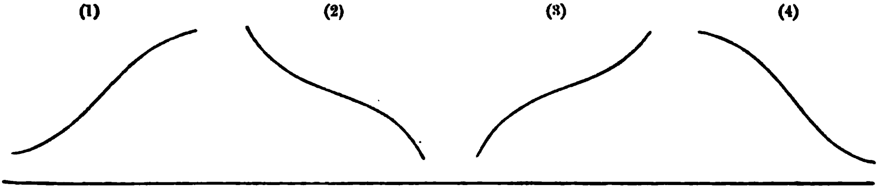


There are thus four cases, which are represented in the above woodcut. As a specimen of the first, we may take the series formed by the squares or cubes of the natural numbers; for the second, we may take the series formed by the square or cube roots of those numbers; for the third, the series formed by the reciprocals, either of the numbers or of their squares or cubes; and those students who are not already familiar with the propositions I have stated, cannot do better than verify them by taking numerical examples. It will be found an instructive exercise to extract from Barlow's very useful tables, numbers of the kind I have described, then to calculate their first and second differences, and lastly to plot down on cross-ruled paper the curve that corresponds to each different series of numbers. As a specimen of the fourth form of curve, we may take the series of numbers obtained by subtracting the squares of the natural numbers from a fixed number: for instance, by subtracting the squares of 1, 2, 3, ... 9, from 100, we get the series of numbers 99, 96, 91, 84, 75, 64, 51, 36, 19: the first differences of these are, −3, −5, −7, −9, −11, −13, −15, −17; and the second differences are all equal to −2.

If we find that the series of first differences changes from positive to negative, then the curve, which was receding from the base line, has a maximum point, and begins to approach that line; and, on the contrary, if the change is from negative to positive, the curve, which was approaching the base line, has a minimum point, and begins to recede from that line. These two forms of the curve are shown in the appended woodcut, *A* and *B* being the maximum and minimum points respectively. The student will find several examples of this kind in the two series of graduated probabilities given by Mr. Higham on pages 20 and 21 of his paper.



If the second differences change sign, the curve has what is called a point of inflection, or a point of contrary flexure; that is to say, up to a certain point it is convex to the base line, and at that point it changes its direction and becomes concave; or the contrary. If the second differences change sign from positive to negative, the curve is first convex to the base line and afterwards concave, as is shown in Figures (1) and (2) of the following woodcut; while if the differences change from negative to positive, the curve passes from concave to convex, as shown in Figures (3) and (4) of the woodcut. Many examples of this kind occur in Mr. Higham's second series of adjusted probabilities.

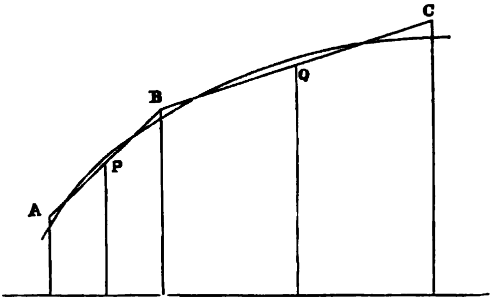


The student will easily be able to satisfy himself that there must always be a point of inflection between a maximum and a minimum point in the curve. In the neighbourhood of a maximum point the curve is concave to the base line, and the second differences are negative; while in the neighbourhood of a minimum point the curve is convex to the base line, and the second differences are positive. Hence, in passing from a maximum point to a minimum, the second differences change sign from negative to positive, which proves that there is a point of inflection between them. Thus, in the diagram above, there is a point of inflection, *C*, between the maximum point, *A*, and the minimum point, *B*.

When, as is the case with our two unadjusted series of probabilities given in Tables L and M, the intervals at which the ordinates are taken are not all equal, all that has been said regarding first differences still holds good, but the statement requires modification as regards the second differences. When the interval is constant, the first difference forms a measure of the rate at which the curve recedes from the base line (or approaches it); but when the intervals are of unequal magnitude, it is clear that, in order to get a proper measure of the rate at which the curve recedes from (or approaches to) the base line, we must divide each first difference by the interval to which it relates. In Table L (ages 19–70) the number of years in each interval is stated in column (6), and the quotients obtained by dividing the first differences by these numbers, are set down in column (7). These numbers, then, which I call *divided first differences,* measure the rate at which the curve recedes from (or approaches to) the base line. So long as this rate increases, the curve is convex to the base line; and when this rate diminishes, the curve is concave to the base line. We have therefore to take the differences of the divided first differences, as shown in column (8); and when these are positive, the curve is convex to the base line; and when they are negative, the curve is concave. The number of changes of sign, therefore, indicates the number of points of inflection in the curve; and we see that from age 19 to 70 there are 7 such points. If we take the second differences, as contained in column (5), we find that they exhibit 9 changes of sign, which is larger than the number of points of inflection.

Pursuing the same course with the figures in Table M, which relates to ages over 70, we see that the final column exhibits 15 changes of sign, corresponding to an equal number of points of inflection in the curve; and, consistently with what was said above, we have now to substitute for this irregular curve one which shall present no points of inflection, so that when we take the corresponding probabilities of dying at each age and form the second differences, these shall exhibit no changes of sign. It is here that the graphic method comes in. We have, in Diagram (4), 22 points marked on our sheet of cross-ruled paper; and we must actually draw the best curve we can, that shall pass as near those points as practicable, above some and below others, but without exhibiting any point of inflection.

I may here remark that, when our points indicate a curve free from sudden changes of direction, and with its curvature in the same direction throughout, theoretical considerations teach us that the curve should not pass exactly through the points, but a little outside them. It does not seem desirable to give here a formal proof of this proposition; but, for the benefit of those who may be disposed to investigate the matter for themselves, it may suffice to state that, having regard to the way in which our points are obtained, if we assume that the rate of mortality increases by constant first differences through each of the intervals we have dealt with, the unadjusted mortality curve will form a polygonal figure *ABC* .... the sides of which will be bisected by our points, *P*, *Q*, ...; and our adjusted curve must be drawn so that its area is approximately the same as that of the polygon. The curve must therefore cut off the corners of the polygon, as shown in the subjoined figure. The distance, however, between the curve and the points is so small, that it may be generally disregarded, and the curve may be drawn through the points.



The curve drawn in accordance with these principles is the dotted line in Diagram (4); and the next step is to note the points in which it cuts the vertical lines corresponding to the various ages, and to estimate the length of the ordinate for each age. Thus, taking the age 75, I see that the curve cuts the ordinate between the seventh and eighth line from the base, and the adjusted rate of mortality for age 75 will therefore lie between ·07 and ·08. The next decimal place is got by estimating the fraction of an interval; and, as I estimate the fraction in the present case to be $\frac{7}{10}$ths of an interval, I get the probability ·077. Proceeding in this way for each age, I get the probabilities shown in the following table:

Table N.—*Ages* 70–102. *Mr. Sprague's Graduated Probabilities of Dying, and Expected Deaths.*

Age x (1)	Probability of Dying in a Year q_x (2)	Number of Deaths		Differences (3)—(4) (5)
		Expected (3)	Actual (4)	
70	·044	368·4	396	−27·6
71	·050	414·6	401	+13·6
72	·056	456·7	432	+24·7
73	·063	498·0	518	−20·0
74	·070	531·1	532	− ·9
75	·077	555·5	592	−36·5
76	·084	563·9	567	− 3·1
77	·091	564·7	548	+16·7
78	·099	564·7	565	− ·3
79	·108	561·2	562	− ·8
80	·118	550·3	549	+ 1·3
81	·129	534·3	558	−23·7
82	·140	500·6	479	+21·6
83	·152	466·6	481	−14·4
84	·166	427·5	399	+28·5
85	·183	392·7	404	−11·3
86	·200	341·2	342	− ·8
87	·217	289·0	299	−10·0
88	·234	238·2	210	+28·2
89	·250	197·0	208	−11·0
90	·265	150·7	160	− 9·3
91	·283	110·7	115	− 4·3
92	·300	79·5	72	+ 7·5
93	·320	60·5	63	− 2·5
94	·339	41·0	39	+ 2·0
95	·360	28·1	31	− 2·9
96	·382	18·0	12	+ 6·0
97	·410	13·9	18	− 4·1
98	·439	7·0	9	− 2·0
99	·466	3·3	4	− ·7
100	·500	1·5	...	+ 1·5
101	·545	2·2	2	+ ·2
102	·605	·6	...	+ ·6

It is right that I should mention that a slight correction has been applied to these figures. There is always a liability to error in estimating the tenths of an interval, and small errors may also arise from inequalities in the ruling of the paper, or from the curve being drawn unsteadily. In order to remove these errors, I difference the quantities; and, when I find the series of differences presents irregularities, I remove these by inspection. Thus, in the present case, the figures I actually got in the first instance from the ages 71 to 83, are shown in column (2) of the following table:

TABLE O.

Age x (1)	q_x (2)	Δ(2) (3)	Col. (3) Corrected (4)	Col. (2) Corrected (5)
71	·050	6	6	·050
72	·056	7	7	·056
73	·063	7	7	·063
74	·070	7	7	·070
75	·077	8	7	·077
76	·085	6	7	·084
77	·091	8	8	·091
78	·099	10	9	·099
79	·109	10	10	·108
80	·119	11	11	·118
81	·130	10	11	·129
82	·140	12	12	·140
83	·152			·152

The differences are given in column (3). These are then corrected by inspection in column (4), and the corrected probabilities deduced from the corrected differences, are given in column (5). It will be noticed that in no case do these differ by more than a unit in the third decimal place from those originally obtained. In other cases there may be a difference of 2 in the third decimal place, or possibly of 3. The probabilities of death thus obtained evidently proceed with sufficient regularity, and we have next to determine whether they will reproduce the original number of deaths shown by the observations. For this purpose the probability at any age is to be multiplied into the number at risk at that age according to the observations. The products thus obtained, or the expected deaths at each age, are given in column (3) of Table N, the actual deaths being given in column (4), and the differences between them in column (5). Adding the deaths at ages 71 to 101, the general result is that we have 9,164·2 expected deaths, against 9,171 actual, or a difference of only 6·8, which must be admitted to be sufficiently close. Choosing suitable groups of ages, as indicated either by our figures or by the differences in Table N, we get the results given in columns (2), (3), (4), of the following table:

TABLE P.—*Ages* 71–101. *Expected and Actual Deaths, and the differences.*

Ages	Actual Deaths	By Sprague's Graduation		By Finlaison's Graduation	
		Expected Deaths	Difference (3)–(2)	Expected Deaths	Difference (5)–(2)
(1)	(2)	(3)	(4)	(5)	(6)
71– 77	3,590	3,584·5	–5·5	3,604·5	+ 14·5
78– 82	2,713	2,711·5	–1·9	2,710·2	– 2·8
83– 86	1,626	1,628·0	+2·0	1,613·6	–12·4
87– 90	877	874·9	–2·1	870·4	– 6·6
91– 95	320	319·8	– ·2	328·9	+ 8·9
96–101	45	45·9	+ ·9	52·7	+ 7·7
All Ages	9,171	9·164·2	–6·8	9,180·3	+ 9·3

It may, I think, be fairly said that this is a remarkably close agreement, which leaves nothing to be desired. The general conclusion is, that the probabilities of dying I have obtained satisfy both the conditions of a good graduation. They proceed with sufficient regularity, and they reproduce with as great accuracy as is practicable, the actual number of observed deaths.

I may be permitted now to compare my results with Mr. Finlaison's, which have been obtained by the use of Mr. Woolhouse's formula. Table Q contains in column (2) the adjusted probabilities given by Mr. Finlaison, cut down to 4 decimal places; and the first and second differences of these probabilities are given in columns (3) and (4). It will be observed that these differences exhibit great irregularities, there being no less than 22 changes of sign in the series of second differences. The expected deaths, calculated from the graduated probabilities* are given in column (5), the actual deaths in column (6), and the differences in column (7). Collecting these in the same groups as formerly, the expected deaths at various groups of ages are given in column (5) of Table P; and the differences between these and the actual deaths in column (6). On comparing these with the corresponding figures according to my graduation, it will be seen that my results, besides proceeding more smoothly, agree more closely with the original facts than Mr. Finlaison's obtained by the application of Mr. Woolhouse's formula. In both respects, therefore, the graphic method affords more satisfactory results, even when, as in this case, the observed facts are very numerous.

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* In this calculation the probabilities here set down are not used, but those given by Mr. Finlaison to 5 decimal places.

TABLE Q.—*Ages* 71 *to* 101. *Mr. Finlaison's Graduated Probabilities of Dying, and Expected Deaths.*

Age	Rate of Mortality	Δ(2)	Δ(3)	DEATHS		Differences (5)—(6)
				Expected	Actual	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
71	·0511	·0054		423·7	401	+22·7
72	·0565	64	·0010	461·0	432	+29·0
73	·0629	68	4	497·3	518	−20·7
74	·0697	74	6	528·8	532	− 3·2
75	·0771	72	− 2	556·1	592	−35·9
76	·0843	78	6	566·1	567	− ·9
77	·0921	75	− 3	571·5	548	+23·5
78	·0996	89	14	568·3	565	+ 3·3
79	·1085	91	2	563·7	562	+ 1·7
80	·1176	109	18	548·4	549	− ·6
81	·1285	106	− 3	532·3	558	−25·7
82	·1391	130	24	497·5	479	+18·5
83	·1521	131	1	467·0	481	−14·0
84	·1652	158	27	425·4	399	+26·4
85	·1810	141	− 17	388·4	404	−15·6
86	·1951	191	50	332·8	342	− 9·2
87	·2142	177	− 14	285·2	299	−13·8
88	·2319	196	19	235·0	210	+25·0
89	·2515	156	− 40	198·2	208	− 9·8
90	·2671	249	93	152·0	160	− 8·0
91	·2920	154	− 95	114·2	115	− ·8
92	·3074	206	52	81·5	72	+ 9·5
93	·3280	155	− 51	62·0	63	− 1·0
94	·3435	363	208	41·6	39	+ 2·6
95	·3798	209	−154	29·6	31	− 1·4
96	·4007	450	241	18·8	12	+ 6·8
97	·4457	445	− 5	15·2	18	− 2·8
98	·4902	1637	1192	7·8	9	− 1·2
99	·6539	1239	−398	4·6	4	+ ·6
100	·7778	2222	983	2·3	...	+ 2·3
101	1·0000			4·0	2	+ 2·0
				9180·3	9171	+ 9·3

I now pass on to consider the ages from 50 to 70. Here I infer from a study of the figures, and of the course of the mortality curve, as indicated by Diagram (3), that between the ages of 57 and 70 there must be no point of inflection, but the curve must be constantly convex to the base line. Between the ages of 50 to 57 we have what Mr. Higham describes as a wave, which represents an accelerated increase in the rate of mortality at about the age of 50, and a slackened rate of increase for some time afterwards. He considers that it would be a mistake to obliterate this wave from the table. I am inclined to think, however, that the slackened rate of increase is due to the great influx of newly selected lives that takes place at the age of 50 and upwards, the numbers of entrants at these ages being much larger than at younger ages. These entrants will be included in the observations we are dealing with after the lapse of four years from entry, and will tend to reduce the rate of mortality at ages 54 and upwards; and if we made a more exact determination of the time for which the effect of selection endures, and calculated the ultimate rate of mortality more accurately, we should probably find this feature of the curve of mortality disappear. This opinion is confirmed by an examination of other observations of female life; namely, the females of the British Peerage as given by Messrs. Bailey and Day (*J.I.A.*, ix, 321) and the females of the *Institute Experience*. The numbers at risk and the deaths at various quinquennial ages according to these observations, are given in Table R; and the corresponding mortality curves in Diagram (5).

TABLE R.

Ages	INSTITUTE EXPERIENCE: H^F									PEERAGE FEMALES		
	All Durations			5 years and upwards			10 Years and upwards					
	Number at Risk	Deaths	Rate of Mortality	Number at Risk	Deaths	Rate of Mortality	Number at Risk	Deaths	Rate of Mortality	Number at Risk	Deaths	Rate of Mortality
20–24	5,058	43	·0085	1,497·5	12	·0080	...	...	...	8,221·5	65	·0079
25–29	9,054	107	·0118	2,961·5	36	·0122	905	11	·0122	7,716·0	67	·0087
30–34	13,242	150	·0113	5,569·5	73	·0131	1,719·5	18	·0105	7,206·5	62	·0086
35–39	16,242·5	196	·0121	8,038	111	·0138	3,088·5	39	·0126	6,613·5	65	·0098
40–44	17,826	229	·0129	10,084·5	145	·0144	4,566·5	73	·0160	5,888·5	72	·0122
45–49	17,824·5	248	·0139	11,107·5	174	·0157	5,813·5	96	·0165	5,003·5	57	·0114
50–54	17,363	272	·0157	11,363·5	186	·0164	6,624·5	115	·0174	4,259·5	58	·0136
55–59	15,732	317	·0202	11,113	256	·0230	6,899·5	165	·0239	3,479·5	72	·0207
60–64	13,028·5	373	·0286	9,932	307	·0309	6,607	199	·0301	2,705·5	82	·0303
65–69	9,382	410	·0437	7,834·5	362	·0462	5,705·5	282	·0494	1,993·5	79	·0396

In these no such slackening of the rate of increase is seen above the age of 50 as Mr. Higham describes. Nevertheless, as this slackening is a feature of the observations which we are graduating, I will retain it. I accordingly draw the curve represented by the dotted line in Diagram (6); and then reading off the ordinates, as previously explained, I get the figures shown in column (2) of the following table:

TABLE S.—*Ages* 48–73. *Mr. Sprague's Graduated Probabilities of Dying, and Expected Deaths.*

Age (1)	Rate of Mortality (2)	Difference (3)	Corrected Difference (4)	Corrected Rate of Mortality (5)	Expected Deaths (6)	Actual Deaths (7)	Difference (6)—(7) (8)
48	·0120	·0009		·0120			
49	·0129	9		·0129			
50	·0138	8	9	·0138			
51	·0146	9	8	·0147			
52	·0155	8	8	·0155	41·8	43	− 1·2
53	·0163	7	7	·0163	47·5	51	− 3·5
54	·0170	8	7	·0170	56·6	59	− 2·4
55	·0178	8	7	·0177	65·2	57	+ 8·2
56	·0186	6	7	·0184	75·4	83	− 7·6
57	·0192	10	8	·0191	85·4	86	− ·6
58	·0202	8	9	·0199	97·3	90	+ 7·3
59	·0210	11	11	·0208	109·8	123	−13·2
60	·0221	12	13	·0219	123·6	138	−14·4
61	·0233	17	17	·0232	140·1	116	+24·1
62	·0250	18	21	·0249	159·9	157	+ 2·9
63	·0268	30	24	·0270	182·5	182	+ ·5
64	·0298	22	27	·0294	213·0	209	+ 4·0
65	·0320	30	29	·0321	244·0	258	−14·0
66	·0350	30	30	·0350	275·2	254	+21·2
67	·0380	33	30	·0380	306·3	317	−10·7
68	·0413	29	30	·0410	336·1	353	−16·9
69	·0442	28	30	·0440	365·5	343	+22·5
70	·0470	30		·0470	393·5	396	− 2·5
71	·0500	60		·0500			
72	·0560	70		·0560			
73	·0630			·0630			
					3,318·7	3,315	+ 3·7

The differences of these, as shown in column (3), exhibit several irregularities, which are corrected by inspection and trial, with the result of substituting for them the differences shown in column (4); and from these we get the corrected probabilities in column (5). It will be seen that the second differences of these exhibit one change of sign, corresponding to the feature of the mortality curve above mentioned, as against 3 in Mr. Higham's adjustment, and 6 in Mr. Woolhouse's.

The next step is to calculate the expected deaths: these are set down in column (6), the actual deaths at each age being given in column (7), and the differences between the actual and the expected, in column (8). Collecting the results for the groups of ages shown in Table L, we get the following comparison :

TABLE T.—*Ages* 52–70. *Comparison of the Actual and the Expected Deaths.*

Ages	Actual Deaths	SPRAGUE		FINLAISON		HIGHAM	
		Expected Deaths	Difference	Expected Deaths	Difference	Expected Deaths	Difference
52–55	210	211·1	+ 1·1	212·3	+ 2·3	212·2	+ 2·2
56–58	259	258·1	− ·9	262·8	+ 3·8	262·3	+ 3·3
59–61	377	373·5	− 3·5	374·8	− 2·2	369·6	− 7·4
62	157	159·9	+ 2·9	159·1	+ 2·1	158·5	+ 1·5
63	182	182·5	+ ·5	180·0	− 2·0	181·2	− ·8
64	209	213·0	+ 4·0	211·4	+ 2·4	211·5	+ 2·5
65, 6	512	519·2	+ 7·2	516·5	+ 4·5	516·1	+ 4·1
67	317	306·3	−10·7	302·5	−14·5	301·4	−15·6
68	353	336·1	−16·9	330·3	−22·7	328·7	−24·3
69, 70	739	759·0	+20·0	751·8	+12·8	748·1	+ 9·1
52–70	3,315	3,318·7	+ 3·7	3,301·5	−13·5	3,289·6	−25·4

The general result thus is that the expected deaths, 3,318·7, exceed the actual, 3,315, by 3·7. It would, I think, be nearly impossible to improve on these results. The expected deaths according to the graduations of Mr. Finlaison and Mr. Higham are also given in the above table, and it will be seen that they do not agree so closely as mine with the actual deaths; also that Mr. Higham's adjustment, which has the advantage in smoothness over Mr. Finlaison's, does not represent the aggregate mortality so well.

I now come lastly to the ages 19 to 51; and for these ages the values given in Table E give us the 5 points marked by the crosses in Diagram (7). These indicate a minimum of mortality between the ages of 30 and 35; and the first point we have to consider is whether there should be any point of maximum mortality between 35 and 50, in which case there must be a second point of minimum mortality. Bearing in mind that the mortality curve must not change its direction abruptly, we may give it either the direction shown in Diagram (7) or that in Diagram (8). The former of these shows no maximum of mortality, but does not run so near our 5 points as the second curve, which shows 2 minimum points and one maximum. In order to assist us in deciding which of these two is to be preferred, I refer to Diagram (5), and from this I notice that the mortality of the Peerage females shows a maximum at the age of 42; and that, although the mortality of the $H^{F(10)}$ table shows no maximum at the same age, it shows a great diminution in the rate of increase. I therefore give the preference to the form of curve shown in Diagram (8), and reading off the ordinates of the mortality curve and correcting them by inspection, I obtain the probabilities shown in column (2) of the following Table U:

TABLE U.—*Ages* 19–51. *Mr. Sprague's Graduated Probabilities of Dying, and Expected Deaths.*

x	q_x	Δq_x	Expected Deaths	SECOND APPROXIMATION	
				q_x	Expected Deaths
(1)	(2)	(3)	(4)	(5)	(6)
19	·0180	− 2	·2		
20	·0178	− 2	·2		
21	·0176	− 3	·3		
22	·0173	− 4	·3		
23	·0169	− 7	·5		
24	·0162	− 9	·6		
25	·0151	− 8	·8		
26	·0143	− 8	1·0		
27	·0135	− 7	1·2		
28	·0128	− 7	1·3		
29	·0121	− 6	1·6		
30	·0116	− 6	1·9		
31	·0110	− 5	2·2		
32	·0105	− 2	2·4		
33	·0103	0	2·5		
34	·0103	+ 3	2·9		
35	·0106	+ 3	3·4		
36	·0109	+ 3	3·9		
37	·0112	+ 6	4·7		
38	·0118	+ 2	5·5		
39	·0120	+ 2	6·8		
40	·0122	− 1	7·9		
41	·0121	− 3	8·9		
42	·0118	− 5	9·8	·0119	
43	·0113	− 2	10·6	·0116	10·9
44	·0111	− 1	12·2	·0114	12·5
45	·0110	+ 1	13·8	·0113	14·2
46	·0111	+ 2	15·7	·0113	16·0
47	·0113	+ 6	17·9	·0114	18·0
48	·0119	+10	21·0	·0115	20·3
49	·0129	+ 9	25·3	·0121	23·7
50	·0138	+ 9	29·8	·0130	28·1
51	·0147	+ 8	35·0	·0140	33·4
52	·0155	+ 8			
53	·0163	+ 7			
54	·0170	+ 7			
55	·0177				
			252·1		

The expected deaths calculated from these probabilities, are given in column (4). Summing these, we get the comparison with the actual deaths shown in the following table:

Table V.—*Ages* 19–51. *Containing the figures in Table U suitably groupt.*

Ages (1)	Deaths: Actual (2)	Deaths: Expected (3)	Difference (3)—(2) (4)	Second Approximation: Expected Deaths (5)	Second Approximation: Difference (5)—(2) (6)
19–29	9	8·0	−1·	37·4	− ·6
30–35	15	15·3	+ ·3		
36–38	14	14·1	+ ·1		
39–44	57	56·2	− ·8	56·8	− ·2
45–51	150	158·5	+8·5	153·7	+3·7
19–51	245	252·1	+7·1	247·9	+2·9

The agreement between the actual and expected deaths is sufficiently close, except during the last interval; and we must now correct our curve, so as to show a less mortality during that interval. I accordingly draw the dotted curve in Diagram (8), and from this I obtain the corrected probabilities given in column (5) of Table U. To these correspond the expected deaths in column (5) of Table V; and as these agree sufficiently with the actual deaths, our graduation is complete.

It may now be convenient to present the graduated values in the following table:

TABLE W.—*Female Government Annuitants: 4 years and upwards after purchase. The probability of dying in a year according to Mr. Sprague's adjustment.*

Age	Probability	Age	Probability	Age	Probability	Age	Probability
19	·0180	40	·0122	61	·0232	82	·1400
20	·0178	1	·0121	2	·0249	3	·1520
1	·0176	2	·0119	3	·0270	4	·1660
2	·0173	3	·0116	4	·0294	5	·1830
3	·0169	4	·0114	5	·0321	6	·2000
4	·0162	5	·0113	6	·0350	7	·2170
5	·0151	6	·0113	7	·0380	8	·2340
6	·0143	7	·0114	8	·0410	9	·2500
7	·0135	8	·0115	9	·0440	90	·2650
8	·0128	9	·0121	70	·0470	1	·2830
9	·0121	50	·0130	1	·0500	2	·3000
30	·0116	1	·0140	2	·0560	3	·3200
1	·0110	2	·0155	3	·0630	4	·3390
2	·0105	3	·0163	4	·0700	5	·3600
3	·0103	4	·0170	5	·0770	6	·3820
4	·0103	5	·0177	6	·0840	7	·4100
5	·0106	6	·0184	7	·0910	8	·4390
6	·0109	7	·0191	8	·0990	9	·4660
7	·0112	8	·0199	9	·1080	100	·5000
8	·0118	9	·0208	80	·1180	1	·5450
9	·0120	60	·0219	1	·1290	2	·6050

It cannot be denied that the graduation I have thus obtained, is better, both as regards freedom from irregularities and adherence to the original facts, than the graduations of Mr. Higham and Mr. Finlaison; and I say that this proves that the graphic method of graduation is preferable to the methods they have adopted. Its superiority is perhaps most marked at the ages where the observations are most numerous; and I therefore take exception to the sentiment expressed by Mr. Finlaison, as quoted in the early part of this paper, and held, I believe, by many other persons, that Mr. Woolhouse's is the most satisfactory method of adjustment when the observed facts are sufficiently numerous.

I now proceed to a different part of my subject, and will prove that it is undesirable to employ such formulas as Mr. Woolhouse's, Mr. Higham's, or Mr. Ansell's, not only because, as already mentioned, they will never entirely get rid of the irregularities in our observations, but also because they all have a tendency to introduce an error even into a regular series of numbers. I start with the proposition which I think will command universal assent, that, if we attempt to graduate a perfectly regular series of numbers, the result should be to leave it unaltered; and that, if our method of procedure alters the law of the series, and substitutes for the original series one following a different law, this proves that our method of procedure is faulty. Mr. Higham shows (*J.I.A.*, xxv, 249) that, when the series we are graduating proceeds by constant third differences, his formula reproduces the original series. The same will be found to be the case with Mr. Woolhouse's formula; but Mr. Ansell's formula has not the same property. This may be very simply proved as follows. By one of the fundamental propositions of the Calculus of Finite Differences, we have

$$u_{x+r}=(1+\Delta)^r u_x=\Big(1+r\Delta+\frac{r.r-1}{1.2}\Delta^2+\frac{r.r-1.r-2}{1.2.3}\Delta^3 +\frac{r.r-1.r-2.r-3}{1.2.3.4}\Delta^4+\ldots\ldots\Big)u_x$$

$$=\Big(1+r\Delta+\frac{r^2-r}{2}\Delta^2+\frac{r^3-3r^2+2r}{6}\Delta^3 +\frac{r^4-6r^3+11r^2-6r}{24}\Delta^4+\ldots\ldots\Big)u_x$$

Hence $$u_{x-r}=\Big(1-r\Delta+\frac{r^2+r}{2}\Delta^2-\frac{r^3+3r^2+2r}{6}\Delta^3 +\frac{r^4+6r^3+11r^2+6r}{24}\Delta^4-\ldots\ldots\Big)u_x$$

and $$u_{x+r}+u_{x-r}=\Big(2+r^2\Delta^2-r^2\Delta^3+\frac{r^4+11r^2}{12}\Delta^4+\ldots\ldots\Big)u_x$$

If we apply this result to transform Mr. Woolhouse's formula into one involving the differences of the function instead of its successive values, we have

$$\begin{aligned}
U_x={}&{\cdot}2u_x+{\cdot}192\,(u_{x+1}+u_{x-1})+{\cdot}168\,(u_{x+2}+u_{x-2})\\
&+{\cdot}056\,(u_{x+3}+u_{x-3})+{\cdot}024\,(u_{x+4}+u_{x-4})\\
&-{\cdot}016\,(u_{x+6}+u_{x-6})-{\cdot}024\,(u_{x+7}+u_{x-7})\\
={}&{\cdot}2u_x+{\cdot}192\,(2+\Delta^2-\Delta^3+\Delta^4-\ldots\ldots)u_x\\
&+{\cdot}168(2+4\Delta^2-4\Delta^3+5\Delta^4-\ldots)u_x\\
&+{\cdot}056(2+9\Delta^2-9\Delta^3+15\Delta^4-\ldots\ldots)u_x\\
&+{\cdot}024(2+16\Delta^2-16\Delta^3+36\Delta^4-\ldots)u_x\\
&-{\cdot}016\,(2+36\Delta^2-36\Delta^3+141\Delta^4-\ldots)u_x\\
&-{\cdot}024\,(2+49\Delta^2-49\Delta^3+245\Delta^4-\ldots)u_x\\
={}&u_x-5{\cdot}4\Delta^4u_x+\ldots\ldots
\end{aligned}$$

This result proves that, if the original series proceeds by constant third differences, that is to say, if the fourth and all the subsequent *ı repres* *ıts the*

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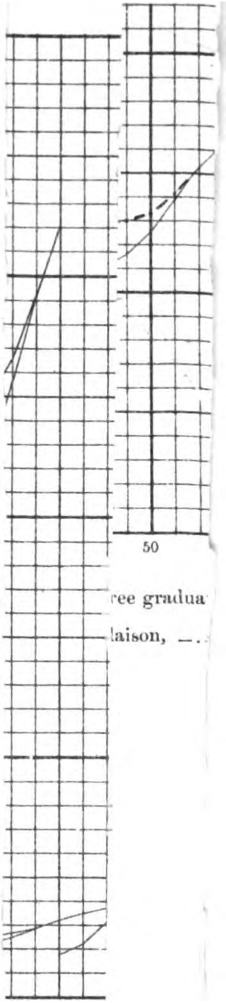

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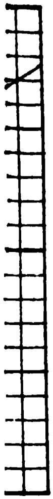

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of u_x vanish, then Mr. Woolhouse's formula reproduces series; but if the fourth differences are constant, then introduces a constant error, $-5{\cdot}4\Delta^4 u_x$, into the eries.

transform in a similar way Mr. Higham's formula, e get

$$U_x = u_x - 6.404\Delta^4 u_x + \ldots\ldots\ldots ,$$

e constant error introduced when the fourth differences constant, is larger than that given by Mr. Woolhouse's Treating similarly Mr. Higham's other formula, $\frac{4S}{300}$, we get

$$U_x = u_x - 6\Delta^4 u_x + \ldots\ldots$$

g in the same way with Mr. Ansell's formula, we have

$$U_x = u_x + 2{\cdot}3(\Delta^2 u_x - \Delta^3 u_x) + 3{\cdot}4\Delta^4 u_x + \ldots\ldots$$

oves that, even when the second differences are constant, ll's formula introduces a constant error into the graduated It is therefore not so good a formula as the others.

ill next suppose the original series to follow a law of a kind, so that $u_x = ca^x$, and will show that in this case f the formulas we have been considering, instead of cing the original series, will give us a new series, in which ms bear a constant ratio to those of the original series. In e we have $u_{x+r} = ca^{x+r} = a^r . ca^x = a^r u_x$. Hence, Woolhouse's la gives us

$$= {\cdot}2u_x + {\cdot}192(a + a^{-1})u_x + {\cdot}168(a^2 + a^{-2})u_x + \ldots\ldots$$

$$= \{{\cdot}2 + {\cdot}192(a + a^{-1}) + {\cdot}168(a^2 + a^{-2}) + {\cdot}056(a^3 + a^{-3})$$

$$+ {\cdot}024(a^4 + a^{-4}) - {\cdot}016(a^6 + a^{-6}) - {\cdot}024(a^7 + a^{-7})\} u_x$$

h proves that U_x bears a constant ratio to u_x. The graduated s will be identical with the original one if a has such a cular value as makes the multiplier of u_x always equal to 1. suggests the idea that there may be a particular law, such , if the terms of the series follow it, the application of olhouse's formula will leave them unaltered, or, in other words, roduce the original series. If this is the case, we shall have

$$U_x = u_x = {\cdot}2u_x + {\cdot}192(u_{x+1} + u_{x-1}) + {\cdot}168(u_{x+2} + u_{x-2}) + \ldots\ldots$$

is is a linear equation of differences which can be solved by ethods explained in all treatises on the Calculus of Finite

Differences. For this purpose we have first to find the roots of the algebraic equation

$$1=2\cdot+\cdot192\,(a+a^{-1})+\cdot168(a^2+a^{-2})+\cdot056(a^3+a^{-3})$$
$$+\cdot024(a^4+a^{-4})-\cdot016(a^6+a^{-6})-\cdot024(a^7+a^{-7})$$

which may be reduced to

$$3(a^{14}+1)+2(a^{13}+a)-3(a^{11}+a^3)-7(a^{10}+a^4)-21\,(a^9+a^5)$$
$$-24(a^8+a^6)+100a^7=0$$

It is not difficult to prove that this equation has four roots equal to 1, or has a factor $(a-1)^4$; that it has two real negative roots; and that the remaining eight roots are imaginary and of the form $\rho_1(\cos\theta_1\pm i\sin\theta_1)$, $\rho_1{}^{-1}(\cos\theta_1\pm i\sin\theta_1)$, $\rho_2(\cos\theta_2\pm i\sin\theta_2)$, and $\rho_2{}^{-1}(\cos\theta_2\pm i\sin\theta_2)$. If we put $-p$ for one of the two real negative roots, the other will be $-1\div p$, and the complete solution of the equation of differences is

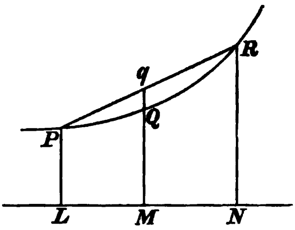
$$u_x=\mathrm{A}+\mathrm{B}x+\mathrm{C}x^2+\mathrm{D}x^3+\mathrm{E}(-p)^x+\mathrm{F}(-p)^{-x}$$
$$+(\mathrm{G}\rho_1{}^x+\mathrm{H}\rho_1{}^{-x})\cos x\theta_1+(\mathrm{I}\rho_1{}^x+\mathrm{J}\rho_1{}^{-x})\sin x\theta_1$$
$$+(\mathrm{K}\rho_2{}^x+\mathrm{L}\rho_2{}^{-x})\cos x\theta_2+(\mathrm{M}\rho_2{}^x+\mathrm{N}\rho_2{}^{-x})\sin x\theta_2,$$

where the coefficients A, B, &c., are arbitrary constants. It follows that, if the terms of the series follow this law, then the application of Woolhouse's formula will leave the series unaltered; and, on the contrary, if the terms follow any other law, then Woolhouse's method of graduation will disturb that law, and give us a new series following a different law. This confirms the result we obtained above; for, putting E, F, and the subsequent coefficients all zero, we have

$$u_x=\mathrm{A}+\mathrm{B}x+\mathrm{C}x^2+\mathrm{D}x^3;$$

or the third differences are constant, and the fourth differences vanish; and we have already seen that this is a function that gives a series of terms which are left unaltered when Woolhouse's formula is applied to them. It is obvious that the remaining terms are quite unsuitable for the representation of any function depending on human mortality; and as neither the rate of mortality nor the numbers living can be represented through the whole of adult life by a series proceeding by constant third differences, it follows that Mr. Woolhouse's formula is not suitable for graduation purposes, because it has a tendency to distort the law which the facts follow. The same is the case with both Higham's formulas and Mr. Ansell's, and with all formulas nilar kind.

e formulas which we have considered, Mr. Ansell's is the most open to objection. In this formula all the terms are positive, and it can be proved that if any formula of this kind is applied to graduate a series of values which, when plotted down as equidistant ordinates, give a curve without any point of inflection, then the ordinates corresponding to the graduated values always give a series of points which lie on the concave side of the curve. The simplest case of this proposition is when a new series is obtained in which each term is the arithmetical mean between two terms of the original series; for instance if $U_x = \frac{1}{2}(u_{x-1} + u_{x+1})$. Let *PL, QM, RN,* in the annexed curve, represent the three terms u_{x-1}, u_x, u_{x+1}; then, if *PR* cut *MQ* produced in *q*, *qM* will represent U_x, and *q* will evidently lie on the concave side of the curve. Let us next suppose that



$$U_x = au_x + b(u_{x+1} + u_{x-1}) + c(u_{x+2} + u_{x-2}) + \dots + l(u_{x+n} + u_{x-n}),$$

where $a, b, c, \dots l$, are all positive. It is clear that, if $u_{x-n}, \dots u_{x-1}, u_x, u_{x+1}, \dots u_{x+n}$, are all equal, then we ought to have U_x equal to each of them; and this gives us the condition $1 = a + 2(b + c + \dots + l)$. Now if the curve, of which u_x is the ordinate, has no point of inflection between the limits $x-n$ and $x+n$, and is always convex to the base line, u''_x will be always positive between those limits. But by a theorem of the Differential Calculus

$$\left.\begin{aligned} u_{x+r} &= u_x + ru'_x + \tfrac{1}{2}r^2u''_{x+\theta_1 r} \\ u_{x-r} &= u_x - ru'_x + \tfrac{1}{2}r^2u''_{x-\theta_2 r} \end{aligned}\right\} \text{ where } \theta_1 \text{ and } \theta_2 \text{ are each } < 1.$$

Hence $\quad u_{x+r} + u_{x-r} = 2u_x + \frac{1}{2}r^2(u''_{x+\theta_1 r} + u''_{x-\theta_2 r})$

Now since u''_x is always positive between the limits $x-n$ and $x+n$, $u''_{x+\theta_1 r}$ and $u''_{x-\theta_2 r}$ are both positive, and therefore $u_{x+r} + u_{x-r} > 2u_x$.

Hence
$$\begin{aligned} U_x &> au_x + b(2u_x) + c(2u_x) + \dots + l(2u_x) \\ &> (a + 2b + 2c + \dots + 2l)u_x \\ &> u_x \end{aligned}$$

which proves the proposition above stated.

The inference I draw from these investigations is that we must include in one general condemnation all such graduation formulas as Mr. Woolhouse's, Mr. Higham's, and Mr. Ansell's. They all have a tendency to distort the true law of the facts; or, adopting

Mr. Higham's metaphor, they do not simply reduce a stumbling-block in the road, to a fraction of its original height, and spread the material in a curve on either side; but they also alter the level of the road throughout, and disturb the gradients where they were already properly adjusted. No objection of this kind can be made against the graphic method; for if it be applied to any series free from irregularities, as soon as we have ascertained this fact, the fundamental principle of the method informs us that the series does not require any adjustment, and must be left unaltered.

I trust that the explanation here given of the method will lead many of the members of the *Institute* to regard it in its true light, as a simple and strictly practical method, that may be applied to observations of any kind, whether they follow a simple or a complicated law; and a method which, with a little practice, will give satisfactory results in all cases.

Discussion.

The Chairman (Mr. G. Humphreys) having briefly invited discussion,

Mr. A. J. Finlaison said that the members of the Institute were much indebted to Mr. Sprague for the very full way in which he had explained the graphic method. The table marked P (p. 100) furnished a demonstration that in skilful hands the graphic method was an instrument capable of effecting a graduation that would closely agree with the standard he (Mr. Finlaison) laid down in the report on the Mortality of Government Annuities, namely, that "The criterion which should be applied to whatever system of adjustment may be adopted, consists in the derived series being found to return an equivalent number of deaths to the original facts, when the adjusted death-rate is assumed to be experienced by a body of persons exactly similar to that upon which the table is founded." Mr. Sprague had shown in his table of comparison that the actual deaths in the original observation numbered 9,171, and the expected deaths by his graphic method of adjustment, 9,164. That was not the whole result of the comparison, for in each group the departures from the original number of deaths were very small. Mr. Sprague had contrasted the extent of the departure from the original facts, with the departures shown by the adjustment he (the speaker) had adopted; but if he (Mr. Finlaison) had had the arrangement of the facts, he thought he could have formed the groups in such a manner that the differences between the actual and expected results, according to the two methods, might have been reversed. In Table N the departures of the expected deaths at individual ages, were greater than the difference shown in the totals; and similarly, in the graduation he had himself adopted, departures from the actual facts at individual ages might be found as high. He thought that either of the graduations was sufficiently accurate, as there was a departure, in one, of only nine deaths out of 9,000, or one in 1,000 deaths; in the other, of no more than seven. In making an adjustment by the graphic method, he should think a preliminary adjustment would be of considerable service to the adjuster. In diagram No. 3 the final graphic curve did not seem to be laid down. [Mr. SPRAGUE—No. It is given in Diagram (8).] The curve would, no doubt, show fewer ups and downs, than the line drawn by joining the successive points by Woolhouse's adjustment; but the practical results on the monetary values which would be obtained from the use of either of the two curves would not differ in any material degree. The statement that the graphic method resulted from a very old idea, was correct; for he believed Descartes himself was its originator. Every delineation of a curve on a plane surface referred to co-ordinates, was a growth from his idea. The graphic method of adjusting tables of mortality might be said to arise by evolution from the method by which Milne graduated the Carlisle Table. The irregularities shown in the adjusted figures at the younger ages, in the Female Life Annuitant Table, were, he thought, due solely to the small number of facts. When the number of deaths shown in a collection of statistics was so small as 2 or 3 in a year of age, no deduction of value could be obtained from them; and for that reason he purposely omitted the ages under 40 years in his graduation of the table. The method Mr. Sprague had expounded was one that every future adjuster of mortality tables must bring under his consideration; but if the method were adopted, the results should be accompanied by a table which would give the means of making a comparison of the number of deaths according to the adjusted table, with the original facts, so that it could be ascertained whether there was a sufficiently close agreement. In laying down curves on paper drawn on the scale adopted by Messrs. Layton, he had found a difficulty in making fractional divisions between the closely-ruled lines. He could not see any objection to using larger sheets of paper so as to form a larger scale. By that means a curve on a very much larger scale could be laid down, and its deviation from any standard of symmetry more satisfactorily shown.

Mr. G. KING thought that the paper would be valuable as long as actuarial science lasted. He would first refer to the following sentence at the close of the paper: "The inference I draw from these investigations is, that we must include in one general condemnation all such graduation formulas as Mr. Woolhouse's, Mr. Higham's, and Mr. Ansell's." The question naturally arose how it was that two great authorities like Mr. Sprague and Mr. Woolhouse differed, and one great reason probably was that they were not speaking of the same function in the graduation. His (Mr. King's) feeling strongly was, that they must suit the formula of graduation to the function to be graduated. The same formula or the same method would not suit all functions in all circumstances. Mr. Sprague was operating upon the rate of mortality, whereas Mr. Woolhouse, in his various papers, while he discussed his formula in a general way, graduated only the column of living, or, what was the same thing, the column of dying. He quite admitted that, in many instances, Mr. Woolhouse's formula would not apply even to such columns as these. In fact, where the rate of change in a function was rapid, or where there was more than one law apparently operating at the same time, Mr. Woolhouse's formula was inapplicable. But, in dealing with the column of living, or with the column of dying, the law generally lay very close indeed to what Mr. Woolhouse assumed. Had Mr. Sprague fully considered the question that Mr. Woolhouse fitted a different curve to each small section of his table? He did not fit one curve throughout, and consequently the so-called "constant" error that Mr. Sprague spoke of, would vary at one point and another, and become insignificant in the results. For practical purposes, in almost all cases Mr. Woolhouse's formula gave excellent results, and it was so very convenient to use, that he (Mr. King) should have no hesitation, although there might be a slight theoretical error in it, in still employing it. One thing was fortunate, namely, that the errors arising from different good systems of graduation were infinitesimal, and they were really discussing theoretical points that were practically not of consequence. They might be apt, however, in doing this, to overlook the question, what was the real object of graduation? Probably the reply would be, To get a smooth curve; but he did not think that quite correct. To his mind, the reply should be, to get the most probable deaths; and it was certainly a fact that a well-graduated table was more in accordance with the original facts, he might almost say, than the original facts themselves. They must remember that when they translated the "exposed to risk", and "deaths", into the l_x column, they derived a mortality table that departed to a very large extent from the original facts. They assumed that the same rate of mortality that occurred in the "exposed to risk", would also occur in the different number of lives, l_x; and therefore, in passing from the "exposed to risk" to the l_x column, they departed from the original facts. His opinion was that, by properly graduating the l_x column, they really arrived at more probable deaths than in retaining the ungraduated column; and their object was so to interpret the original facts as to get the most probable deaths. In all functions they had to deal with, they did get a smooth curve by a good system of graduation, and that was one test of a good system; but in his opinion the first object was not to get a smooth curve, but to get the most probable curve. One reason why they would very often use Mr. Woolhouse's method instead of the graphic method, was that it gave much less trouble. If they wanted to compute a very important table, a table that was to be historical, it would be worth while to graduate and re-graduate, to try this and try that, as the graphic method did; but if they wanted rapidly, for practical purposes, to make use of the figures before them, it would be too laborious to make all the tests necessary to see that the graphic method had been accurately performed; whereas Mr. Woolhouse's formula would give results sufficiently accurate. He would ask Mr. Sprague why he resorted to certain grouping before applying the graphic method? Would it not be sufficient to take the rates of mortality without grouping, and try and run the curve through them? The various groupings which gave such contradictory results as Mr. Sprague had pointed out, might, he thought, be somewhat dangerous. Mr. Ansell's method of graduation had been spoken of, and considerable prominence—undue prominence, he thought—had been given to it; because, after all, it was an entirely empirical method, and one that no one would think of resorting to: in fact, Mr. Ansell himself did not keep to it, but varied it arbitrarily here and there. He (Mr. King) did not think it should be included among the scientific formulas of graduation.

Mr. A. H. **Bailey** remarked that Mr. King was perplexed that Mr. Woolhouse and Mr. Sprague, whose mathematical attainments were so well known, should differ on a purely mathematical problem; but it seemed to him that this was not a purely mathematical problem. Mr. Sprague was suggesting a method of graduation, and for that purpose he used mathematical, graphical, and arithmetical processes. But as this was not a question of pure mathematics, it was quite intelligible that different solutions of the same problem should have been arrived at by different mathematicians. Of the many papers on the subject, he thought Mr. Sprague's was the most complete; and he was sure it would be very useful to all. Although he had not given the subject sufficient study to pronounce a confident opinion upon it, the graphic method commended itself to his judgment as the best in practice. He was of opinion, however, that graduation was a necessary evil. De Morgan said* that it could not be too strongly condemned; for, by using the graduated results, they sometimes lost sight of the lessons taught by the original facts. That they did graduate away some peculiarities which it was necessary to keep in mind, one illustration would show. There was abundant evidence that the rate of mortality from 20 to 25, was in excess of that from 25 to 30; and yet in all the graduated tables of mortality of which they made use, this peculiarity disappeared. He had always thought, although Mr. King seemed to take exception to it, that the object of graduation was to represent the law of mortality by a regular curve; and sometimes the hope had been entertained that the methods of analytical geometry might be applied to this law, and an equation found to the curve—like the equation to the parabola, or the ellipse, or the logarithmic curve, or any other known curve. He did not, however, think that that hope would be realized. De Morgan had also said, and he agreed with him, that it would be better to apply the process of graduation to the money results, and not to the original facts. It might be necessary to insert in prospectuses a table showing that the premium for one year steadily increased from the age of 20 upwards. But it was well to bear in mind that this regular progression was not borne out by observation. There were climacteric periods in human life, and the law of mortality did not follow a regular curve. Therefore, although he knew this was a subject which had always had a fascination for mathematicians, the process was one, he thought, which must be used with a good deal of caution.

Mr. A. F. **Burridge** remarked that the graphic method had suffered in the estimation of some, by the feeling that it required a particularly skilful person to handle it. But as one who had used the method, he thought it could be employed by a far less skilful operator, with a fair average amount of success. Although Mr. Sprague was not the originator of the method, he distinctly was the modernizer of it; for he had brought it before the Institute in a complete and scientific form. With reference to Mr. Sprague's statement, that they had no right to expect that a grouping according to quinquennial ages, or in any other definite way, would be successful, he pointed out that, in census returns, the quinquennial groupings were those alone that were given, so that they had of necessity to be used. He had roughly grouped the figures in Table E in quinquennial groups; and the quinquennial grouping followed very much the same law as Mr. Sprague's much closer and more accurate grouping, that is, the rate of mortality reached its minimum between the ages of 30 and 35. Mr. Bailey had referred to the point of the increased mortality in early manhood, and he (the speaker), in graduating his table, was led to consider the question, which Mr. Sprague had said must occur in a graphic graduation, as to what discrepancies from the regular curve should be obliterated and what irregularities retained. Mr. Sprague had adopted a thoroughly sound view in his treatment of the case: when he saw an unexpected peculiarity in the tables, he consulted other standard tables, with the object of seeing whether the same peculiarity was there to be found. The difficulty of the graphic method certainly lay in the fact that many successive trials were necessary before one could arrive at a satisfactory result; but he did not think one required to be an eminent mathematician in order to be able to use this method with advantage, especially after Mr. Sprague's interesting and useful exposition of the subject.

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* *Essay on Probabilities*, p. 162.

Mr. C. D. Higham observed that he did not think Mr. J. A. Higham had ever claimed that his formulas were the best in existence, certainly not the best possible; but he maintained that the results his method brought out were as good as, or perhaps a little better than, Mr. Woolhouse's, while they could be applied with a very great deal less labour and with great facility. Nor did the work demand experienced skill, for it could be done by a junior clerk. Moreover, any number of people using such a formula as his, would bring out the same results, whereas by the graphic method there would be as many different graduations as operators. Then, again, he advised that, even if the graphic method be adopted for the perfect table, it was very desirable that the rough results should also be graduated for purposes of comparison, by what he would call a mechanical method, so as in some measure to put a check on the figures, if not the fancies, of the pencil-worker.

Mr. T. G. Ackland thought that Mr. Sprague's paper treated not only of the graphic method, but also of the whole theory of graduation, in a way that had not been previously attempted. Mr. Sprague had expressed disapproval of certain methods of graduation, because they made assumptions as to the law of mortality; but he could not, in reading through the paper, help asking himself whether the graphic method did not somewhat err in the same direction. Mr. Sprague had grouped the facts by different methods, and had shown very clearly that the graphic method required a skilled and careful grouping, to get out even fairly accurate results; and in this grouping he had been very anxious to discern the law underlying the facts. He (the speaker) imagined, however, that it must be in many cases a matter of opinion as to what the precise law might be, and what astronomers called the "personal equation" would largely enter into the matter. This appeared to him to be one objection to the graphic method. Mr. Woolhouse never proposed in his method of graduation to go beyond third differences, so that the fourth differences would be constant; and Mr. Sprague had shown that by his method a constant fourth difference error was introduced. Mr. Woolhouse had, however, never proposed to do more than that. He had stated in the introduction to the Institute of Actuaries' Life Tables, that, if the curve be originally of the third order, his method of graduation would reproduce it precisely. With regard to the considerable difference of opinion that arose in drawing the Government Annuity curve for ages younger than 40, he thought these differences due to the fact there were only 40 deaths at these ages, and it was almost a waste of time to attempt to draw a curve based upon such a small number of deaths. Mr. Finlaison had reminded the meeting that Milne was practically the first to deal with the graphic method. In many ways his methods were closely identical with Mr. Sprague's, particularly where Mr. Sprague made the polygonal correction, by passing the curve, not through the points on the diagram, but a little inside or outside, so as to make the area of the polygon approximately equal to the area included by the curve. The latter part of the paper, where Mr. Sprague mathematically demonstrated a test by which different methods of graduation might be judged, was, in his opinion, at least as valuable as the earlier part. Nothing in this direction had been so fully done before. It enabled them at once to test any given method, and see how far it gave correct results. For instance, taking the method that Mr. John Finlaison has given in his Report of 1829, he found, as a matter of fact, that Mr. Finlaison's method of summation in fives, when applied either once, twice, or thrice, affected second differences, and, therefore, introduced an error even where the original curve was of the second order. The further this process of graduation was carried, the more the error increased; so that the third graduation had an excess of error as compared with the first or second. Taking a series of squares of the natural numbers, and summing them in fives by Mr. Finlaison's method, he found that the graduated central value of 256 (the square of 16) was subject to a constant error affecting the second differences, so that, as a result of the first summation, the graduated value of 256 was brought out at 258, by the second summation it was brought out at 260, and by the third at 262, and so on, showing a steadily increasing error. [Mr. Finlaison—That method particularly assumes that the curve is a curve of the first order.] If they graduated by that process a series of the natural numbers, or of numbers in arithmetical progression, they would find the series not affected by the graduation, however far they might proceed. Mr. Ansell's formula was, he thought, hardly worthy of serious consideration. There appeared to be no law about it at all. Mr. Sprague did not give a summary of his results. He gave them in sections, from 70 to the end, 51 to 70, and 19 to 51; but nowhere brought them together, so as to enable them to judge how near the actual deaths, as a whole, were to the expected. He had found that the actual deaths were 12,731 at all ages, and the expected deaths by Mr. Sprague's method are 12,735·6, an error of only 4·6. This illustrated his entire success in graduating the facts. Again, Mr. Sprague had not given from 19 to 51 the expected deaths by Mr. Finlaison's method (see Tables U and V), so that they were not able to compare them in the same way as at other groups of ages.

Mr. H. W. **Manly** said he became a convert to the graphic method after trying nearly all the other methods with indifferent success. He was at one time a firm believer in Mr. Woolhouse's method. The first thing that shook his faith in it was that, when plotted out in a diagram, the graduated results presented the most remarkable zigzags. Considering that these could not be natural, he graduated these graduated figures, and plotted the new results out on the same sheet of paper; and he then found to his surprise that some portions of the original curve which appeared to be fairly perfect, had got upset by the second graduation and formed a new zigzag. Then he tried various experiments, such as Mr. Sprague's plan of grouping, so as to smooth down the worse features; but with no success. Other processes which gave smooth curves, obliterated the features of the original facts. At last he came to the conclusion that probably the best method was to adopt Mr. Sprague's plan and draw a line through the original facts plotted out on diagrammatic paper. From the nature of the tables he was constructing, he graduated the d_x from an unadjusted table of the l_x constructed from the original facts; and one conclusion he came to during the work was that, in order to obtain a curve which should embody all the features of the original facts, not only the sum of the d_x, in special groups of ages, should correspond with the sum of the original groups of d_x, but that the sums of the l_x in the same groups should correspond with the original groups of l_x, and that by that means they would get the most perfect graduation possible. Mr. Sprague had given a very excellent account of this method, but he had not imparted the secret of how his curves were drawn. It was very well to say that they were to draw a curve through the zigzag lines, representing the original facts plotted out on diagrammatic paper, that is to say, on squares of about one-eighth of an inch in size, and to say that these lines were to be drawn so carefully that they could read off accurately the tenths of one-eighth of an inch. No hand could possibly do it, and they must have recourse to some mechanical process. He used very largely the French curves, but there was a very excellent set of curves which might be used in drawing large diagrams, called ship's curves. It would afford them useful information if Mr. Sprague would say how he drew his curves—they were not done by the hand and the eye alone. Some objection had been raised to Mr. Sprague's irregular groupings. After using this method a little time that would wear off, and they would begin to adopt it naturally. With reference to Mr. Finlaison's suggestion of having larger squares, he had tried that, and found it so difficult to draw a large curve, so as to survey its general course, that he gave it up, and now always reduced the diagrams to the small size represented in Diagram I. The difficulty of reading off the heights of the curve from a small diagram, was more than counter-balanced by the great advantage of seeing the work in one compact whole; and any irregularities that might arise from imperfect reading, could always be adjusted afterwards.

Mr. T. B. Sprague, in reply, said he was pleased to find that the method had made considerable progress since it was last under the consideration of the Institute, and hoped that his present paper would contribute further in the same direction. Mr. Finlaison had suggested a preliminary adjustment by Woolhouse's or Higham's formula, but he did not see the necessity for anything of the kind. He had not found it necessary, and doubted whether it would be of any real advantage. On the contrary, if he was right in saying that these formulas had a tendency to introduce an error into even a regular series of numbers, then the subsequent graduation by the graphic method would first of all have to get rid of that error. Therefore he preferred to take the crude facts and group them tentatively, thus making them tell their own tale and show their own law. Mr. Finlaison spoke about drawing a curve on a larger scale. His experience, however, agreed with Mr. Manly's, that, if they had a very large curve, they could not follow the sweep of it so well as when they had a manageable curve that they could take in at a single glance. He therefore thought it best to have a small and compact curve. There was a great deal in what Mr. King had said, to the effect that the formula that was used should be one that was suitable to the function to be graduated. In connection with this, he should mention that Mr. Finlaison's probabilities of dying, as given in Table "Q", were obtained by applying Mr. Woolhouse's method to the unadjusted number living, or rather to the unadjusted number dying, in the same way as it was applied by Mr. Woolhouse himself. He did not quite understand Mr. King, when he said that their primary object was not to get a smooth curve. The fundamental principle he (the speaker) went upon was, that there were no jumps and discontinuities in nature, that all nature—human mortality included—proceeded by regular laws. There might be ups and downs, but there would not be sudden and abrupt irregularities. When the numbers were sufficiently large, they would certainly find that nature proceeded without a jump. Therefore he held that their object was to get a smooth curve. As regards the grouping of the original numbers, whether it was better to take the numbers for individual ages, or to group them as he had done, that was a point upon which he would not speak dogmatically. What he had undertaken to do, was to show how he himself went to work. He did not say, with regard to the matters of detail, that his method was absolutely the best; other operators might find some other method preferable, but he found the method of grouping was the best for him. The same applies also to the drawing of the curve. At one time he had used the French curves that Mr. Manly had referred to, but he afterwards laid them aside, and now always drew his curve by hand. He did not perhaps get it accurate enough the first time, but he touched it up and managed in that way to get it sufficiently regular. He did not think Mr. Ackland was quite correct when he said the object of Mr. Woolhouse's method was to draw a curve of the third order. Also Mr. Ackland had remarked upon successive applications of the same method, and Mr. Manly also spoke of the same thing. He (Mr. Sprague) understood, however, that Mr. Woolhouse himself did not think that permissible; and this agreed with his own conclusions. He had shown that the method introduced an error the first time it was applied, and that it introduced a similar error in the same direction a second time, and each time it was applied an additional error in the same direction was introduced. Therefore, the oftener they applied it, the larger would be the error introduced into the law of the numbers.

History of Life Assurance in the United Kingdom. By Cornelius Walford, F.I.A.

(*Continued from* p. 24.)

Globe Insurance Company.—Early in the year 1799 a plan was laid before Mr. Pitt for forming a Chartered Insurance Office, for granting Insurance against Fire, and for Insurance of Lives; for buying and selling Annuities, and for receiving deposits from Friendly Societies, and the Industrial Classes: to be called the *Globe or General Insurance Office.* It was understood that the plan received that Minister's approval, and a Bill was introduced to Parliament embodying these objects. The measure encountered some opposition. In the first place, the Bank of England objected to the Deposit branch, and the clauses relating thereto were accordingly struck out. Then it was opposed by several of the Insurance Offices; but the Bill finally passed through both Houses, and received the Royal Assent.

A special feature of this Company—which was finally constituted under a deed of copartnership—was that of having *one million* of paid-up Capital, divided into shares of £100 each, treated as stock. This large paid-up Capital was supposed to lend confidence to the new undertaking, which transacted Fire business also, and had designed to embrace Marine Insurance. The Capital in course of time became a great incumbrance, and finally led the Office into amalgamation.

Ireland.—I have heretofore said nothing regarding Life Assurance in the Sister Kingdom beyond the mere mention of a work treating (*inter alia*) of Life Assurance, published in Dublin in 1770. Almost the first indication of the practice of Life Assurance here was the passing in 1786, by the Irish Parliament, of 26 Geo. III, c. 3, *whereunder a stamp of* 2*s.* 6*d.* was imposed upon all Life policies. But by 56 Geo. III, c. 56 (1816, U. K.), the rate was increased as follows: policy not exceeding £500, 10*s.*; but exceeding £1,000, 20*s.*; not exceeding £3,000, 30*s.*; exceeding this, 40*s.* In 1799, two Insurance Companies were founded in Dublin, the *Commercial* and the *Royal Exchange,* each of which transacted Life Assurance business in connection with Marine and Fire Insurance. These were proprietary Companies. In 1808 the *Hibernian* Office was founded; in 1822 the *National Assurance;* and in 1823 the *Royal* and the *Shamrock;* both of the latter were Joint-Stock Companies, and carried on Life as well as Fire business. In 1824 the *Patriotic* Fire and Life was founded.

First National Census.—In 1801 the first National Census of the inhabitants of Great Britain was taken—a significant step in regard to Life contingencies. The number of inhabitants was found to be—England, 8,331,434; Wales, 451,546; Scotland, 1,599,068; Army, Navy, &c., 470,598—total, 10,942,646. The census of *Ireland* was first taken in 1811, when the population was 5,937,856.

Policy Stamps.—In 1804 another exemption was made in favour of Life Assurance—I suspect also by Pitt, who returned to office that year,—this was from the stamps on the policies. Life policies had never been specifically named in the early Stamp Acts, but had been classed under the general head "Policies of Insurance", and they carried the same stamps as Fire and Marine Insurance policies. This year they were exempted entirely; but the favour was a brief one, lasting only until 1808. I propose, later in this history, to give a succinct enumeration of the fiscal changes which have occurred in Life policy stamps.

New Life Offices.—In 1805 there was established another proprietary Insurance Company, the *Albion,* Capital £1,000,000, in 2,000 shares of £500, of which but 10 per-cent was paid up. The business embraced Fire and Life, and in the following year (1806) three important Life Offices were organized, namely:—

1. The *London Life,* a Mutual Office, the distinguishing feature of which was that its profits were to be divided amongst its members during life, by means of a reduction of the annual premiums. It had a guarantee fund, subscribed by the founders for protection in the early years of the Society. The affairs of the young Society were conducted with the greatest economy, on an upper floor in St. Paul's Churchyard.

When the Society had become firmly established, the premiums were increased—I believe on two occasions—on the principle that it was more advantageous for members to join a firmly-established than a new Assurance Office.

2. The *Provident Life* Office, with a Capital of £250,000, in shares of £100. The founder was Mr. J. T. Barber Beaumont—a gentleman largely connected with philanthropic movements—and the object of the Office was to "enable the industrious and economical to "appropriate their savings most beneficially to their "different objects of prudence and affection."
3. The *Rock* Life Insurance Office, with a Capital of £1,000,000, in shares of £25, of which £5 was to be paid. "To be established by a deed of agreement, to be enrolled in the Court of King's Bench." The rates of the *Equitable* Society were adopted.

Industrial Assurance.—In 1807 the first steps in the direction of what is now known as "Industrial Assurance"—unless the *Provident* Life of the preceding year can be deemed the first—was propounded. The scheme was to establish an Office, to be called *The Poor's Assurance Office.* Commissioners were to be appointed by the Crown to take the management of the enterprise; and the persons to be entitled to the benefits of the Office were those "who subsisted wholly or principally by the wages of their labour." A Bill was introduced to Parliament "for establishing "a Fund and Assurance Office for investing the Savings of the "Poor." This was referred to a Committee, and rejected by Parliament.

The machinery of the Post Office was to be employed. The project was more than half-a-century too soon.—*See* 1861.

In this same year the *Amicable* Society obtained powers, under a new Charter, to issue policies for short periods, or on joint lives, or any other contingency of life, thus becoming for the first time a Life Office, as ordinarily understood. It adopted a graduated scale of premiums; but no more than 8,000 persons were to be members of the Society at any one time.

In this year also (1807) there were founded the following Life Offices:

1. *Eagle* Insurance Company, with a Capital of £2,000,000, in shares of £50, of which £5 was to be paid up. The business was to extend to Fire as well as Life—hence, probably, the large Capital. The limit to be insured on any life was £5,000. This Company has played an important part in the history of Life Assurance.
2. *Hope* Insurance Company, with a Capital of £2,000,000, in shares of £50. The business was to extend to Fire Insurance. It was a provision of the Deed that every person holding Stock in this Company was to insure either his own life or the life of another therein, according to a graduated scale stated: "And the " sooner the insurances are effected agreeably thereto, " the more valuable will be their shares, as from the " date thereof the profit of the Life Fund will " commence."
3. *West of England* Insurance Company, with a Capital of £600,000, founded at Exeter, for carrying on the business of Fire and Life Insurance—being the third provincial Assurance Company founded in England. The rates of premium were advertised to be 10 per-cent below those deduced from the *Northampton* Table of Mortality.

In 1808 Mr. Francis Baily, "of the Stock Exchange", published the first edition of his work: *The Doctrine of Interest and Annuities Analytically Investigated and Explained*—a work which attracted much attention, and deservedly so. A second edition, in two 8vo. volumes, was published in 1813, and long remained a standard work.

Two other important Life Offices were founded this year:

1. The *Atlas* Assurance Company, with a Capital of £1,200,000, in shares of £50, of which £5 was to be paid up. The business was to embrace Fire as well as Life. The Company a few years later obtained a special Act of Parliament, to the powers of which reference will hereafter be made.—*See* 1825.
2. The *Norwich Union,* founded in the City of Norwich as a Mutual Life Association; and which, after a very stormy youth, settled down in a respectable and useful middle age.

In 1810 Mr. Francis Baily published *An Account of the several Life Assurance Companies established in London, containing a View of their respective Merits and Advantages.* This was a publication of a popular character; and as it was written in a peculiarly outspoken manner, claimed much attention, and was, I believe, productive of good results. In the preface the author says:

> "Numerous offices have lately sprung up in the Metropolis for the purpose of granting Assurances on every possible contingency amongst lives in general; and it therefore becomes every one engaged in the public business of life to study the subject with attention. But, notwithstanding the importance and utility of these enquiries, I fear that Life Assurances are too often effected in a very loose and careless manner; and which can arise only from an ignorance of the true nature and constitution of the several Societies that have been formed for this purpose. A person is apt to imagine that because the rates are the same in all the offices, it can be of little consequence at *which* of them he effects his policy. If he is himself in doubt upon the subject, he applies to some of his acquaintance, who, influenced by partiality (or perhaps by a more disgraceful motive), recommend him to that office with which they are more immediately acquainted or connected; or who, equally ignorant with himself, confirm him in the indifference of his choice. Thus blindly driven to a hasty decision, he discerns (perhaps too late) the error into which he has either inadvertently fallen or been insidiously betrayed."

These remarks are not without force even in the present day. This same year there was founded:

1. The *Sun* Life, a proprietary Office, worked in association with the *Sun* Fire (founded a century previously), but having a distinct capital and constitution.
2. The *Birmingham* Life Assurance and Annuity Office, which transacted but a small business; and this, in 1826, was incorporated with that of the *Provident* Life.

In 1814 there was published a pamphlet: *Life Insurance—Important facts showing the successive reductions that have taken place in the terms for the Insurance of Lives, and the probability of failure in some recent Schemes: recommended to the serious consideration of Persons interested in the Permanent Stability of such Establishments.* By Philanthropos—namely, Mr. Beaumont, the founder of the *Provident* Life. Some of his remarks in this pamphlet were almost prophetic; but it is a fact that no Life Office has at present failed from the reason of its rates being too low; there have always been more cogent reasons.

The year 1815 is characterized by two marked events. The first was the establishment of the *Scottish Widows' Fund* Life Office in Edinburgh—being the earliest Life Assurance Company founded in *Scotland*. The Society was professedly constituted on the model of the *Equitable* Society, embracing its supposed best features; and while its progress was very slow in the period of its youth, in its more mature age it has, in volume of business at least, far distanced its prototype.

Carlisle Table of Mortality.—But the event of most general interest characterizing the "Waterloo" year, from an Insurance point of view, was the introduction of the *Carlisle Table of Mortality*—a Table which in course of time produced very marked results in the practice of Life Assurance. The Table was promulgated in: *A Treatise on the Valuation of Annuities and Assurances on Lives and Survivorships; on the Construction of Table of Mortality; and on the Probabilities and Expectations of Life: wherein the Laws of Mortality that prevail in different parts of Europe are determined, and the Comparative Mortalities of different Diseases, &c., of the two Sexes are shown. With a variety of new Tables.* By Joshua Milne, Actuary to the *Sun* Life Office. (2 vols. 8vo, pp. 783.) One of the most important works ever issued from the press bearing upon Life Assurance.

It was this work of Milne's which first showed the various underlying causes affecting the law of human mortality, as far as any such law can be admitted. It showed that the influences of climate, of location, of parentage and nationality, all have a bearing upon the probable duration of life. It affiliated the entire range of Vital Statistics into the Science of Life Contingencies; it suggested considerations of Public Health, and made its readers familiar with the importance of periodical enumerations of the people: drawing attention to the great problems of Infant Mortality and Pauperism. Students of Life Assurance will find in this work a perpetual source of instruction.

For a full half-century the Carlisle Table was almost the *vade mecum* of the actuary.

At this date (1815), when the *Northampton* Table was receiving its quietus, a dread was expressed that the *Equitable* Society would become unmanageable by reason of its magnitude, and a resolution was passed limiting participation in its surplus to the 5,000 policies of longest duration. This was almost an act of temporary confiscation, as regarding the later entrants beyond that limit; and there was room to doubt if the motive alleged for this act was the true one. The new Life offices were benefited by the step.

In 1818 the *Kent* Life and Annuity Society was founded; and in the following year (1819) the *European* Life and Annuity Company, with a Capital of £1,000,000, in shares of £20. This is not the company which reached such evil fame in recent times, although, by a roundabout process, the remaining business of this Company, by reason of its amalgamation with the *People's Provident* Office in 1858, became involved in that notorious collapse. I shall have some remarks to offer on the events which are supposed to have led to the downfall of this latter office at a later period in this history.

In 1820 several Life Offices were founded, viz.: the *British Commercial,* proprietary (Capital £1,000,000); the *General Benefit* (a species of superior friendly society); the *Star,* not the solid office now known by that name; and the *Imperial,* a proprietary office with a Capital of £750,000, founded under powerful auspices, and still flourishing.

In 1821 there were founded two Life Offices. One, the *Commercial,* in Glasgow, which had but a short career, and the other, the *Guardian,* "for Insurance against Fire, and on Lives "and Survivorships, Endowments for Children, Immediate, De-"ferred, and Progressive Annuities, and for the purchase and sale "of Reversions and Annuities", with a Capital of £2,000,000, in shares of £100, 10 per-cent of which was to be paid up. No person was to hold more than fifty shares. The profits were to be applied towards paying up the shares in full. Local committees were to be founded to give information concerning the progress of the Company in the provinces.

The year 1823 became famed for its Insurance progeny. The following are the Life Offices then founded:

1. The *Bombay* Life.—Founded, I believe, in London, for the purpose of assuring lives proceeding to India. The Company issues no whole-term Life policies. It had three classes of short-term insurances, viz.: For one year, not renewable without a fresh certificate of health, premium at age 30, £3. 8*s*. 0*d*. per £100; for three years, renewable without fresh certificate of health, premium, age 30, £3. 14*s*. 0*d*.; for five years, renewable without fresh certificate of health, premium, age 30, £4. This was an early attempt at Life Assurance in India, to which so much attention has since been paid.

2. *Economic* Life.—Mutual, with a guarantee fund long since paid off. Feature, very low rates.
3. *Law Life.*—Proprietary Capital £1,000,000; shares issued exclusively to members of the Legal Profession. The earliest of the class offices, and it has met with the most abundant success.
4. The *Edinburgh* Life.—Capital £500,000, in shares of £100; founded upon the model of the *Law Life.*

The year 1824 was marked by great financial activity throughout the country—such as had not been experienced for more than a century. Joint Stock enterprise ran rampant. The following is a complete list of the Life Offices then founded.

1. *Alliance* British and Foreign Fire and Life Insurance Company.—Capital £5,000,000, in 50,000 shares of £100. This Company was established under most powerful auspices, and speedily obtained a considerable Life business in Germany and other parts of Europe. A detailed history of the Company is already published. The *Alliance* Marine is a distinct Company.
2. *Asylum* Foreign and Domestic Life Assurance Company. —With a Capital of £240,000, in 2,000 shares of £120. "The *Asylum* Life confines its business to "assuring the lives of persons going beyond the "limits of Europe; predisposed to hereditary or "other constitutional maladies; of delicate health; "of peculiarity of form, whether natural or accidental; "labouring under mania, melancholia, or any kind of "chronic disease, unaccompanied with immediate "danger; females in a state of pregnancy; persons "engaged in unhealthy occupations; and those who, "from inadequate testimonials, uncertainty of date of "birth, or other causes, would be subject to rejection "or an exorbitant rate of premium at other offices." Here was a wide departure from all that had gone before. The details of the history of this Company are also published.
3. *Berkshire, Gloucester,* and *Provincial* Life and Fire Company.—Capital £500,000, in 10,000 shares of £50. This was founded under influential local auspices.

4. *Herts, Cambridge,* and *County* Fire and Life Office.—A local proprietary Company.

5. *Landlord* and *Tenant* Life Office.—Probably abortive from its commencement.

6. *Leeds* and *Yorkshire* Fire and Life Insurance Company.—Capital £1,000,000.

7. *Manchester* Assurance Company.—Capital £2,000,000, for Fire and Life Insurance.

8. *Medical* and *Clerical* (became Clerical, Medical, and General) Life Assurance Society.—With a capital of £500,000, in Shares of £100. "The common usage "of excluding from the benefits of Life Assurance, or "exposing to a forfeiture of their policies individuals "who may have been afflicted with gout, asthma, "fits, rupture, hæmorrhage, complaints of the liver, "spitting of blood, vertigo, or any other disease; "being in many cases a source of fraud, on the one "hand, and of litigation on the other," was to be obviated by the plan of this Company, the promoters of which were the first to support the scheme for the assurance of under-average lives. The Company is still flourishing.

9. *Palladium* Life and Fire Assurance Society.—Capital £2,000,000.

10. *Protector* Life Assurance.—Capital £1,000,000.

11. *Patriotic* Assurance Company of Ireland.—Capital £1,500,000; Fire, Life, and Marine.

12. *St. Patrick* Assurance Company of Ireland.—Capital £2,000,000; Life and Marine.

13. *Scottish Union* Insurance Company of Edinburgh.—Capital £5,000,000; Life and Fire.

14. *South Devon* Marine, Life, and Fire Insurance Company.—Capital £2,000,000.

15. *Sussex County* and *General* Assurance Company.—Believed to have been proprietary.

16. *United Empire.*—Amount of capital not stated.

17. *United Kent,* Life.—Capital £100,000, in Shares of £50.

18. *Yorkshire* Fire and Life Insurance Company.—Capital £500,000; Fire and Life.

Here were eighteen, mostly solid, Assurance Offices founded—eleven of them being provincial—representing a capital not far remote from twenty millions sterling, but not limited to Life Assurance business. There had been no such rapid creation of Insurance for a full century, since the worst days of the South Sea mania. The character of these new Companies differed very much from those of earlier date. The early mutual Life Offices had conducted their business on very economic principles. They had a chamber in some unobtrusive locality, where the books were kept and the details of the business were transacted. The principal officer attended at one of the most frequented coffee-houses with which the City, as also this district beyond Temple Bar, abounded; places where merchants and insurance brokers attended for the transaction of their ordinary business, and of which "Garroway's" was the last relict. Here they frequently jostled against the wits and beaux of the period. Persons desirous of effecting assurances knew where to meet the authorized officials of the offices; and, indeed, the times and places of meeting were often fixed by public advertisement. Thus the first meeting of the *Equitable* Society was held at the "White Lion" on Cornhill, when only four assurances were effected. The *Eagle* was founded at Cole's Coffee-house.

But these new companies, with enormous capitals, deemed it necessary to be seen. They commenced to erect fine buildings in prominent positions in public thoroughfares; and so contributed to the improvement of London, while securing, in many cases, eligible investments in house property.

How Business was obtained.—It may be wondered how so many principal Offices, being brought upon the scene simultaneously, managed to secure a paying share of business. Those who are familiar with the Deeds of Settlement of these Companies will know that in the case of the proprietary offices, in nearly every instance, it was a condition of obtaining an allotment of the Stock that the holder of such Stock should either effect an insurance upon his life for an amount regulated by his holding of Stock; or if his own life could not be accepted, others for equivalent amounts must be insured; while the office of Director involved usually a policy for a considerable amount. In this manner a sufficient number of lives was secured to give average mortality results, and a sufficient premium-income was obtained to look satisfactory in the accounts when presented to the proprietors—for they were not presented to the public in those days.

Agency Commission.—Another practice came into being with the advent of these proprietary Companies, and this was the practice of giving Commissions to Agents for introducing business. Leading firms of solicitors in the provinces, as also country bankers, were eagerly looked after, and their duties were responsible; for before these agents appeared persons who could not personally present themselves before the Board. There was a great outcry raised at this "immoral" practice of paying a Commission to influence the introduction of business. Mr. Francis Baily was particularly severe and outspoken upon the subject. The rates of Commission allowed were particularly moderate. At first only 5 per-cent alike upon new and renewal premiums, but later 10 per-cent on the first premium and 5 per-cent on renewals, and they remained so for a full half-century.

Plague.—The old dread of the recurrence of pestilential visitations had not yet died out. The founders of the *Equitable* Society, in 1762, by the 22nd clause of the Deed, provided that, during any such visitation, three Directors might constitute a Court; and, should the affairs of the Society require it, might "reduce the payments of the several and respective sums of "money, which may become due by reason of the deaths which "may happen in such a time of public calamity, to any sum not "less than one-quarter part of what shall have so become due." While the Deed of the *London Life* (1806), and those of some of the Companies of still later date, embodied like provisions.

Mutual v. *Proprietary Offices.*—The competition of the Proprietary Offices led to some conflict regarding the relative advantages of the two systems. The advocates of the Mutual plan said: "Since the fluctuations in the price of life, arising "from the natural uncertainty of its duration, is considerably less "than that which occurs in the price of most commodities, a "person who deals in securities dependent on lives, requires less "capital to carry on his business than one who trades to an equal "extent in any other species of merchandise to which we have "referred"—*ergo,* capital should not be paid into the business. (*Vide* Babbage's *Comparative View,* 1826, p. xxi). To which it was replied by the advocates of Proprietary Offices: "To Mutual "Assurance Societies it has justly been objected that, from the "want of an original capital, the assured are compelled to contribute to the policies of others, while they obtain no effectual "indemnity until the premiums have accumulated to a sum "more than equal to the risks incurred; and that, consequently, "such establishments are driven to so large a reserve of their "estimated profits that, for a considerable period, the sums paid "on claims are greatly below the fair proportion; and thus the "early members create a fund for the benefit of their successors, "without improving the value of their own policies." (*Vide* Prospectus of *Palladium* Life Office, 1824.) I have not quoted the extreme views advanced on either side.

The discussion so raised had the effect of inducing many of the Proprietary Offices to set apart a portion of the profits realized for division amongst their policyholders of the "participating class"—*i.e.,* those who paid a higher scale than the ordinary rates for the benefit; while those who required the protection of a fixed amount of assurance at the lowest possible premium were insured in the "non-Participating" Class. The Proprietary Offices so granting participation in profits came to be designated "Mixed Offices": they were proprietary in constitution, but shared the profits with their policyholders in an agreed proportion, and so granted mutuality.

The proportions of profits, which at this date the Offices of each class distributed to their policyholders, were:

MUTUAL SOCIETIES.	PROPRIETARY OFFICES.
Amicable—seven-eighths.	*Imperial*—two-thirds.
Economic—three-fourths, after deducting £2,500 interest on Guarantee Fund.	*Law Life*—four-fifths. *Palladium*—four-fifths. *Provident*—share with Proprietors.
Equitable—two-thirds.	*Rock*—two-thirds.
London Life —one-fifth annually.	*United Empire*—optional.

This was the state of things existing in 1824; but changes were continually taking place, the Mutual and the proprietary Offices alike increasing the proportions of profits divided. The periods of division varied, according to the terms of the Deed of Settlement.

In the following year, two of the new Offices declared their proportions, as follows:

MUTUAL SOCIETIES.	PROPRIETARY OFFICES.
	Crown—two-thirds.
	University—four-fifths.

Days of Grace.—The days of grace allowed within which to pay the annual or other premiums were then usually 15; some Offices allowed 20; some of the newer Offices 30; the *Amicable*, 3 months; but persons dying during these days were not understood to be assured. The period was simply allowed for renewing the contract, if alive. The change in practice, by which most of the Offices agreed to remain liable during days of grace, occurred as recently as 1858 (*vide* their advertisements in the *Post Magazine*).

Limits of Foreign Travel.—Most of the Offices at this date allowed the assured to go by sea from one part of Great Britain to another, without payment of any extra premium; they, however, required that the voyage should be performed in decked vessels, King's packets, regular packets, or steam boats; some permitting one, some several or all of these modes of conveyance. During peace the assured were allowed also to pass, in the vessels already named, from England to several ports in the Channel—usually those between the *Texel* on the north, and Brest on the south. The *Atlas*, "any ports in the Channel." The *United Empire*, "any port of Europe to another port." The *Sun*, "from Hamburg to Bordeaux." The *Pelican*, "only to and from *Ireland*, in Government packets." The question of *Residence* abroad was then hardly contemplated as being within Life Assurance probabilities. "I had occasion, a short time since, to enquire the " price of assuring the life of a friend who had an intention of " going to India, and I found the addition charged for the risk " from climate to be at some offices *nearly three times* as great as " at others: and the total charge at the most expensive Office, " was to that of the cheapest very nearly as eleven to six."—(*Vide* Charles Babbage, 1826.)

Payment to Trustees.

THE two cases reported below deal with the important point whether, on the sale of land to trustees, the consideration-money should be paid to all the trustees personally or into a banking account opened in their joint names, or whether the trustees may require payment to be made to an agent duly authorized by them to receive the amount. In detail, the actions differed in one respect; for whereas in the former—*in re* Bellamy—the agents appointed by the trustees were their own solicitors, in the latter—*Flower* v. *the Metropolitan Board of Works*—the agent was one of the trustees themselves. As a matter of law, this distinction does not appear to be material; indeed the express responsibility, according to the ruling of Mr. Justice Kay in the last-mentioned case, rests upon trustees of preventing any one of themselves having exclusive control of the trust-moneys, and there would accordingly appear to be a stronger reason for the purchase-money being withheld from a trustee than from the solicitors to the trustees. In each case, however, the decision was the same—namely, that the purchasers had the right to insist upon the consideration-money being paid either to all the trustees in person, or to a banking account standing in their joint names. It may be noticed that the question whether the trustees were competent to delegate authority to receive payment to any person was also discussed, Lord Justice Cotton and Lord Justice Bowen holding that, as a general rule and under ordinary circumstances, they were not justified in doing so.

The cases appear to have had their origin in a doubt whether the 56th Section of the "Conveyancing and Law of Property Act, 1881", altered the legal regulations concerning the powers of trustees in matters of sale, which before prevailed. On this point the Court of Appeal decided, *in re* Bellamy, by a majority of two to one, that the Act did not extend the powers of trustees as to the employment of a solicitor, but that "its only effect was to make a "vendor's receipt equivalent, for the protection of the vendee, to a "written direction of the vendor." It must, therefore, we imagine, be accepted as definitive that, in point of strict legality, trust-moneys should under all ordinary circumstances be paid to the trustees personally, or to their banking account; and if this were not done, the responsibility for any loss that might arise would fall upon the purchaser or other person who, acting upon a power of attorney or other authority, had paid the moneys to third parties.

It may, however, be of interest to mention that one assurance company has recently, after full consideration of these judgments, adopted the following courses in the settlement of claims:

(1) Where the payment is to be made to executors or administrators, the existing practice is confirmed of drawing the cheque to the order of the person formally deputed to receive the amount by the legal representatives, whose discharge is, of course, required.

(2) Where the policy is vested in trustees, the cheque is drawn payable to the orders of the several trustees, and then handed to the solicitor or other person authorized by the trustees to arrange the settlement.

This we consider is a practical solution of the matter which, while greatly facilitating the payment of claims under policies in trust, does not involve the company in more than the ordinary risks of business.

LAW REPORTS.

IN THE COURT OF APPEAL.

Baggallay, L.J.; Cotton, L.J.; Bowen, L.J.

In re BELLAMY.

8, 9, *and* 26 *June* 1886.

Vendor and Purchaser—Trustees for Sale—Payment of Purchase-Money to Solicitor of Trustees—Production of executed Conveyance with Receipt indorsed—" The Conveyancing and Law of Property Act, 1881," *ss.* 8 *and* 56.

Per Curiam.—The 8th and 56th sections of "The Conveyancing Act, 1881", were framed with reference to the views expressed by the learned Judges who decided Viney *v.* Chaplin (2 De Gex, F. & J., 468), and the effect of the 56th section is to substitute the production of the executed conveyance, with a receipt in the body of the deed, or indorsed thereon, for the written authority of the vendor to his solicitor to receive the purchase-money, which would otherwise have been necessary.

That section, therefore, does not confer on vendors any greater authority, nor afford to purchasers any greater protection than they respectively would have had if the section had not been passed.

On a sale of land by trustees for sale the purchaser made a requisition, which was in accordance with the usual practice of conveyancers, that the vendors should, on completion, either attend personally to receive the purchase-money, or should give the purchasers a written authority to pay the purchase-money into some bank to their joint

account, but the vendors' solicitors claimed to have the purchase-money paid to them on production of the executed conveyance with a receipt indorsed.

Held (per Cotton, L.J., and Bowen, L.J., dissentient Baggallay, L.J.,) that as a general rule, under ordinary circumstances, trustees are not justified in authorizing their solicitor or other agent to receive purchase-money which ought to be paid to them personally, and that, therefore, the purchasers were entitled, as a matter of prudence, to insist on the requisition.

This was an appeal from the decision of Kay, J. (reported 52 Law J. Rep., p. 89), holding that the 56th section of "The Conveyancing and Law of Property Act, 1881", applied to the case of a sale by trustees as much as to that of an ordinary vendor, and, therefore, that where the solicitor of trustees for sale, who have a power of giving receipts, produces to the purchaser a deed duly executed by the trustees, having in the body thereof, or indorsed thereon, a receipt duly signed by the trustees, the purchaser ought to pay the purchase-money to such solicitor, unless he has reason to suspect an intended misapplication of the money.

W. Pearson, Q.C., and *Pounall*, for the Appellants, the purchasers.
G. Henderson for the Respondents, the vendors.

For the arguments see the report below; and, in addition to the authorities there cited, the following were referred to:—Bourdillon *v.* Roche⁽¹⁾; Bostock *v.* Floyer⁽²⁾; Speight *v.* Gaunt⁽³⁾; Essex *v.* Daniel⁽⁴⁾; Osborn *to* Rowlett⁽⁵⁾; *in re* Burroughes⁽⁶⁾; and Bacon *v.* Bacon⁽⁷⁾.—*Cur. adv. vult.*

The considered judgments of the Court were delivered on 29 June, their Lordships differing.

Baggallay, L.J.—This is an appeal from an Order of Mr. Justice Kay, dated 29 November 1882, and it raises the question of the extent of the operation of the 56th section of "The Conveyancing and Law of Property Act of 1881." The appeal has been brought under the following circumstances:—

On 6 May 1881, the Metropolitan Board of Works contracted to purchase certain real estate from the trustees of the will of the Rev. William Tylden, for the sum of £500. The solicitors for the purchasers required that, on the completion of the purchase, the vendors should either attend personally to receive the purchase-money, or should give the Board a written direction, signed by the vendors, to pay the purchase-money into some bank to the joint account of the vendors. On the other hand, the solicitors of the vendors insisted that having regard to the 8th and 56th sections of

(1) 27 Law J. Rep., Ch. 681.
(2) 35 Law J. Rep., Ch. 23, Law Rep. 1, Eq. 26.
(3) *Ante*, p. 503; Law Rep. 22, Ch. D. 727.
(4) Law Rep. 10, C. P. 538.
(5) 49 Law J. Rep., Ch. 310; Law Rep. 13, Ch. D. 774.
(6) 46 Law J. Rep., Chan. 528; Law Rep. 5, Ch. D. 601.
(7) 5 Ves. 331.

"The Conveyancing and Law of Property Act of 1881", it would be sufficient if they attended and produced the deed of conveyance, duly executed by the vendors, and having in the body thereof, or indorsed thereon, a receipt duly signed by them, and that, therefore, the purchase-money should be paid to them. This not being assented to by the purchasers they took out a summons, under "The Vendor and Purchaser Act of 1874", asking for a declaration that the requisition so made by them ought to be complied with. On the return of the summons Mr. Justice Kay held that unless the purchasers had reason to suspect an intended misapplication of the purchase-money, the vendors were intended to avail themselves of the provisions of the Act of 1881, and he dismissed the application. From such dismissal the present appeal is brought.

On the part of the Appellants it has been contended before us, as it appears to have been contended before Mr. Justice Kay, that an authority given by trustees for sale to their solicitor to receive the purchase-money would be a breach of trust, and that if the purchaser paid his purchase-money to the solicitor upon such an authority, he would be implicated in the breach of trust, and might be called upon to repay the purchase-money in the event of its being misapplied by the solicitor. The former of the two propositions thus affirmed is, in my opinion, well founded, and as a general proposition it has not been disputed by the Respondents.

Lord Langdale's decision in Ghost *v.* Waller (9 Beav. 847) has always been recognized and followed. But I am not aware of any authority that supports the latter proposition, as applicable to any cases other than those in which, from the surrounding circumstances, knowledge, or at least a suspicion of an intended misapplication of the purchase-money has been attributed to the purchaser. The cases of Webb *v.* Ledsam (1 Kay & J. 385) and Hope *v.* Liddell (21 Beav. 180), so far as they have any bearing on the general proposition, appear to me to be adverse to the Appellant's contention. In Webb *v.* Ledsam, the purchaser, by arrangement with the trustees, retained in his own hands for eight years a considerable portion of the purchase-money, and eventually it was one of the trustees who misapplied it. Here there was a clear breach of trust, of which the purchaser was not only cognizant, but took an active part in carrying it out. The Vice-Chancellor held the purchaser responsible by reason of his participation in what he knew to be a breach of trust, but in the course of his judgment he said: "I know of no "authority for holding liable to pay over again purchase-money paid "to one of several trustees upon a receipt signed by all", and then proceeded to distinguish the case that was under his consideration. In Hope *v.* Liddell Lord Romilly said: "The purchaser having "obeyed the direction of the trustee to pay it"—that is the purchase-money—"to another person has, in my opinion, thereby made a "payment of it to the trustee himself, and is exonerated from the "consequences of misapplication, unless the purchaser had express "notice that the person to whom he was directed to pay it was about "to commit a breach of trust in such a way as to make the purchaser, "in fact, a party to the wrongful act." The case of Viney *v.* Chaplin (2 De Gex & J., 468; 4 Drew, 237; 27 Law J. Rep.;

Ch. 434), though alluded to by Mr. Justice Kay in his judgment, does not appear to have been discussed in the arguments of counsel before him; but I agree with Mr. Justice Kay in thinking that the 8th and 56th sections of "The Conveyancing Act of 1881" were passed with reference to the views expressed by the learned Judges by whom Viney *v.* Chaplin was decided. In that case two questions were raised—the first, whether a purchaser has a right to insist upon having the conveyance executed in the presence of himself or of his agent; and in the second, whether he has a right to require that the money should be paid to the vendor personally or in his presence. Upon these two questions the Court (composed of Lord Chancellor Cranworth and Lords Justices Knight-Bruce and Turner) declined to give an opinion in the abstract; but in the course of his judgment the Lord Chancellor—after stating that under ordinary circumstances the purchaser's solicitor had no hesitation in accepting the conveyance, though he had not witnessed its execution, or in permitting his client to pay the purchase-money to the solicitor of the vendor—said that circumstances might arise in which the purchaser might feel that he ought to be armed with the most complete proof of the transaction, both as to the execution of the conveyance and as to the receipt of the purchase-money; and, after further observing that, if the purchaser paid his purchase-money to a person not authorized to receive it he would be liable to pay it over again, and that the possession of the executed conveyance, with the signed receipt for the purchase-money indorsed, was not in itself an authority to the solicitor of the vendor to receive the purchase-money, he proceeded as follows:—"The difficulty is to see how the purchaser can be safe at " all times from the danger of paying his money to an unauthorized " person without the vendor being present, or having expressly given " to the purchaser or his solicitor authority to pay it in a particular " manner"—and after further observing that in many cases it would be extremely unreasonable for the purchaser to require the presence of the vendor, he added: "It will be sufficient for this case to say " that if a purchaser has not the right in every case to insist upon " the vendor being present when the purchase-money is to be paid, " neither is a vendor to refuse compliance with a request of this " description when circumstances arise which are sufficient to justify " it." And in the same case Lord Justice Turner, after stating that a solicitor was not entitled by virtue of his office to receive purchase-money, and that if a vendor asserted the authority of his solicitor to receive it, it was incumbent upon him to produce that authority, proceeded to express himself as follows: "I think that the " purchaser has the right of insisting, either that the vendor should " attend and himself receive the purchase-money, or of requiring " that there should be written authority to pay the solicitor." Much reliance has been placed by the Appellants upon the opinions of text writers, and we have been told in the course of the argument that Hope *v.* Liddell has not been approved by conveyancers, but nearly thirty years has elapsed since the decision in that case, and I am not aware that its authority has ever been impugned. It was quoted in Viney *v.* Chaplin, and no disapprobation of it was expressed, and the decision in Viney *v.* Chaplin was perfectly consistent with it. We are all aware of the advice given by Lord St. Leonard's, that when the vendors are trustees for sale the purchasers should, for their own protection, either pay the money to the vendors, or into a bank to the joint account of the trustees. That advice was given many years ago and was, in my opinion, sound advice for general guidance, and for this reason—that if at any future time the authority of the agent should be disputed, or it should be thought to affect the purchaser, with knowledge or suspicion of an intended misapplication of the purchase-money, it might be difficult for the purchaser or his representatives to prove the one or disprove the other. That advice has been repeated in the books of other writers, and it appears to have been adopted in practice by conveyancers generally. It was doubtless the foundation of the requisition made on behalf of the Appellants in the present case; but it was not advice which ought in all cases to be acted upon, if effect is to be given to the views expressed in Viney *v.* Chaplin; and Lord St. Leonards, in the 14th section of his great work, published some two or three years after the decision on the appeal in Viney *v.* Chaplin, first refers to the decision of Vice-Chancellor Kindersley (4 Drew, 237) as supporting the proposition, "that as a "general rule the purchaser had a right to pay the purchase-money to "the seller himself," and adds that, "on appeal it was considered "that a written authority would be sufficient, but the solicitor, as "such, had no right to receive the money." Whatever may have been Lord St. Leonard's opinion of the views expressed by Lord Cranworth and Lord Justice Turner, in Viney *v.* Chaplin, he expressed no dissent from them. In the last edition also of Mr. Lewin's work "On Trusts," the following passage occurs:—"Trustees as between "themselves and the purchasers are not bound to receive the purchase-"money personally, but may give a written or other express authority "to their solicitor or agent to receive it on their behalf; but payment "to a solicitor or agent, without a written or other express authority "from them, will not be sufficient." In my opinion, these comments of Lord St. Leonard's and Mr. Lewin correctly represent the principles recognized and acted upon by Courts of Equity previously to the passing of the Act of 1881, when dealing with questions such as are involved in the present appeal. It is impossible to state them more concisely, but with the view of applying them to the case now under consideration, I would enunciate them as follows:—That in the absence of any reason to suspect a misapplication of the purchase-money, a purchaser from trustees for sale would be bound to pay them to the solicitor of the trustees upon the production by him of a written authority signed by the trustees to receive them.

Let us now examine the 8th and 56th sections of the Act of 1881. The 8th is in the following terms:—"On a sale the purchaser shall "not be entitled to require that the conveyance to him be executed "in his presence, or in that of his solicitor as such, but shall be "entitled to have at his own cost the execution of the conveyance "attested by some person appointed by him, who may, if he thinks "fit, be his solicitor." This deals with the subject matter of the first question raised in Viney *v.* Chaplin, as to which their Lordships, while declining to lay down any general rule applicable to all cases in effect, held that the question whether a purchaser in any particular case had a right to insist upon having the conveyance executed in the presence of his solicitor or his agent must depend upon the circumstances of that case. The 8th section gives to the purchaser the right of having, but at his own cost, the execution of the conveyance attested by some person appointed by him, who may be his own solicitor, but negatives any right in him to require that the conveyance shall be executed in his presence or in that of his solicitor as such. The 56th section of the Act in like manner deals with the subject matter of the second question raised in Viney *v.* Chaplin. It provides as follows:—" Where a solicitor produces a deed having in " the body thereof or indorsed thereon a receipt for consideration- " money or other consideration, the deed being executed or the indorsed " receipt being signed by the person entitled to give a receipt for that " consideration, the deed shall be sufficient authority to the person " liable to pay or give the same for his paying or giving the same to " the solicitor without the solicitor producing any separate or other " direction or authority in that behalf from the person who executed " or signed the deed or receipt." It has been suggested, though it has hardly been pressed, that this 56th section does not apply to the case of a sale by trustees, but I can see nothing either in the section itself or elsewhere in the Act to justify so limited an application of its provisions. Treating the section as applicable to the present case, and assuming the views which I have expressed as to the effect of the decision in Viney *v.* Chaplin to be correct, the 56th section, in my opinion, substitutes the production of the deed as mentioned in the earlier part of the section for the written authority, which, according to the decision in Viney *v.* Chaplin, would have been sufficient. I will only add that, in the arguments before us, the vendors have been throughout treated as trustees for sale, with power to give receipts; but it appears that they are not trustees for sale, as the expression is commonly understood. The estates are strictly settled by the will with a power to the trustees upon request, as therein mentioned, to sell and to revoke uses and trusts, to give effect to such sales with a declaration that their receipts shall be effectual discharges for the purchase-moneys thereby expressed to have been received, and that the persons taking such receipts shall not be obliged to see to the application, or be in anywise answerable or accountable for any loss, misapplication, or non-application thereof. The distinction does not appear to me material, but the provision as to the purchaser not being liable for misapplication of the purchase-moneys brings the case more completely within the decision in Hope *v.* Liddell. I will only add that, in my opinion, the 56th section of the Act of 1881 was intended to meet the circumstances of such a case as the present, and to prevent, in like cases, such a requisition being made, as has been made, by the Appellants. I think the appeal should be dismissed.

Cotton, *L. J.*—This case comes before us on appeal from an Order made by Mr. Justice Kay upon a summons taken out under "The Vendor and Purchaser Act." The purchasers made a requisition that the vendors should either personally receive the money or name some bank into which it should be paid to the joint account of the vendors, who were trustees. Their solicitors, without any explanation of any circumstances which rendered it desirable or necessary that neither of those alternatives should be adopted, said:—"We will "produce a conveyance in which there shall be a receipt for the "purchase-money—that is sufficient authority. We claim under "that to receive the purchase-money, and to disregard your requisi-"tions." The question which we have to consider is, whether upon the answer so made by the vendors they are entitled to specific performance. Now the first point, which was argued a good deal, was upon the 56th section of "The Conveyancing and Law of Property Act of 1881", and it was contended that however the matter would have stood in the absence of that section, that section prevented the requisition made by the purchaser from being insisted on in the case where the trustees are vendors. In my opinion that is not a correct rendering of that section. In my opinion the simple effect of that section is to make a deed in the form pointed out by that section equivalent to a special authority given to the solicitor to receive the money. What is the section? [His Lordship read the section, and continued.] Well, that on its fair interpretation points simply to the absence of a separate or special written direction or authority, and means that the production of the deed shall be equivalent to and obviate the necessity of a separate or other written direction or authority, and shall operate as a good discharge to the purchaser. If it was meant that it should be a good discharge under all circumstances to the person who so paid the money, that would have been in different language, and although I do not say that this section does not apply to the case of trustees being vendors, where possibly they may be right in authorizing their solicitor under certain circumstances to receive the money, yet, in my opinion, that applies only to vendors who have power to authorize their solicitors to receive the money, and does not give vendors any additional power of authorizing their solicitors to receive the money, or give to the purchaser any additional protection in payment to a solicitor who produces the deed, except this, that the deed shall be equivalent to and have the same force and authority as a separate and special written authority or direction.

If that is so—and that, in my opinion, is the true construction of this section—what one has to deal with are two questions. First, we must consider whether trustees have power to authorize their solicitor to receive purchase-money payable to them, the vendors; and then whether a purchaser is bound to accept the receipt of the solicitor instead of paying the money to the trustees personally or into a bank to their account. Now, I think, it may be safely stated, as a general rule, that, under ordinary circumstances, trustees are not justified in authorizing their solicitor or other agent to receive purchase-money which ought to be paid personally to them. But that is not in any way, in my opinion, a universal rule. There may be circumstances which would justify and render it necessary for trustees to grant power to somebody else to receive purchase-money for them; but, as a general rule, I think it may be safely laid down that it is their duty not to authorize or delegate to an agent or solicitor power to receive the trust-money. It was argued that that would be wrong, that it was part of the ordinary course of business to employ a solicitor in completing a purchase; and then it was said that the case of Speight *v.* Gaunt was an authority that trustees would be justified in doing what was usual, and in giving their solicitor authority to receive the money. That case I entirely agree with; but it has no application to the present case. That case simply decided this—that when, according to the ordinary course of business, it is necessary for trustees to employ an agent—an agent having, from his very employment, power to receive money—the trustees are not committing any breach of trust in employing an agent having those powers where it is necessary, as a matter of business, nor are they answerable for his misapplication of the money. The case there was that of a broker on the Stock Exchange, who was to invest trust-money. But here it is an entirely different case. A solicitor as such, and from his employment in completing a purchase, has not authority to receive the money. Viney *v.* Chaplin settled that; and the question is as to special authority given to him, not incidental to his employment, in the completion of a purchase, but a special authority given to him under the terms of this section. That being so, taking the general rule, can the vendors, being trustees, require the purchaser to complete in this way, simply by saying to the purchaser, "I require completion on payment of the purchase-money to him;" not deigning to give any explanation of circumstances which would justify or render it necessary for them to authorize him to receive the money, but simply saying, "There is the conveyance; there is the act. You are bound to pay the money to him." In my opinion, that is not sufficient. Assume there may be circumstances where the trustees would be justified in authorizing the solicitor to receive the purchase-money, yet the 56th section does not say that the purchaser shall be bound to pay. But, in my opinion, when the vendors' solicitor requires the purchase-money to be paid to him simply, the purchaser has a right to say: "I may be safe, and I by no means say that I am "not, but I do not choose to mix myself up with any investigation "which may hereafter arise between the *cestui que* trust, and the "vendors, the trustees, and myself, as to the validity of the sale or "the distribution of the purchase-moneys"—instituted it may be with the object of fixing him with knowledge of a breach of trust committed by the trustees. I think a purchaser is justified in saying, "I will be prudent. It may be I am safe in taking the conveyance "from you, but I decline to be mixed up in the question in any way, "and I require—for the refusal of which there is no reason here—"payment of the moneys to the trustees themselves, or to their joint "account with the bankers designated by them." It is said there is authority against that. Now undoubtedly high authorities on conveyancing have advised trustees to adopt the course laid down by these requisitions. It is said there have been alterations in the later editions, and possibly there have; but I do not think the authority of Lord St. Leonards can be vouched for the proposition which was involved in the decision of Mr. Justice Kay, of which I speak with great respect, as of course I need hardly say I do also of the opinion of the Lord Justice who has preceded me. One must look to see whether the case of Viney *v.* Chaplin—which was referred to by Lord St. Leonards, not as laying down a doctrine he approved of, but as altering the practice—decided anything of the kind. Now, that the Lord Chancellor did not intend to decide what should be done where there were trustee-vendors, and the purchaser desired to be prudent and safe, is clear, for he says at p. 79, "Neither is it necessary to "consider whether Mr. Chaplin was or was not a trustee, although "the cases show that where there was a trust a purchaser may incur "considerable risk in paying to a solicitor for a trustee, for this is "merely a question of degree;" so that he certainly did not intend by his judgment in anyway to give an opinion on this question we have now to consider; nor, as far as I see, did Lord Justice Turner, because the facts there were these, that the solicitor had no written authority at all, but insisted on his right from his mere position as a solicitor, or from a verbal authority given him. It is very true that he does refer to the non-existence of a special authority, but that is when he is dealing with the question whether the solicitor was then in a position to say that, as between him and his employer, the vendor, he could give a good discharge, which would prevent the vendor from afterwards saying, "I insist on my vendor's lien and that payment has never been effectually made to me." Here it is different. It is not a question between vendor and purchaser whether the vendor can afterwards object to the payment to the solicitor, but whether the purchaser would be safe in paying to the solicitor; whether he is not prudent, and justifiably prudent—not over cautious, but justifiably prudent—in saying, "I will not specifically perform until I am made "safe from any future question by your allowing me to pay the "money to the trustees themselves, as the vendors, or to an account "in their names at a bank." In my opinion, the purchasers here were justified in taking that course; and, therefore, in my opinion, the decision of Mr. Justice Kay must be reversed.

Bowen, L. J.—I agree with the opinion of Lord Justice Cotton in this matter. It seems to me the questions we have to decide are two. First, whether "The Conveyancing and Law of Property Act, 1881", affects the position of the solicitor of the trustee in respect to the question which has been raised; and, secondly, what really was the law of the Court as to payment made to solicitors of trustees before the Act.

I begin by saying what, in my opinion, is the true construction of section 56 of "The Conveyancing and Law of Property Act, 1881." I think its only effect is to make a vendor's receipt, of the character described in the section, equivalent, for the protection of the vendee, to a written direction of the vendor. I do not think the section is meant to compel the purchaser to accept a written receipt, where he would not, before the Act, have been compelled to act on a written direction.

It seems to me that the whole framework of the section points to its being intended for the protection of a vendee who acts on a receipt without having any written authority, that is to say, to place the vendee who acts on such a receipt in the same position as if a written authority had been handed to him by the vendor. I do not say, and I do not think it can be said, that this section does not apply to the case of trustees. It appears to me that it applies to the case of any trustee who would be right in authorizing his solicitor to receive the purchase-money upon a written direction. All that appears to me to be clear is, that it does not enlarge the powers of trustees as to the employment of a solicitor, but leaves it exactly where it was before.

Next comes the question—What was the law before as to the employment by trustees of a solicitor to receive the purchase-money? It appears to me there can be no invariable rule on the subject. The employment of a solicitor by a trustee to receive purchase-money from purchasers must depend on the general principles which regulate the conduct of trustees. Sometimes the trustee would find it essential to employ a solicitor to receive the purchase-money. For example, a case was put in argument which was an apt illustration. Suppose a trustee is abroad, you cannot expect a purchaser to be hunting all over the globe for the purpose of finding the man to whom he is to pay the money. That would be unreasonable, and I could see many cases in which it would be reasonable that a trustee should employ a solicitor to receive the purchase-money; but still a solicitor—to use the language of a judge in a well-known case—is not employed to receive the purchase-money except in the case of moral necessity. It must depend upon the circumstances of each case whether he was properly so employed or not, and, therefore, I do not propose to lay down any general rule as to whether the purchaser will or will not be safe in cases where a trustee employs a solicitor to receive the purchase-money. It seems to me sufficient to say, speaking with great consciousness of my own inferiority to judge as compared with that of the learned Lords Justices who have preceded me, that a purchaser would not have been compelled before the Act to pay to the solicitor of the trustee, if the trustee simply wrote to say he wished him to do so, and the purchaser simply said that he wished to run no risk. Then if that be so, what is there in section 56 of this Act which alters the position of the vendor and vendee? If I am right in the construction I put on section 56, that it is not meant to enlarge the power of trustees to employ a solicitor, but only to warrant the purchaser in paying where he could before have done so only on a written direction, I think it would not justify the Court in insisting on a purchaser now doing in respect to the purchase-money what the Court would not have insisted on his doing before.

Appeal accordingly allowed; but without costs, the Appellants not asking for them.

SOLICITORS:

R. Ward, for Appellants.

Hunters, Gwatkin & Co., for Respondents.

IN THE HIGH COURT OF JUSTICE, CHANCERY DIVISION.

MR. JUSTICE KAY.

25, 26 *July* 1884.

In re FLOWER AND THE METROPOLITAN BOARD OF WORKS.

Vendor and Purchaser—Sale by Trustees—Payment of Purchase-money to one Trustee on written authority from Co-Trustees—Breach of Trust—"Conveyancing and Law of Property Act, 1881." (44 *&* 45 *Vict. c.* 41, *s.* 56).

On a sale of land by trustees with power of sale, the purchasers made a requisition that either the vendors should attend personally to receive the purchase-money or the purchase-money should be paid into a bank to the joint account of the vendors, under a written direction to be signed by them and given to the purchasers. The vendors refused to comply with the requisition, alleging that it was inconvenient so to do, and proposed that one of them should attend and receive the purchase-money under a written direction to that effect to be signed by them all. Held, that the purchasers were entitled to insist upon their requisition.

In re Bellamy (52 Law J. Rep., Ch. 870; Law Rep. 24, Ch. D. 387) applied.

The Metropolitan Board of Works contracted to purchase certain lands at Limehouse, known as London Wharf, belonging, as to one undivided moiety, to the trustees of the will of J. W. Flower, and, as to the other undivided moiety, to the trustees of the will of P. W. Flower, and held by these trustees respectively as to a part in fee-simple, as to other part under a renewable lease, and as to the remainder under a lease for a term of 500 years from 1586. The purchase-money for each moiety was £4,350.

In each case the trustees were three in number, and had power to sell the land and to give receipts for the purchase-money. A draft conveyance from them, as trustees selling under their respective powers of sale, had been prepared and approved. The conveyance contained the usual receipt clause.

The solicitors for the purchasers in each case made a requisition that the vendors, being trustees, should attend personally on the completion of the purchase to receive the purchase-moneys, or that the purchase-moneys should be paid by the purchasers into a bank to the joint account of the vendors, under a written direction to be signed by them and given to the purchasers.

In answer to this requisition the solicitors for the vendors stated, that Mr. Wickham Flower, a trustee of both estates, would obtain from his co-trustees a written direction for payment of the purchase-moneys to him.

The solicitors for the purchasers, however, insisted on their requisition, and the solicitors for the vendors, in the case of the trustees of the will of P. W. Flower, sent an amended reply, which, so far as material, was in the following terms:—"The vendors in this case are the executors of Mr. P. W. Flower, deceased, and they are selling the property as executors. They cannot, without great inconvenience, all attend personally at the office of the Board to receive the purchase-money. They have accordingly signed a formal authority and direction to the Board (which is sent herewith) to pay the purchase-money to one of them, Mr. Wickham Flower, and he will attend at the office of the Board to receive the money under that direction and authority. As regards the suggested payment of the money to a bank to a joint account, this suggestion cannot be acceded to. We should explain that for some time past the accounts of Mr. P. W. Flower's estate have, as a matter of family arrangement and convenience, and by direction of the beneficiaries, who are all of age and have long since attained vested interests in their shares of the estate, been kept by the firm of Messrs. P. W. Flower & Sons, and that it has for some time past been the practice to pay all purchase-moneys of land belonging to that estate direct to that firm, and that course will be pursued with regard to the purchase-money of London Wharf, when received. We submit that the above-mentioned direction is amply sufficient authority to the Board for payment of the purchase-money; but we are willing, if the Board require it, to get the direction confirmed in writing by all the beneficiaries, at, of course, the expense of the Board."

Mr. Wickham Flower was stated to be a solicitor. It was also stated that in one of the cases one of the trustees was unwell and resident at Devizes, in Wiltshire, and another was resident in South Wales.

Correspondence took place between the parties, and ultimately the Metropolitan Board of Works took out in each case a summons under "The Vendor and Purchaser Act, 1874", asking that it might be declared that the requisition of the Board ought to be and must be complied with.

The two summonses were adjourned into court, and came on to be heard together.

W. Pearson, Q.C., and *Pounall*, for the Summonses:

The decision *in re* Bellamy⁽¹⁾ applies to this case. The only authority that trustees may depute one of their number to receive the purchase-money is a dictum of Lord Hatherley, when Vice-Chancellor, in Webb *v.* Ledsam.⁽²⁾ The question is not merely whether the trustees could give such authority, but whether, as was said *in re* Bellamy,⁽¹⁾ a prudent purchaser is bound to act on it.

[They were stopped by the Court.]

Graham Hastings, Q.C., *J. G. Wood*, and *Manly*, for the Respondents, the vendors:

In re Bellamy⁽¹⁾ has no application. That was the case of a delegation of their duty by trustees to an agent, and the decision

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(1) 52 Law J. Rep., Ch. 870; Law Rep. 24, Ch. D. 387.

(2) 1 Kay & J. 385.

went upon the ground that such a delegation is a breach of trust. Where there are several trustees, if only one receives the money, and the others join for the sake of conformity, there is no breach of trust. (Brice *v.* Stokes).[3] All that *in re* Bellamy[1] decided, was that as the delegation to an agent is *prima facie* a breach of trust, the purchaser, by paying the agent, may involve himself in a breach of trust, and is therefore "justifiably prudent" in declining so to pay.

[*Kay*, J., referred to Styles *v.* Gay.][4]

The money can only be received physically by one of the trustees. If all attend, and the money is laid upon the table, it will be taken up by one of them only.

Even if the money is paid into a bank to a joint account there is a delegation of authority. The bankers to whom the money is paid are the agents of the trustees to receive it.

They referred also to Lewin on Trusts[5] and Lord St. Leonards on Vendors and Purchasers.[6]

No reply was called for.

Kay, J.—I think the purchasers have a right to insist on this requisition, the object of which is that they may make themselves perfectly safe, and not embarrass themselves with anything that may arise between the vendors. The vendors are three trustees for sale, and they have sold certain property. Now the purchase of that property is to be completed, and the vendors say, "Hand the "purchase-money to one of our number; he will produce to you the "conveyance, with the receipt on it." The purchasers say, "No, we "are not satisfied with that; we would rather pay it to your joint "account at a bank. We do not want to be embarrassed with any "questions which may arise hereafter, and we want to make ourselves "perfectly safe."

Now, if it had not been for the decision of the Court of Appeal in *in re* Bellamy,[1] I should have said, on the authority of Webb *v.* Ledsam,[2] that what was proposed to be done would make the purchasers perfectly safe, and that they would not be reasonably justified in requiring anything else to be done. But I have tested the arguments that have been addressed to me in this way—I have asked first this question: Suppose the requisition were that payment should be made to one of these trustees without his producing any receipt from the others or any authority by the others to receive the money, would that be a good payment? It is admitted frankly that it would not. The purchaser might then possibly be obliged to pay his money over again. Then I asked, secondly, would this do? Suppose the one trustee had said, "I will produce a written authority from my co-trustees that I alone should receive the money." Would that do?' Yes, it is said, that would do, and that would be a good payment. Why? Because in that case there would be an authority from the co-trustees to the one trustee to receive the money, and because he, who without that special authority had no right to receive the money, would be enabled by the authority so given to him to receive the money and misapply it, and it might be lost to the trust. I do not suppose that anything of that kind would happen here, but I am looking at the matter in a legal point of view. The theory of every trust is that the trustees do not allow the trust-money to get into the hands of one of them, but that they all exercise control over it, taking care that it is in the hands of all of them, or properly invested, or placed in a proper bank in their joint names; and it is quite clear that if, by the act of the trustees, they enable one of themselves to receive the money, they are liable then for that receipt just as much as if they all received it, because then they have enabled the one trustee to do that which, but for the special authority, he would not have been enabled to do. The reason of appointing more than one trustee is that they shall take care that the trust-money is always under the power and control of every one of them, and that they have no right, as between themselves and their *cestuis que* trusts, unless the circumstances are such as to make it imperatively necessary so to do, to authorize one of themselves to receive the trust-money; and the case of their authorizing one of themselves to receive the trust-money does not, for the purposes of the decision in *in re* Bellamy,(1) differ in the least degree from the case of their authorizing any other agent. *In re* Bellamy(1) decided this. In that case I was of opinion that the 56th section of the Conveyancing Act, under which the production by a solicitor of a deed having in the body thereof, or indorsed thereon, a receipt for the consideration-money—the deed being executed or the indorsed receipt being signed by the proper parties—is a sufficient authority for the payment to him of the money, applied to the case where the vendors were trustees. The case went to the Court of Appeal, and the Lords Justices, as I understand their judgment, agreed in that opinion, that the section applied to the case of trustees selling; but then they said this—and, as this is the reason of the decision, I will not give it in my words, but in the words of the Lord Justice Cotton. "I think", he said, "it may be "safely stated, as a general rule, under ordinary circumstances, that "trustees are not justified in authorizing their solicitor or other agent "to receive purchase-money which ought to be paid personally to "them." I never doubted that for a moment. That certainly is the law. But does not "other agent" include one of themselves? Suppose there are three trustees, and there is a sum of £1,000 to be paid, are they justified in allowing one of themselves to receive that money any more than they are justified in allowing any other agent not one of themselves to receive it? Most certainly not. It is the duty of trustees to prevent one of themselves having exclusive control over the money—certainly not by any act of theirs to enable one of themselves to have the exclusive control of the money. That is contrary to their duty; and although it differs in degree it is precisely the same kind of breach of trust as is committed by authorizing their solicitor, or any other person outside themselves altogether, to receive it. Therefore, if the purchasers are not bound to pay to the agent of the trustees, not being one of themselves, upon that agent producing a power of attorney from the trustees, or the deed with the receipt on it, which is now equivalent to a power of attorney in the case of a solicitor being the agent—if they are not bound, when such power of attorney or receipted deed is produced by the solicitor, to pay to the attorney or solicitor, how are they bound when one of the trustees says, "I have got authority from my co-trustees to receive the money: "I am their agent to do that which but for a special authority I "should not have any power to do"? How are they bound to pay to that trustee any more than to a solicitor? I confess I do not see that they are. It seems to me quite plain that the purchasers have nothing to do at all with the question whether the authority is a good one or not. They are not bound to investigate that. All that is before them is this: "Our vendors are three trustees. We are "not exonerated unless the money gets to their hands. We are not "going to trouble ourselves by looking into the question whether "they have given a proper authority to their co-trustee to receive it. "We do not choose to be embarrassed by any enquiry on the subject. "We will see that that money gets to their hands—or that which "will satisfy us equally well—we will pay it into a solvent bank, "approved by us, to their joint account; and then we shall be "completely exonerated." Are they, or are they not, to use the language in Bellamy's case,(1) "justifiably prudent" in doing that? In my opinion they are. It might happen, for anything they know, that the trustees never gave any authority to their co-trustee to receive the money. It might happen, for anything they know, that the circumstances were such that the payment was a bad payment altogether as between them and the trustee. The purchasers do not choose to be embarrassed with that question. They say: "We want "to make ourselves safe. Bellamy's case(1) has decided that we "have a right to make ourselves safe, and to pay this money in such "a manner as frees us from any question as to the authority of the "person who receives the money." In my opinion this case comes within the principle of that decision entirely, as I understand it, and therefore I am bound by that decision, and I must hold that the purchasers have a right to insist either that the trustees do all attend, and that they do pay the money down on the table in their presence, or that they should pay the money to their joint account at the bank as they have proposed to do.

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[1] 52 Law J. Rep., Ch. 870; Law Rep. 24, Ch. D. 387.
[2] 1 Kay & J. 385.
[3] 11 Ves. 319.
[4] 1 Mac. & G. 422; 19 Law J. Rep., Ch. 185.
[5] 5th Ed., p. 239; 7th Ed., p. 246.
[6] 14th Ed., p. 667.

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(1) 52 Law J. Rep., Ch. 870; Law Rep. 24, Ch. D. 387.

But then it is said that in no case can there be a payment to all the trustees. That if they did all meet and the money were laid upon the table before them all, that money would be taken up by one, and the payment would not be to all. I absolutely dissent from that proposition. If the money were laid down on a table in the presence of all the trustees, that is a payment to all of them if they accept the money; and what is done with it afterwards is a matter with which the purchaser has no concern in the world. If the trustees say to one of their body, "Will you take the money to the bank or to the broker?" that is a subsequent act to the receipt. The receipt is the acceptance, by their presence in the room, of the money which is laid on the table before them. What they do with the money afterwards does not concern the purchaser.

Then it is said that this is a case in which the circumstances are such that the purchasers might be satisfied with the authority given to the one. Upon that there seems to be some difference of opinion. On one side it is said that only one of the trustees is in London. On the other side it is said that that is not so, and that the deed of conveyance describes two trustees as resident in London. Why should the purchasers trouble themselves with all these circumstances? A course is proposed which will put the trustees to very little inconvenience if they cannot meet, namely, that the money should be paid to their joint account at the bank. It seems to me no case is made out which can compel the purchasers to take any kind of risk, or possibility of risk, in the matter, and that they are justifiably prudent in insisting that the money should be paid as they propose, either to all the trustees meeting in a room, or to their joint account at some bank to be approved by them. My order will be that the requisition must be complied with, and that the Respondents do pay the costs of the application.

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(1) 52 Law J. Rep., Ch. 870; Law Rep. 24, Ch. D. 387.

SOLICITORS:

R. Ward, for the Board.
W. & J. Flower & Nussey, for Respondents.

A Note on Determinants.

THE Theory of Determinants may be said to have no direct application to an actuary's pursuits. In his work, however, he occasionally requires the solution of simultaneous equations; and it has occurred to me that, notwithstanding the very restricted character of the theory in relation to his ordinary work, a presentation of the theory in the aid it affords in solving these equations (enabling us to write down the values of the unknown quantities in terms of those known, almost by inspection) may, by a practical example, induce students to devote attention to this valuable field of mathematics, not merely as a scheme of mental discipline, but as furnishing a really interesting subject of investigation and study.

With this preliminary apology I propose, before exhibiting the application of the theory to simultaneous equations, involving several unknown quantities, to briefly indicate the nature of a Determinant and its mode of expression and use.

Take the general expression—

$$xy'z''+x'y''z+x''yz'-x''y'z-x'yz''-xy''z'$$

formed from the quantities

$$x, y, z, x', y', z', x'', y'', z''.$$

Exhibited in the form

$$\begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix}.$$

the expression is called a Determinant, and the complete expression is obtained from it by

- (i) Multiplying together the elements contained in the diagonal sloping downwards from x to z'', in order to form the first term, $x\,y'z''$;
- (ii) Multiplying together the elements in the line sloping downwards from x' to y'', and the last element, z, in the top row, in order to constitute the second term, $x'y''z$;
- (iii) Multiplying x'' into the elements in the line sloping upwards from z' to y, to form the third term, $x''y\,z'$.

The terms thus obtained are positive.

To obtain the negative terms, we

- (iv) Multiply together the elements contained in the diagonal sloping upwards from x'' to z, thus forming the fourth term, $x''y'\,z$;
- (v) Multiply together the elements in the line sloping upwards from x' to y and the end element z'' of the bottom row, obtaining thus the fifth term, $x'y\,z''$; and
- (vi) Multiply together the first element x in the top row and the elements shown in the line sloping downwards from z' to y'', forming therefore the final negative term, $x\,y''z'$.

Thus

- (i) The algebraical expression developed from the Determinant

$$\begin{vmatrix} a & b & -c \\ d & -e & f \\ g & h & -k \end{vmatrix}$$

is, by the preceding method, obtained as

$$aek-dhc+gfb-gec+dbk-afh.$$

- (ii) Similarly, the single number equivalent to the Determinant

$$\begin{vmatrix} 4 & -1 & -2 \\ 0 & 3 & 0 \\ 3 & -7 & 4 \end{vmatrix}$$

is ascertained to be

$$48+0-0+18+0+0=66.$$

(iii) The value of x derived from the equation in Determinant form

$$\begin{vmatrix} x & -4 & 1 \\ -6 & 3 & -2 \\ x & 2 & 1 \end{vmatrix} = 0$$

is

$$3x - 12 + 8x - 3x - 24 + 4x = 0$$
$$\textit{i.e.,} \quad 12x = 36$$
$$x = 3$$

(iv) And the following expression—

$$bfj + eid + hcg - hfd - ecj - big$$

reduced to Determinant form, becomes

$$\begin{vmatrix} b & c & d \\ e & f & g \\ h & i & j \end{vmatrix}$$

The quantities in the Determinant are called *elements* of the Determinant; those standing in the horizontal lines together constitute a *row* of the Determinant; and those in the vertical lines form a *column* of the Determinant. The rows are numbered first, second, and third, reckoning from the top; and the columns are similarly numbered respectively from the left. The elements contained in the line sloping from the left top corner to the right bottom corner constitute the Principal Diagonal of the Determinant.

Take now the simultaneous equation

$$bx + cy = d$$
$$b'x + c'y = d'.$$

Solving it in Determinant form we obtain, by the rule furnished below,

$$x = \frac{\begin{vmatrix} d & c \\ d' & c' \end{vmatrix} \text{ = numerator;}}{\begin{vmatrix} b & c \\ b' & c' \end{vmatrix} \text{ = denominator:}}$$

and

$$y = \frac{\begin{vmatrix} b & d \\ b' & d' \end{vmatrix}}{\begin{vmatrix} b & c \\ b' & c' \end{vmatrix}}$$

Again, to find by means of Determinants the values of x, y and z, which satisfy the following equations:

$$\begin{aligned}3x-4y+2z&=1\\2x+3y-3z&=-1\\5x-5y+4z&=7\end{aligned}$$

To find x:

$$x=\frac{\begin{vmatrix}1&-4&+2\\-1&+3&-3\\7&-5&+4\end{vmatrix}}{\begin{vmatrix}3&-4&+2\\2&+3&-3\\5&-5&+4\end{vmatrix}}$$

and working out the Determinants as above explained we obtain for the numerator

$$12+10+84-42-16-15=33$$

for the denominator —— $=1$

$$36-20+60-30+32-45=33.$$

Thus $x=1$.

And similarly for the remaining unknown quantities.

The rule for solution is—

The denominator is the Determinant whose elements in order are the coefficients, with their given signs, of the unknown quantities in the equations; and the numerator, representing the value of any of the unknown quantities, differs from the corresponding denominator only in the substitution of the right-hand members of the equations (with their given signs) in the place, in order, of the coefficients (with their given signs) of the unknown quantity whose value is to be obtained.

The books in which the student can thoroughly study this interesting department of mathematics are (among others) Scott's *Theory of Determinants*, Muir's *Theory of Determinants*, and Salmon's *Lessons in the Modern Higher Algebra*. Examples of the application of the theory in algebra and mathematical physics may be found in Prof. Clifford's *Collected Mathematical Papers*, and also in his *Elements of Dynamic*.

T. E. YOUNG.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1885.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	140,750,808	2,947,249	143,698,057
Adjustment for one Industrial Return discontinued (Deficiency Balance)	...	+ 1,572	+ 1,572
	140,750,808	2,948,821	143,699,629
Premiums	12,555,797	3,289,181	15,844,978
Consideration for Annuities . .	644,274	...	644,274
Interest and Dividends (less Tax)	5,918,058	120,296	6,038,354
Increase in value of Investments .	227,718	228	227,946
Fines, Fees, &c.	7,085	183	7,268
Capital Paid-up	6,122	5,254	11,376
Customs Timber Measuring, &c. .	2,547	...	2,547
Donations (Itinerant Methodists) .	3,282	...	3,282
Transfers from other Accounts .	20	...	20
Miscellaneous	219	16	235
	160,115,930	6,363,979	166,479,909

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	10,475,536	1,250,250	11,725,786
Cash Bonuses and Reduction of Premiums	896,225	44	896,269
Annuities	633,091	15	633,106
Surrenders	772,905	5,705	778,610
Commission	579,992	870,593	1,450,585
Expenses of Management . .	1,195,134	528,198	1,723,332
Bad Debts	18,534	537	19,071
Decrease in value of Investments .	69,592	...	69,592
"Losses, Bad Debts, &c." (Briton Medical and General) . .	226,207	...	226,207
Interest on Capital and Dividends and Bonuses to Shareholders .	576,222	5,847	582,069
Transfers to other Accounts . .	20,673	280	20,953
Miscellaneous	2,567	...	2,567
Balance* at the end of the Year .	144,649,252	3,702,510	148,351,762
	160,115,930	6,363,979	166,479,909

* This Balance includes the whole of the Life and Annuity Funds (£143,133,223), and, in addition, the Capital of those Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1885).

LIABILITIES	Ordinary Companies	Industrial Companies	Total
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,409,805	117,027	11,526,832
Life and Annuity Funds	139,546,012	3,587,211	143,133,223
Fire Funds of Companies transacting Life business	7,839,560	...	7,839,560
Marine Funds of Companies transacting Life business	601,026	...	601,026
Reserve Funds	3,513,368	...	3,513,368
Other Funds	419,891	505	420,396
Profit and Loss Balances	2,151,145	...	2,151,145
Depreciation and Investment Balances	186,189	100,000	286,189
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,634,969	19,562	3,654,531
Outstanding Accounts	390,127	404	390,531
Temporary Loans	114,740	...	114,740
Sundries	12,338	10,000	22,338
	170,921,970	3,834,709	174,756,679

ASSETS	Ordinary Companies	Industrial Companies	Total
	£	£	£
Mortgages	74,934,342	104,941	75,039,283
Loans on Policies	7,998,896	5,143	8,004,039
„ Rates (and Rent-charges)	21,323,681	1,270,310	22,593,991
British Government Securities	4,848,447	494,101	5,342,548
Indian and Colonial Government Securities	10,700,167	4,510	10,704,677
Foreign Government Securities	3,897,809	...	3,897,809
Debentures	12,412,326	216,386	12,628,712
Shares and Stocks	10,398,198	...	10,398,198
Companies' own Shares	613,057	...	613,057
Land and House Property, and Ground Rents	9,384,463	1,340,570	10,725,033
Life Interests and Reversions	3,214,410	48,611	3,263,021
Loans on Personal Security	1,305,355	18,872	1,324,227
Agents' Balances and Outstanding Premiums	3,568,799	206,260	3,775,059
Outstanding Interest	1,515,130	43,171	1,558,301
Cash, Bills, Stamps, &c.	4,601,825	78,801	4,680,626
Customs Timber Measuring Balances, &c.	2,252	...	2,252
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Outstanding Accounts	9,043	...	9,043
Deficiencies, Preliminary Expenses, &c.	118,770	3,033	121,803
	170,921,970	3,834,709	174,756,679

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's* SUMMARY (1885), *when compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
	£	£
INCOME		
Premiums	+ 248,645	+ 229,917
Consideration for Annuities	+ 7,764	...
Interest and Dividends (less Tax)	+ 152,496	+ 28,305
Net Increase in Value of Investments	+ 60,190	− 275
OUTGO		
Claims	− 143,409	+ 125,628
Annuities	+ 39,940	...
Surrenders	− 426	+ 819
Commission	+ 31,717	− 9,505
Expenses of Management	+ 54,155	+ 66,725
LIABILITIES		
Paid-up Capital (including sundry Shareholders' Balances)	+ 40,146	+ 2,738
Life and Annuity Funds	+ 3,941,380	+ 748,454
ASSETS		
Mortgages (including Loans on Rates and Rent-charges)	+ 879,882	+ 340,523
Life Interests and Reversions	+ 264,548	+ 2,153
Loans on Policies	+ 192,724	+ 551
British Government Securities	+ 185,929	+ 146,683
Indian and Colonial Government Securities	+ 1,319,350	− 3,760
Foreign Government Securities	− 168,724	...
Debentures	+ 207,457	+ 74,012
Shares and Stocks	+ 468,670	...
Companies' own Shares	− 5,633	...
Land and House Property and Ground Rents	+ 494,306	+ 190,982
Loans on Personal Security	− 83,695	+ 2,375

During the year one name has been removed from the official List of Companies, namely, that of the City (Limited), which went into Liquidation. One new name has been added, namely, that of the Blue Ribbon (Limited).

The total number of Companies appearing in the above Summary is 104, of which 95 have been classed as Ordinary, 8 as Industrial, and 1 appears in both Classes, this Company's Return showing its Ordinary and Industrial business separately.

THE INSTITUTE OF ACTUARIES.

PROCEEDINGS AT ANNUAL MEETING.

WE give below a *résumé* of the proceedings at the Annual General Meeting of the Institute, held on the 5th June last, which considerations of space compelled us to omit from the last number of the *Journal*. The occasion, as was aptly remarked by the retiring President, was a historical one; and we accordingly present our readers with a fuller account than usual of the interesting speeches delivered.

The **PRESIDENT** (Mr. T. B. Sprague) pointed out that it was the first meeting of the new Institute—that is, the first meeting after the adoption of the new bye-laws. It was, therefore, a turning point in the history of the Institute, which had obtained its charter of incorporation, had its constitution thoroughly settled, its bye-laws approved by the Privy Council, and had now fairly started upon its new career. The Council hoped that the new Institute would be as successful and as prosperous as its predecessor, and that it would be even more prosperous, as it certainly would occupy a more important public position. In connection with this subject, he knew he should be expressing the sentiments of all the members if he paid a tribute to the services of the gentleman who had done most to bring about the change in the constitution of the Institute—Mr. Sutton. Mr. Sutton had taken an immense amount of trouble in assisting to obtain the charter, and but for his exertions the charter would probably never have been obtained. He had drafted the charter and the bye-laws, which were then carefully considered by the Council and the various committees appointed to take charge of the subject. Why Mr. Sutton was led to take so much interest in the matter was, he understood, that, in his official position as actuary to the Registrar of Friendly Societies, he had found the necessity of having a precise legal definition of an actuary, inasmuch as he constantly had reports submitted to him which were made by unqualified persons professing to advise friendly societies, and styling themselves actuaries; and he saw the great benefit which would result, not to the actuarial profession, but to the large number of poor men who are interested in friendly societies, if a legal definition, an unmistakable definition, could be given to the term actuary. The Council of the old Institute took up the matter warmly, with the result that the Institute, after an existence of more than thirty years, had at last obtained a charter of incorporation, and an actuary could now be defined as a person who holds the diploma of the Incorporated Institute of Actuaries in England, or of the Incorporated Faculty of Actuaries in Scotland. This was a result which he was sure they would all heartily congratulate themselves upon. The number of members continued to be very much the same, having increased by seven during the year. The new members elected were 25, of whom 24 were students. These students formed a new class of members, established in terms of the charter. In certain cases he (the President) understood that the letters A.I.A., which the Associates of the Institute put after their names, had been supposed to imply that the persons using them were necessarily qualified actuaries; and in order to prevent, as much as possible, any misunderstanding of this kind in future, all persons hereafter admitted members of the Institute without having any knowledge of the special subjects of the Institute would be admitted as students simply, and would not be entitled to place any letters after their names. During the past year the Institute had been, to a considerable extent, in a state of transition. Until the bye-laws were completed and certain regulations drawn up and approved by the Council, it was not practicable to admit members in the usual way, and that would account for the fact of only one Associate being elected, and not a single Fellow, from outside. The hon. secretary had informed him that at the present time there were a great many candidates for admission on the lists, and that, in particular, the number of students applying for admission was considerable. In connection with the financial position of the Institute, mention had been made of the *Text Book*. Personally he was somewhat at a loss to know what they were to do with their capital—£3,000—and that was a point which he commended to the attention of the new President. Now that the Institute was incorporated and its *Text Book* well advanced, it seemed time to consider whether any further accumulation of capital was desirable. He was quite sure they would be pleased to hear the announcement made by the Council that the first volume of the new *Text Book* was nearly ready, and would probably be published during the current year. In the course of preparing this work, the gentleman who had undertaken it, Mr. George King, had found the task grow so much under his hands that the book would be larger than was originally intended. In consequence of that, the Council had resolved that the publication should not be delayed until the whole work was completed, but that the first part of it should be published as soon as finished. This first part would, practically, cover the ground of the second examination of the Institute; and the publication of this part of the *Text Book*, therefore, would be of great assistance to students preparing for that examination. As to the papers read during the year, he would only say that they were very glad indeed to have had a paper by one of the former Presidents, Mr. Bailey, on the income-tax. That was a subject of great practical importance and interest to all, and he should like to express their sense of indebtedness to him (Mr. Bailey) for his exertions in bringing the question of the liability of assurance companies to pay income-tax upon certain items under consideration of the law courts. The next subject mentioned in the report was that of the examinations of the Institute. Thirty-two candidates had presented themselves for the first part, of whom only seven passed. It was much to be deplored that so small a proportion of those who presented themselves succeeded in passing, and the Examiners thought it desirable to make special reference to that subject in their reports to the Council. They expressed their opinion that the papers of many of the candidates showed clearly that sufficient time had not been given to study, and that their preparation for the examination was too hurried. The Council would have to consider what they could do in the way of inducing or assisting students to prepare themselves more thoroughly, by devoting a longer time to their preparation for the different examinations. There were two tutors appointed by the Institute to prepare students for the first and second examinations, but the Council would consider whether it would not be practicable to spread the course of study for each of these examinations over two years, instead of one, for those students who wished to avail themselves of the services of the tutor for a longer time. With regard to the third year's examination there was no tutor at present, and consequently the Council would consider whether they could give assistance to the students preparing for this examination by appointing a tutor, or by indicating in some way the course of study that should be followed. These questions would not be lost sight of by the Council. In connection with this subject, he would mention that the practice of giving a certificate to the students who had passed the final examination had been, by a resolution of the Council, abolished, and in future no certificates would be given. There would be an authoritative printed list of the members of the Institute, and that list would show what examinations had been passed by each of the members, and the printed official list would take the place of the certificate. The only other point he had to mention, was to join in the regret that the Council had expressed at the loss of two of its prominent members—Mr. Hodge, who was formerly one of the prominent members, and Mr. Walford, who almost up to the day of his death took an active part at the meetings. It was a melancholy thing to see their old friends removed by death in this way, but when a society had been established as long as the Institute, such things must be expected in the ordinary course of nature, and they could only hope that younger men would strive to fill worthily the places of those whom they had lost. He then moved the adoption of the report.

Mr. G. HUMPHREYS seconded the adoption of the report; and the motion was agreed to unanimously.

The PRESIDENT, in asking the meeting to elect the members recommended by the Council, said he wished to express the great pleasure he felt at seeing his old and valued friend, Mr. Archibald Day, proposed as President. He (Mr. Day) was one of the earliest members of the Institute, and had passed the first public examination held for the certificate of competency. Among other services he had rendered to the Institute, he had trained up, in the comparatively small office over which he formerly presided, no less than four actuaries, of whom at the present time three were the principal officers of companies of high standing. He would be a worthy representative of the Institute, and would, he was sure, give unqualified satisfaction to everybody with whom he might be associated. Amongst the Vice-Presidents he was glad to see the name of one of the recently-joined members, Mr. Edward Docker, who belonged to the Actuaries' Club. It was a great satisfaction to see the amalgamation of the two sections of the profession thoroughly completed in this happy way. With regard to the Council generally, about one-half of those recommended were new members—14 out of 30. On former occasions it had been customary to have about five new members only in the place of old members. Last year there was no change of members of the Council, in consequence of the adoption of the new charter. For that and other reasons, it had this year been thought desirable to make a larger change than usual by bringing in a greater proportion of new men; and he trusted the change would be found to work well in practice.

The office-bearers proposed having been unanimously elected,

Mr. ARCHIBALD DAY thanked the members for the compliment which they had paid him in electing him to the position of President of this Institute—an Institute of which he had been a member for so many years. He felt conscious that his election was due more to the feelings of personal friendship which existed between himself and many members of the Institute than to any merits of his own, and he fully appreciated the distinction which the office of President conferred upon him. But, at the same time, he was aware of many defects and shortcomings, which he feared might preven this proving so efficient a President as they could wish or he desire. His actuarial practice was of a somewhat limited character, and his knowledge of the higher mathematics—acquired in what would now be considered the dark ages —was not up to the mark at the present day. This he would feel all the more in having to follow so distinguished an actuary and scholar as his friend Mr. Sprague. But the Council must take the responsibility upon themselves; and perhaps it was just as well that the Presidents should not always be formed in the same mould. The Institute would hardly expect from him those learned addresses and scientific papers which Mr. Sprague had been in the habit of contributing, but it was possible that he might be able to help the Institute in practical matters. He had had an experience of some thirty years on the Council, and during that long period, by the kindness of his colleagues, he had never been off it a single year. This experience, and the intimate association he had enjoyed with the distinguished Presidents who had preceded him, might give him a little knowledge how to manage the business department of the Institute. With regard to the more abstruse actuarial matters, he would have to trust very much to the assistance of the distinguished Vice-Presidents now added to the list. He was sure that the Council could not have chosen names which would be more acceptable than those which had been approved, and he had no doubt that they would all be glad and willing to assist him. If it had rested with him, he should have been exceedingly happy to yield precedence to any one of those gentlemen. A great number of new members had been elected on the Council, and he had no doubt they would lend him valuable aid. With new men they might hope for new papers, and he felt sanguine that some of the new members of the Council would contribute them. It was unfortunate for him that Mr. Manly and Mr. Finlaison, who had so admirably performed the duties of honorary secretaries for four years, were retiring—for the secretaries were, after all, the real executive,— but their newly-elected successors were both so well-known, and so constant in their attendance at the Institute, that he had not the least doubt they would be able to work harmoniously, and keep the Society going for another twelve months. Anything he could possibly do for the Institute would be small in comparison with the advantages he had derived from it—he being now one of the oldest members, and feeling that almost all his success in life had been due to his connection with the Institute.

The auditors having been unanimously re-elected by the associates,

Mr. B. Newbatt proposed that they should give their thanks to the outgoing President, Vice-Presidents, Council, and officers of the Society. In saying that such a resolution at such a meeting would be its own best advocate, he by no means wished to imply that it should be accepted as a matter of course, because everybody was too indifferent to put a vote of this kind to the challenge. They would all agree that no higher compliment could be paid to earnest men than to watch their work closely, and to criticize it or reward it as its merits might seem to deserve. On an occasion so exceptional as this—he might even say so historical—when, to adopt a phrase now being largely used in another place, sometimes with exultation and sometimes with despondency, they were "at the parting of the ways",—a resolution of this kind would gain by some small special comment. This comment he thought himself not unfitted to offer; for having for nearly half a generation merely looked on the proceedings of this Institute from a distance, he had enjoyed, for this purpose, the advantage which the old adage allots to those who watch the game rather than to those who play it. In his view this resolution, though chiefly directed to those who were retiring, had a wider sweep and a larger embrace, for in some sort, at this moment, he thought they should extend their acknowledgments to those other and earlier guiders of the Institute, who in their day and generation did so much to build up the fame and enlarge the usefulness of this Society. Many of them, unfortunately, were equally beyond praise and blame. But, fortunately, there were still left many of the old officers who had been with the Institute through good and ill report, and they would rejoice that this first chapter of the Institute had been brought to a close by men who had shed so much lustre upon its annals. In the late President especially, he thought he should wound no susceptibilities if he asked them to recognize one who was *primus inter pares.* He had exemplified in many fields of labour the old maxim that "knowledge is power"; and in their domain of work he had united with a profound technical knowledge of assurance principles the power of adapting them to the practical affairs of life, which they and those who followed might endeavour to emulate, but could never hope to excel. He (the speaker) also particularly wished to mention the name of one who might, *par excellence,* be called the maker of the charter, and who had been fittingly chosen as one of the first to reap one of the highest honours which the Institute could confer. For many years Mr. Sutton, by labours known and unknown, had deserved well of their little State, and he trusted he might live to enjoy an even larger measure of the esteem in which they held him, and of the thanks they now awarded him. He would add one word as to the future: In the higher and larger place which the Institute held in the public regard, there had been much wisdom shown in giving it a visible unity by calling to this new Council, men representing every section of the profession—men who had not hitherto been in close association with them, and who, always held in high regard, had very fittingly been referred to by the present Chairman. In the new President they had one who, sharing all the best traditions of the Institute, would bring to its councils a welding influence, and a highly stimulating influence, for notwithstanding all his modest disclaimer of his own merits, he would be able, both by precept and practical example, to teach the rising generation the sometimes needful lesson that, by acts of faithfulness as well as by products of genius, every man, by doing in his day and generation with all his might that which his hand found to do, might be a help and ornament to the profession from which he received countenance and profit, and to which he was in consequence so much the debtor. He moved a cordial vote of thanks to the retiring officers which, Mr. E. Justican having seconded, was at once heartily adopted.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President, ARCHIBALD DAY, ESQ.

[Delivered 29 Nov. 1886.]

IN conformity with the practice which has grown up during recent years, it becomes my duty, in virtue of the office which you have so recently conferred upon me, to offer on this, the first night of the session, some few remarks in the nature of an inaugural address.

The custom was initiated, if I mistake not, by my excellent friend Mr. Bailey during his tenure of the office of President of the Institute, and it was not allowed to fall into abeyance while the Presidential chair was filled by his successor, Mr. Sprague, whose learned and most instructive addresses were looked forward to from year to year by the members of the Institute with ever-increasing interest. I could not hope to achieve any such success, and, indeed, I would gladly have broken through the rule and have thus avoided the comparisons which might be made between my humble efforts and the elaborate manifestoes of my great predecessor; but I comply with the desire of my colleagues on the Council, who hint that this ceremony is part of the duties attaching to the post of honour to which I have been elevated, trusting that you will bear with patience what I fear may not prove very instructive or interesting remarks. These, I am disposed to make rather of a retrospective character, and to comprise within them some matters which are not wholly confined to the finance of life assurance.

It will be freely admitted that any further reference to the new Charter of Incorporation is quite unnecessary. That has been introduced again and again, and the subject is well-nigh exhausted. What remains to be seen is "what we shall do with it." One great advantage which *has* actually accrued to the Institute thereby has been the prolongation of the usual term of office in the case of my distinguished predecessor, while it has at the same time curtailed the period during which the present and future Presidents may retain the dignity. It seems to me fitting that I should on this, the earliest occasion which offers, express the admiration which I feel for Mr. Sprague's genius, and for the hearty zeal which he has ever thrown into all matters which he undertook, both in the theoretical and practical sides of the actuarial profession, for the care which, as President, he has shown for the honour of the Institute and in the direction and supervision of its affairs, and for the mass of learned and original contributions which he has made to the literature of the profession through the medium of the Institute and its *Journal*. *"Nil tetigit quod non ornavit."*

Those whose acquaintance with Mr. Sprague has endured as long as my own—and there are few left who have known him longer—can testify to the devotion which he, throughout his career, has displayed in the encouragement of the exercise of the *educational functions* of the Institute. His own academical distinctions (combined as they were with the knowledge he had acquired of the practical duties of the actuary to a life assurance company), procured for him his election as a Fellow of the Institute without the ordeal of the usual examinations. But this did not prevent his taking the most active interest in the conduct of the examination of the many students who came up in the usual course; and, learning what were the principal difficulties which they had met with in their reading, he volunteered, and with the approval of the Council founded, a class in which to read, with any who might be willing to join, the subjects contained in the syllabus of examinations. This was, indeed, the *ovum* from which sprung the appointment of a tutor, subsidized by the Institute for the assistance of students preparing for the "Intermediate" Examination, the success of which, having been many times publicly recognized, has led, after a few years' experience, to the extension of the privilege to the students reading for what was then known as the Matriculation Examination. Those who are better qualified than myself to form an opinion upon this matter have decided that the time is not ripe for offering the same assistance to those desirous of passing, by the final examination, from the degree of Associate to that of Fellow, desiring to observe first, the results which may follow from the more detailed syllabus which was in course of preparation; otherwise, I should advocate the appointment of a third tutor, under the conviction that this was a most legitimate mode of expending part of the income of the Institute, and as being another step in the direction of its educational objects.

It was as recently as last session that Mr. Sprague contributed his great paper on the Graphic Method of Graduation—an admirable specimen of thoroughness in the mastery of a subject, combined with extreme lucidity; and it may be doubted whether any scientific principle or method was ever advanced so much by a single effort. He has made the subject his own, and it will always henceforth be associated with his name. Mr. Sprague has also advanced upon and widely extended problems which have exercised the minds of others; and as but one instance thereof, I am glad to record the sense of gratitude and relief I feel for his carrying many steps further, an investigation to which I had devoted much attention in former years, namely, the probabilities of marriage and of the birth of issue therefrom, combined with the question of assurance necessary to protect reversionary interests against the contingency thereof. The continued success of this Institute will be assured so long as its most eminent members evince the same unflagging interest in its proceedings, the same perseverance in their efforts to extend the scientific knowledge of our profession, and the same desire to increase its power and influence, as Mr. Sprague so conspicuously has done during the many years of his membership.

Passing from this personal tribute to the merits of the late President, I would refer cursorily to a few changes which have taken place in the conduct of the affairs of the Institute, and principally to a variation introduced into the system of examinations. The new rules of examinations prescribed by the Council in accordance with the Bye-Laws, and which really came into operation in April last, have provided opportunities for candidates obtaining special recognition by the introduction of optional subjects into each part of the necessary examinations, such as Conic Sections in Part I, Differential Calculus in Part II, and the Application of Higher Mathematics in Part III. Those gentlemen who are content simply to qualify for the next stage will find the examination practically very similar to what has hitherto been considered sufficient; but on the publication of the list of successful candidates the names will appear in alphabetical order, while those who take up, in addition and with success, the optional subjects, will gain the distinction of special marking in the published list. It would be gratifying to myself if some scheme of a special honour class could be evolved.

It may not be within the knowledge of all present this evening that on the occasion of the very first examinations held by the Institute, which happened in the year 1850—and the practice obtained for a few years subsequently—the result announced was that Messrs. Bailey, Orchard, and Porter had passed *with distinction*, and that certain other gentlemen had passed. The names were placed alphabetically, the Council considering it inexpedient that any order of merit should be recorded. It thus appears that we are now, in some measure, reverting to the original practice, and it is probable that the idea will meet with general approval, though I, personally, feel that a greater difference might be made between the classes, rather in the direction of the application of a less severe examination test to those who do not aspire to more than an ordinary pass. It is hoped that the appointment of an examination committee (provided for by recent Bye-Laws) to advise with the examiners, &c., may prove valuable, and may further tend to a more consistent relation between the papers of different years, and perhaps, also, between the proportion of the successful candidates to the unsuccessful. The percentage of failures on the last occasion was very marked, and some soreness was exhibited in the letters of certain correspondents in the insurance press — presumably unsuccessful candidates. The members may be assured that every possible consideration will be given by the examination committee to all reasonable complaints.

Some of the statistics of the results of former examinations seem worthy of record, and they are accordingly submitted in the following table:

Date of Examination (Years included)	PRELIMINARY			INTERMEDIATE			FINAL		
	Entered	Passed	Per-cent	Entered	Passed	Per-cent	Entered	Passed	Per-cent
1850–51	(Only one examination required in these two years)						12	11	91·7
1852–56	52	40	76·9	23	22	95·7	7	7	100·0
1857–61	44	30	68·2	19	14	73·7	10	8	80·0
1862–66	63	30	47·6	34	11	32·4	10	8	80·0
1867–71	104	42	40·4	40	20	50·0	11	7	64·2
1872–76	96	32	33·3	56	25	44·6	22	14	64·2
*1877–81	105	39	37·1	52	20	38·5	10	8	80·0
1882–86	116	44	38·0	76	35	46·1	38	19	50·0
	580	257	44·3	300	147	49·0	108	71 11	65·7
								82	

* Four years only are included in this division, the date of examination having been changed in 1878-9 from December to April.

The table requires little explanation, but the grand results appear to be that, from the foundation of the system of examinations, 592 candidates had presented themselves, and 82 successfully passed their final examinations. It may be further stated that 65 remain upon the list of existing Fellows of the Institute, eight only have died, while nine have retired from various causes. It is interesting to notice that 44 per-cent of the candidates at matriculation were successful, 49 per-cent at the intermediate, and 66 per-cent at the final examination, from which it may be inferred that applicants come up for the first less thoroughly prepared than they do when they enter for the two later and more important competitions.

While it must be the aim of the Council of the Institute to keep up a high standard of efficiency for admission to the Fellowship, it must, at the same time, be borne in mind that the candidates at our examinations are, as a rule, drawn from a limited class—the clerks in life assurance offices—and that these are not, in the majority of instances, appointed to such positions on the score of their having any special aptitude for actuarial studies. They are handicapped, too, in a manner not experienced to so great an extent by students in other professions, by many hours of daily official work, and those who have come up for our examinations and proved successful, must have exercised great self-denial in the course of their preparation. I would that the rewards which follow such merit and perseverance were of a more lucrative character. Higher prizes are, no doubt, to be obtained in other professions; still I question whether the position of the actuary to an assurance company of wealth and standing is not one to be envied by a host of successful practitioners in any other profession.

It will have been noticed, as a significant change effected lately in the conduct of the affairs of the Institute, that, at the recent annual meeting, in addition to the introduction on the Council of certain distinguished members of the Actuaries' Club who had previously become, by virtue of the Royal Charter, Fellows of our body, a considerable infusion of new blood had been effected from the ranks of those gentlemen who had become Fellows by examination. This is clearly in the spirit of the new constitution, and much is hoped for from the rejuvenescence of the Council; but a word of recognition is due to the self-denial of several distinguished actuaries who have voluntarily given the members the opportunity of effecting so large a redistribution of seats. It may now be hoped that no more may be heard of the whisperings that widely propagated the notion that the members of the Council were a self-elected clique.

Looking further afield than the limited concerns of our own Institute, I may refer to a few matters possessing general interest for assurance students which have occurred since the President's address of last session, some of which have been of vast significance and importance. Although it has been the practice at our sessional meetings to avoid discussion of the affairs of individual companies, confining our observations mainly to the consideration of theoretical principles, it is impossible to pass over in silence the disaster which has overtaken the Briton, Medical and General Life Association. I will ignore all discussion of the chief causes of the fall of the institution, the dark spot upon the record of the assurance history of the year, but I claim to be entitled to bring before you certain matters and certain lessons which may be learned from the catastrophe.

1. Referring to the now published report to Mr. Justice Kay of Messrs. Deloitte, Dever and Co., the accountants appointed by the Court to investigate the position of the assets, it will be found that they represented that certain amounts "had been abstracted "from the funds of the association *at various times* and in various "manners *since the year* 1873"; and in another place, "these "additions are fraudulent, inasmuch as they render *false* the "published accounts of the association, although they do not add "to the total loss by defalcations." (I may remark, parenthetically,

that for perspicuity and method the report of the accountants can bear no comparison with that of the actuaries employed by the Court). What is the lesson that we may learn here? Is it that a system of Government supervision, glibly recommended by irresponsible writers, should be introduced? Is it that the Act of 1870 has failed? I think not; and for myself would strongly deprecate panic legislation, or any additional interference by Government departments. Frauds happen in other institutions besides life assurance companies, and although this failure may be the cause of much suffering to many unfortunate persons, it is a matter of great congratulation that the vast mass of our life assurance societies, representing approximately assets of 150 millions and sums assured of 450 millions, are beyond suspicion, and indeed are both admirably and conscientiously conducted. The conclusion which it seems to me should be drawn from this catastrophe is, that the auditing of assurance office accounts should be made of a far more thorough character, and that companies should appoint and properly remunerate independent and professional accountants, who might carry on a continuous audit, and who, it may be suggested, should be Fellows of the Institute of Chartered Accountants, just as their actuaries invariably will be Fellows of our Institute. The idea is not novel. It has existed in practice in Scotland where assurance institutions attain gigantic dimensions, and these are almost universally audited by Chartered Accountants who are independent of the officers, and who keep up a regular and continuous audit. It is a credit to Scotland that no assurance failure of this disastrous character has occurred there, and I suggest whether to the system of audit may not be attributed some of the merit.

It is encouraging to be able to point to some signs of an improving system; for instance, one of our most conservative assurance institutions appointed professional auditors at its last annual meeting, and the matter was discussed with a view to future adoption at the meeting of another company which I was privileged to attend.

2. It is fair to note here how inaccurate are the statements which have been made in various journals that certain eminent actuaries gave, as recently as 1875, a certificate of the *solvency of the Briton, Medical and General.* This assuredly they did not do. They were consulted upon its position as exhibited by the statements and valuations made up to December 1872, based upon the Carlisle 4 per-cent Tables and deposited with the Board

of Trade, and by a subsequent valuation made two years later, and they found on the figures submitted to them that there was a balance of some £200,000 available for future expenses and contingencies. They further approved a plan submitted to them for securing for the benefit of the policyholders the existing assets, and for reducing the expenses to a minimum, upon which scheme they reported thus (the document has become public by its appearance in the Parliamentary Blue Book):

> "It is proposed to accomplish this by means of a new company with an adequate paid-up capital, bearing the name of the Briton, and conducting its business on the same premises, which, in consideration of the valuable connection which will at once be available, will agree to work the existing business at a charge of 8 per-cent of the premium income for the first two years, 6 per-cent for the succeeding five years, and 5 per-cent thereafter. Assuming this plan to be carried into effect, *the existing business to be preserved*, and taking the balance of assets over and above the accrued liabilities to be £659,305, *according to the balance sheet last published*, and bearing in mind that the society possesses a large unpaid capital (£142,000) as a further guarantee for the integrity of its assurance contracts, we are of opinion that it will be in a position, *with careful and judicious management*, to discharge all existing liabilities as they mature."

The gentlemen whose signatures were attached were actuaries of the highest reputation in their profession. Whether it were wholly wise to give an opinion at all upon a case in which the figures set out had not had their personal verification is a matter which we need not now discuss. Undoubtedly the sole motive which actuated them was the noble desire to save from shipwreck the institution and its members; and had the facts placed before them been as stated, and had the office been in the future conducted on the terms which had the approval of the actuaries, they would not now be feeling misgivings that they had, though with such praiseworthy motives, lent their authority to its reconstruction. It may not be generally known, but it is no less true, and I take this opportunity of giving it publicity, that these gentlemen, when they found by the annual returns that the management was not acting on the understanding which was the fundamental basis of the new arrangement, addressed a protest to the board against the course which was being pursued, and then and there repudiated all responsibility for the results which might follow.

3. On hearing the petition for winding up the Briton, Medical and General in the early part of this year, the Court remitted the task of making the requisite valuation of its liabilities to Messrs.

Bailey and Hardy (gentlemen exceptionally well qualified to undertake a duty of such delicacy and importance), and they may be complimented on the thorough manner in which they fulfilled the task, as well as upon the perfect distinctness with which, in comparatively few words, they have stated the principles upon which their valuation was made, and the final results at which they had arrived.

A very noteworthy incident in connection with this liquidation is the popularity, as evidenced by correspondence and comments in the insurance press, of the scheme for reconstruction, *with a reduction of contracts,* provided for by the clauses of the Act of 1870, in place of liquidation pure and simple. In the present instance it is probable that many unfortunate victims will have cause to congratulate themselves upon the actuarial foresight which introduced into the Act of 1870 the beneficial clauses which Mr. Justice Kay has seen fit to determine upon applying. But it is still somewhat surprizing that, when the policyholders of the Briton, Medical and General had been allowed the right of voting by proxy on the question of reconstruction or liquidation, so small a proportion should have exercised their privilege. The numbers who voted appear to have been: for winding up, 2,563, assured for £580,000; for reconstruction, 2,539, assured for £796,000; together amounting to 40 per-cent of the whole; and the reconstruction is, therefore, the act of the Court on the recommendation of only one-fifth of the assured. It might be interesting to know how it was determined who should be qualified to exercise the privilege of voting. It is not an unusual thing for a policy to be in mortgage, and between mortgagee and assured the interests in the treatment of a policy may be diverse.

Passing to a wider field, it may not be out of place to refer in a few words to the scheme of national provident insurance, promulgated by the Rev. Canon Blackley. This gentleman, an enthusiastic supporter of a somewhat Utopian idea, has had the rare good fortune to obtain for his plans the consideration of a committee of the House of Commons in two successive Parliaments (with a recommendation for the re-appointment of such committee in the newly-elected chamber); and it must be conceived, therefore, that much could be urged in support of the compulsory system of which the reverend Canon is an advocate. It was no doubt thought, by many of our actuarial friends, that the evidence given by that distinguished expert, the actuary to the Chief Registry of Friendly Societies, before the committee of 1885, had gone far towards extinguishing the chance of a favourable report, had time been available for its preparation before the dissolution. The new committee which sat last spring found its deliberations summarily stopped, precisely in the same manner and at the same critical stage; but the evidence which has since been published in the report of its proceedings contains much that will be of special interest to the members of this Institute, notably the evidence given by one of its veteran members, Mr. Pinckard—and that of Mr. Hardy, replete, as usual, with marks of his genius, freshness, and originality. Both Mr. Pinckard and Mr. Hardy, as well, apparently, as Mr. Blackley himself, attach greater importance to the provision for pension after 70 than to the sick pay prior to that age; and therein lies the essential difference between his plan and the compulsory national insurance of the German Empire, which provides (1) for sickness benefit, and (2) for compensation in case of accident, with no deferred annuity or payment at death, which benefits, Dr. Ashchrott, who was examined as to the German system, considered were distinctly objectionable. Mr. Hardy's evidence, like every actuarial report that he makes, will well repay careful study; while all his friends will cordially appreciate the energetic protest he made against being supposed to approve the *sufficiency of the proposed contribution towards the national scheme,* because he might have given an opinion that, *according to certain tables,* such an amount would mathematically *be the equivalent of sick pay of* 8*s., and pension of* 4*s. per week.*

Much opposition was offered before the committee by representatives of the large affiliated friendly societies; but it may be questioned whether their condemnation of it is especially significant, as the foundation and growth of national provident insurance under Government supervision might, in their view, endanger the very existence of many of these bodies. They have probably taken an exaggerated view of the matter. The discussion of Canon Blackley's pet scheme at our Institute would probably have been considered as treading too closely upon the border-land of politics, otherwise it might be marvelled why the actuaries had not had their say upon it. Whether a minimum sum of £10 *could* be collected within three years from every man or woman who attains the age of 18; whether *every* person should contribute the same sum in respect of the same benefits, while the probabilities of receiving them are vastly different; whether pauperism does not arise from *other* circumstances than *sickness;* whether compulsory thrift will affect an individual's *moral* nature; whether, seeing that the fund for providing the benefits has to be collected from every person, and accumulated until they fall due, it might not be a simpler process to pay them out of the taxes, without all this vast complication of collection and administration; are all questions that enter into the full discussion of the subject, but which might run the risk of being ruled outside the pale of our proceedings. It remains to be seen whether a third House of Commons will be impressed with the importance of coming to a definite conclusion upon a subject which has eluded the grasp of its predecessors. Already, notice has been given that the question will be brought forward in the next session of Parliament.

Those who feared the effect which the patronage of the State, if extended to national insurance, would have upon societies having kindred objects, were perhaps hardly cognizant of the puny results which flowed from the efforts of Government to grant assurances of small sums through the agency of the gigantic system of the Post Office, and backed by the credit of the nation. It is little more than twenty years since the Government of the day, considering it their duty to protect the savings of the provident, though poorer, classes of the community, and to give them absolute security instead of the doubtful solvency (as they considered it) of the companies then transacting assurances of comparatively small sums, inaugurated the system of life assurance through the Post Office. As may, however, be gathered from a return very recently issued, the total assurances granted through the Post Office have not exceeded £610,000; while, to make the failure to collect customers still more conspicuous, a company which was unfavourably referred to by the Chancellor of the Exchequer in introducing his measure (in 1864) counts its annual new assurances by millions. Many will think that it is hardly consistent with the functions of the Government to continue, as a department, a field of business in which they enter into direct competition with private enterprize.

While commenting on the feeble action of one State department in the matter of life assurance, it may be well to remark that there seems reason to hope—from a statement made by Baron de Worms, M.P., in the House of Commons—that the Board of Trade are making arrangements whereby it is hoped that the Blue Book containing the annual returns made by assurance companies may be issued at an earlier period of the year. It is earnestly to be desired that this exhibition of zeal may not be allowed to cool. The Blue Book was circulated this year at the end of July. The accounts contained therein would be those *lodged with* the Board of Trade during the year 1885, and would be those relating mainly to the year 1884; and the effect has been that the public, who by the enterprize of the offices have been put in possession (in most instances long before July 1886) of the figures of 1885, have had no means of comparing them with the corresponding statements of 1884. One would think that a greater work than this might obtain publication in a less period. Whether the modern system of publishing summaries in the appendix—which, be it remarked, are an innovation (and, I freely admit, a laudable one) of the department, and not in literal compliance with the Act of Parliament—may be in part answerable for the delay, we may leave undiscussed. It seems that the Board had, according to their Parliamentary secretary, received no previous complaints on the subject, and it may therefore be confidently anticipated, now that their attention had been drawn to the inconvenience of the delay, that they will speedily recognize the wishes of the profession.

The returns lately published by the Board of Trade have now become familiar to the members of the Institute, and they may be spared any recapitulation of the figures. There has been a total increase in the life and annuity funds of 4½ millions, and in the premium income of little less than half a million—nearly one-half of this being industrial premium income. But, alas! there is an increase in the rate of expenses of management and commission, which, on the ordinary branch of business, is now 14·1 per-cent, against 13·7 per-cent in the previous return. The growth of the ordinary premium income has been small; and the interval may not be long before the new assurances, though they may not fall in actual amount, will be insufficient to supply the waste caused by lapses, surrenders, or death claims.

It may be interesting to attempt an estimate of the amount of existing assurance per head of population. Taking the ordinary assurances in force at £450,000,000, and the population of the United Kingdom at 36,000,000, we have an average assurance of £12. 10*s*. Putting aside the female population, amongst whom the practice of effecting assurances in the ordinary class is very unusual, the male population remaining would be 17¼ millions, and the equivalent average assurance would be £26; while, by confining the basis to that of the adult male population, *i.e.*, above the age of 20, the number of adult males would be about 9 millions, upon which the average rate of assurance would work out about £50 per head. This is, of course, the roughest approximation; and it would be somewhat affected in the direction of a higher average by the amount of the industrial assurances, of which *comparatively* little is at present known. The industrial premium income, according to the last return of the Board of Trade, was £3,289,181—3¼ millions—and if it is assumed that the industrial premiums are about 5 per-cent on the amount assured, the gross amount of industrial assurance would approximate to 66 millions. The assurance upon children's lives and upon minors might probably be 26 millions, leaving 40 millions for policies upon lives of adults; and as it is understood that in this class of industrial assurances the sexes are about equally divided, there would be about 20 millions to spread over the population, restricted as above; and the average amount of assurance arrived at would be about £52 per head of adult male population.

It would appear from the returns that the claims by death over a series of years have shown no startling irregularities. They have increased, as it seemed natural they would, from some nine millions per annum in 1871, to 10½ millions (ordinary assurances) in the returns for 1885, and this is a very considerable sum to be distributed yearly from institutions mainly resorted to by the public for the purpose of making family provision. I am aware that the doctrine has been promulgated that the increase of life assurance did not of necessity imply an increase of provident habits in the community; but that, on the contrary, the effecting of life policies might to some extent be taken as evidence of *im*providence, attested by the numerous assurances required as collateral securities. This might have been deduced from observations upon the business of certain "class" offices; but the experience of some of the larger general assurance companies, and notably of the non-commission-paying offices, would not unlikely present strongly the reverse side of the picture. The opportunity has been afforded me of observing the claim experience of a mutual office of repute, and having extracted the whole facts from a volume of the claim registers taken at random, it was found that out of a total of £425,000, £250,000 had been paid to executors alone, £51,000 to trustees of marriage settlements, £92,000 to assignees, or executors and assignees combined, £32,000 to persons assuring the life of another. From which it appears that three-fourths of the whole had gone to the families of the assured, while probably they would have received in the other classes balances remaining after satisfying lenders or creditors. The example is not absolutely conclusive evidence, because it must be admitted that some of the members had contracted loans on their policies from the office within their surrender-value, and it might be argued that they should have been transferred to the third class in the foregoing statement; but whether conclusive or not as an argument, the facts quoted will not be uninteresting as an illustration. The proportions may be expected to vary according to the nature of the business of various offices.

The latest Blue Book does not reveal any very material changes in the nature of the investments of our companies; and that may, perhaps, be due in some measure to the period to which the accounts published relate. We should hardly expect to see, before the end of the current year, the effect which the probably temporary depreciation in the value of land (and the consequently reduced margins) may have produced under the heading of "Mortgages upon Property"; nor how far a panic may have been excited by amateur writers upon the subject of life assurance as to the nature and eligibility of such securities for the investment of assurance funds. During the last five years the amount of mortgages and the percentage upon total assets have been as represented in the following table:

Blue Book of	Mortgages	Total Assets	Percentage of Mortgages to Total Assets
	£	£	
1882	70,787,079	153,482,645	46·1
1883	71,151,812	157,875,360	45·1
1884	72,634,057	162,622,879	44·7
1885	74,282,393	166,658,597	44·6
1886	74,934,342	170,921,970	43·9

These represent mortgages on property, and do not include loans on security of *rates or rent-charges*. The latter would be more distinctly secured upon land, and should come into the category of mortgages; but the summaries in the Blue Book amalgamate the loans on rates and the rent charges. The total advances on rates and rent-charges five years ago were £19,354,056, and have increased to £21,323,861; but the percentage upon the whole assets remains the same—12½.

There are some mortgages on land which political circumstances have rendered it next to impossible to realize at the present crisis except at a very serious sacrifice. I allude to the Irish mortgages; and as something akin to panic has arisen as to these, I have been at some pains to collect all the information publicly imparted to proprietors and policyholders as to the holdings of their companies in mortgages upon Irish land. Acting on a principle that a proportion of its funds should be invested by any company in the country whence any substantial premium income has been derived, many offices have had to recognize a temporary lock-up of capital in Ireland. The directors of many companies have shown a wise discretion by promptly representing to their constituents the amounts which are so invested; and the information which I have gleaned from those who have confessed to the soft impeachment points to an investment in Ireland of some 8 per-cent of their funds. Making allowance for the restriction imposed by the constitution of some companies, it would appear that many had no such investments at all; and one is disposed to believe that, taking all assurance companies together, the rate would be distinctly lower upon the total funds, and that the entire advances on mortgage of Irish property made by the assurance companies of Great Britain would certainly not exceed 10 millions sterling. It does not seem to be within my province to estimate the probable loss which may arise, but I may call attention to the principle which has been adopted in some Scotch institutions, of forming a reserve fund from the excess of interest received on Irish mortgages beyond that realized upon similar mortgages in England and Scotland. It would be entering too much into the political sphere to offer any opinion upon the probable future of land values, but it is hoped and believed that the margins (which in Irish transactions it has been the habit to require at a higher rate than usual) have not entirely disappeared.

The investment of assurance funds on mortgage of lands in the colonies, and especially in Australia, was a topic of Mr. Sprague's Presidential address last year, and I need not enlarge upon that subject; but it is noteworthy that while British funds were seeking investment there, it occurred to Australian companies, on their part, to seek life assurance business on this side, and that we should have seen the advent of two colonial companies upon British soil. They would, I am convinced, have found a ready welcome, especially in this year of fraternization with all our colonists and the great exhibition of their products at South Kensington. The stay, however, of the more important office was very limited; the feeling of the non-official members, expressed somewhat freely, having led to the withdrawal by the directors from competition with English companies here. Unfortunately, the actuaries had so impressed the colonists with statistics showing the greater value of life in the colonies than in Great Britain, that the members at one special meeting, of which I read a newspaper report, expressed themselves strongly on the impolicy of diluting their greater vitality with the exhausted stamina of the old country. It is certainly extraordinary how greatly the practice of life assurance thrives on Australian soil. Whether this is due to the greater appreciation of the advantages of this mode of making family provision, or to the more energetic mode of canvassing on the part of the officials of the companies, we need not now stop to enquire; but I believe that, on the latter head, we have yet something to learn from our colonial rivals.

But our colonial friends have not been alone in their designs to reap a harvest in the wide assurance field of Great Britain. The last few months have introduced us to a fresh competitor from the novelty-producing continent of America. The foremost exponent of the assessment plan has established itself in this country with a mission to revolutionize the whole system of life assurance in which we have been educated, and which we have continued to teach.

Based upon the assessment principle, which seems to have become more or less popular in the United States, the plan is to cultivate the *insurance*, as divorced from the *investment*, element in the system universally recognized here; to charge rates of contribution after claims have arisen in place of premiums collected in advance; to reduce the small reserve fund which is formed to the minor object of equalizing claims in various years, instead of forming a fund out of which claims are to be ultimately paid; and to collect rates of contribution from members by six instalments per annum.

With true missionary zeal the gauntlet is thrown down to the leviathan company of Great Britain, and we are informed that:

> "The theory that as a company grows older its death-rate "constantly increases is a fallacy abundantly disproved "by the experience of, among others, the largest and "wealthiest British life office in existence, whose death- "rate during its ninth septennial period, when its "losses were admittedly the heaviest it had ever "experienced during its long and prosperous career, "were under $1\frac{3}{8}$ per-cent per annum of its membership

> "in force. To provide for a theoretical death-rate, "*utterly impossible under the circumstances*, an average "annual level premium of over 3 per-cent had been "collected during the whole period."

It is convenient to ignore the fact that the death-rate has been kept down by the immense accretion of new business of recent years, and that instances are not of rare occurrence where the accumulated funds were successfully employed in paying, and would continue to pay, the liabilities as they accrued under contracts of offices which have ceased to transact new business at all, or are doing so at a diminished rate. It seems hardly probable that the assuring community here will prefer to give up the security which the present system provides, and to accept in place thereof the faith that the money remaining in the pockets of the members will be forthcoming at the critical moment. It seems to me that the perpetual calls of uncertain amounts during the course of a year will have a powerful tendency to increase the proportion of lapsed policies; and, by the nature of things, there can be no surrender-values. We have some experience, and that not altogether satisfactory, of the assessment system as in practice in a certain class of friendly societies; and America can furnish statistics, not always of success, in native institutions of the kind. The system of assessment assurance might form an interesting subject upon which a practical paper for this Institute might be founded; and I have some hope that this may shortly be undertaken by the accomplished hand of a leading member.

One noteworthy innovation in the practice of the company upon which I have been commenting is that of a heavy *initial* charge for expenses, reduced to a constant quantity per annum for *remainder of life*, and also the reduction of the rate of that initial expenditure as the amount proposed for assurance is increased. The adoption of the £100 sum assured as the fixed standard for measurement is here overthrown. Its almost sacred character has again suffered in the adoption, by two well-established companies, of fixed scales of premiums with variable sums assured as the basis of a system of *monthly assurances*, which has been introduced quite recently, with the view to reach that large section of the community between the artisan and the well-to-do middle classes, receiving fixed incomes by payments at short intervals. The success which has attended the weekly collection of premiums in the foremost of the industrial companies has promoted the effort to offer in these monthly schemes the benefits of life assurance at a smaller cost than could be secured in that organization. There is room for difference of opinion whether a fresh class of members will be gathered in, or whether it will tend merely to change the character of business that would probably, under the old *régime*, have been obtained.

The drawing closer of the ties between Great Britain and her colonies and dependencies seems likely to result, at no very distant date, in a still closer approximation to one rate of premium for the whole world—or rather, I should say, to the abolition of extra premiums. I well remember a former and a very far-seeing President of this Institute, Mr. Samuel Brown, arguing, many years ago, in favour of freedom from any such imposts, and representing that, as 6*d*. per-cent added to all the ordinary rates of premium would cover the extra risk incurred by members who had exceeded the usual free limits, it was not practically worth while (however theoretically correct) to perpetuate the grievance which all payers of extra premium feel, and which they do not hesitate to express sometimes in a forcible manner.

While referring to this point, it may be appropriate to call attention to the fact that at the recent quinquennial valuation and distribution of profits of the United Kingdom Temperance and General Provident Institution, a sum was retained from the divisible surplus, in consideration of which it was determined that no members of the institution should incur any penalty for exceeding what had previously been considered free limits—except, indeed, members of the military and naval professions in time of war. Another reason for my noticing the circumstance in this address is to call the attention of our members who may not have chanced to study the speech of the chairman, to the fact that the rate of extra premium which should be charged to persons who engaged in such active service, should be determined by the President, for the time being, of the Institute of Actuaries.

It seems probable that the Incorporation by Charter of the Institute of Actuaries may largely tend to increase its power and authority, as well as to extend the influence of the President, who is for the time being its official head. For, now that a condition of permanence, if I may so express it, has been given to the Institute by its Charter of Incorporation, we may assume, with every confidence, that many important references will be made to it on matters falling within the scope of its special subjects.

The liberal policy which has been adopted in recent years by so many companies, of freeing from restriction as to foreign travel or residence policies which have been in force five years, and of granting whole-world policies from the commencement without the charge of any extra premium where the proposer has little probability of exceeding free limits, appeared to me likely to make considerable reduction in the gross amount of extras collected by the assurance companies, and it seemed worth while to extract from the last Blue Book the particulars relating to those offices which had made valuation returns up to 31 December 1884. The result is noteworthy. Twenty-two companies, who have granted assurances for £74,264,790, are receiving extra premiums for foreign travel and residence, amounting in the aggregate to £26,035, which represents an extra charge of 8*d*. per-cent on the sum assured. Looking at it from another point of view, and making the fair assumption of an ordinary rate of premium of £3 per-cent, the above extra charges would amount to 1¼ per-cent of the total premiums only—a proportion insignificant as compared with the total loading.

There remains but one further topic to which it seems appropriate to call your attention before I close. It appears to be generally recognized that the recent fall in the rate of interest upon first-class securities (if it should not proceed any further) may be looked upon as of a permanent character; and, if that be so, it must be clear to all ordinary minds, that (1) seeing that life assurance profits depend in a material degree upon the rate of interest yielded by the assurance fund, a diminution of bonus will be the first and natural result; while (2) it must become necessary to pay attention to the reserves made in respect of policy liabilities with a view to their being strengthened where necessary.

It will be accepted as an indisputable fact that, *taken altogether,* valuations are now stronger and made on firmer bases, than was the case twenty years ago. This, in many instances, is due to the adoption of the Institute of Actuaries' Mortality Tables by the great majority of assurance institutions, replacing in most cases the familiar Carlisle and Northampton Tables; and it is a pleasant evidence of the value placed upon the great work of the Institute that three-fourths of the offices have accepted that basis, either in whole or in part, for their periodical valuations.

It may be interesting to some of our members to learn, that of nearly 400 millions of sums assured and bonuses granted by 79 offices, particulars of which have been extracted from the latest edition of the useful publication of Mr. Monilaws (*The Surplus Funds of Life Assurance Offices: How Arrived At and How Disposed Of*), 253 millions had been valued, alone or in combination, by the Institute Tables, 11 millions by the Northampton, 42 millions by the Carlisle, and 89 millions by special tables of construction unknown to the general profession—English Life, and other combinations. An outsider cannot readily estimate the relative strength of the reserves of the fifteen companies which comprehend this last class, but they number among them some of the most important institutions. The Northampton 3 per-cent Tables are now adopted by two historical offices only, and the Carlisle, which was for many years the leading and most popular table, has had in its turn to give place to its younger rival, the stronger table of the Institute of Actuaries—the former being no longer adopted as sole basis of valuation by more than seven companies.

The rate of interest at which the reserves have been calculated has the more important bearing on the question at issue, and a compilation having been made from the same returns (Monilaw's) —with the same omission as was made before—it appeared that 33 per-cent of the assurances had been valued at 3, 41 per-cent at 3½, and 26 per-cent only at the 4 per-cent rate, which is approximating somewhat closely to that being earned by the funds of assurance companies of the highest class. This 4 per-cent is, of course, mainly combined with the Institute Mortality Table; and Carlisle 4 per-cent is now almost unknown as a basis of valuation. It may be inferred from a consideration of these facts that life assurance institutions are fairly prepared to meet the fall which has taken place in the yield of capital, and that their credit as a whole will be fully maintained.

Gentlemen, while I thank you for your patient attention to my remarks this evening, I trust you will consider that I have neither presumed upon nor abused my position. It has been my desire to interest rather than to instruct; for it is a matter of certainty that on many points some of us seniors might readily be instructed by our recently-examined Associates. But it may not be inappropriate to offer a word of advice to these, my rising friends. Do not consider that your professional education is completed when you have surmounted the difficulties which intervene between the Studentship and the full rank of Fellow; use every opportunity of gaining experience; aid, whenever you are fortunate enough to obtain the chance, senior actuaries who may be engaged on heavy valuations and mortality investigations; contribute to the Institute or its *Journal* notes upon any problems which may have come before you in practice; above all, be careful to cultivate the courtesies of gentlemen, and success in your careers may be predicted. Looking back over a period of 40 years, I fancy I see a distinction in tone between ourselves at the present time and the leading spirits to whom we are indebted for the foundation of this Institute. We have more scholarship amongst us now, but the graceful courtesies of a Jellicoe, Hardy, or Ratray, are not so conspicuous. In their day professional communications were personal in their nature; in the present, we ask favours by lithographed circulars, and do not always find time to thank those who respond. If, too, I mistake not, an actuary of the old school took more active interest in the individuals of his staff than seems practicable in the present day—a heavy loss, indeed, to the younger men. Competition, I am told, is to blame for our shortcomings—if such they be; business is done at a higher pressure, and hours of duty are longer. It may be so; but, whatever may be the views as to the past of one whose professional career has now culminated, I have no fear as to the future. It is not to be believed that the Institute, which has, during the first stage of its existence, passed through, with honour and enhanced reputation, trials which might have overwhelmed weaker organizations, has not left traditions which will inspire the new generation of actuaries—who now virtually control its management. Many great spirits whose fame is enshrined in the history of the Institute have passed away, while others have retired from the active exercise of their profession to comparative and well-earned repose. Their examples it will be the ambition of their successors to emulate, and, starting from the higher status which has been obtained for them through our Charter, and with increased educational advantages, they will in their turn guide the aspirants of the present day in the paths of professional duty and honourable ambition.

On the American Tontine and Mutual Assessment Schemes. By HENRY WILLIAM MANLY, *Actuary of the Mutual Life Assurance Society, and one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute, 20 Dec. 1886.]

MR. T. B. SPRAGUE, in his opening address delivered on the 30th November 1885, when he was President of this Institute, treated, in his usually full and exhaustive manner, of the various theoretical doctrines current respecting Life Assurance. One of these, he said, "seems to me to proceed upon the idea that life "insurance is exclusively a contract of indemnity, the object of "which is to indemnify the family of the life assured against the "loss they will suffer if he should die prematurely, and be deprived "of the opportunity he might otherwise have had of saving money "and making a provision for those dependent upon him. Looking "at the matter more practically, it may be held that life insurance "is also a species of investment; in fact, that an ordinary life "insurance policy contains elements both of indemnity and of "investment. . . . . If an indemnity only were desired, life "insurance should logically cease at the age when the need for the "indemnity has ceast; that is to say, when a man's working years "are past, and he is no longer in a position to earn money by "following his trade or profession. In fact, the logical conclu-"sion is that all insurances should be term insurances expiring at "about the age of 65. If the argument is pusht still further, it "will follow that no man shall be called upon in any year to pay "a larger sum than is necessary to provide the indemnity for the "ensuing year, so that life insurance would resolve itself into a "series of short-term insurances, each for the term of one year, "the premium, of course, increasing with the increasing age of the "life. Such a plan is, I believe, actually in use in America, and "seems to me to be the natural result of giving undue prominence "to the indemnity element in life insurance, and neglecting the "investment element. This plan is only likely to be popular with "those persons who wish to pay at the outset the smallest possible "sum, and are satisfied that they can make better use of their "money than an insurance office is likely to do." Later on, he said, "America, which has already furnisht us with an instance of "a policy in which the indemnity idea has been carried to an

" extreme, furnishes us also with examples of the investment idea " being carried to an extreme, namely, in the tontine policies, of " which we have heard so much during recent years."

We are all of us more or less familiar with the American tontine policies, through the operations of the two large American offices, which have, for the past 18 or 19 years, been seeking business in this country; but until recently very little was known of the other class of insurance to which Mr. Sprague referred, and which, in America, is known as the mutual assessment scheme. The invasion of one of these societies has, however, caused no little excitement, especially among the insurance journals; and it has occurred to me that it would be far better to analyze the scheme in a calm, dispassionate manner, rather than allow the general impression to become current that the leaders of the insurance profession have no argument to oppose to it but abuse—an argument, by the bye, which never hurt anyone.

It had been my intention, when I commenced this paper, to limit myself to a dissertation on the latter scheme; but the principles upon which it is founded, and the regulations governing the conduct of its business, are so completely opposed to those of the tontine scheme, that I subsequently found the remarks I might venture to place before you would be much enhanced by setting the two extremes against each other. Moreover, the members of this Institute, as a body, have never taken the opportunity of considering and discussing the peculiar features of the tontine policies; and I therefore finally decided to include both schemes in this paper.

I shall commence with the oldest, namely:

THE TONTINE SCHEME.

The term "tontine" derives its name from a person named "Tonti", who, about the middle of the 17th century, devised a plan by which a fund was to be raised by subscriptions, the interest on which was to be divided each year among the surviving subscribers, the last survivor taking the whole of the fund. Innumerable variations of this scheme have from time to time been applied to financial, commercial, and benevolent undertakings; but the temptations to fraud, and, even worse, to murder, grew so great, that the various governments had to interfere and suppress by law the use of it in any form. The term "tontine" has been applied to one or two methods of dividing profits in life assurance companies where the largest share of the profits is given to the oldest policyholders; but the device of combining a modification of the original plan with simple life assurance, originated apparently with the Equitable Life Insurance Company of New York, who introduced it about the year 1860. By the scheme then propounded, separate classes were to be formed each year of those who effected policies on the ordinary with-profit tables and subscribed for ten, fifteen or twenty-year tontine policies respectively. A separate fund was to be kept for each class, to consist of the contributions and accumulations of the subscribers to that class; and it was provided that if the assured should die before the end of the tontine period, then his executors were to receive the sum assured by his policy and no more; but at the end of the term the whole of the then existing funds of each particular class were to be divided equitably amongst the survivors of the original subscribers who had successfully maintained their policies in force. This, on the face of it, was too glaringly a lottery scheme to pass unchallenged; and therefore a plan was devised of dividing the fund at the end of the tontine period into two parts; namely, (i) the actual reserve values of the policies, which could be withdrawn by the members remaining in the class; and (ii) the surplus, which was to be divided equitably amongst the same members. This enabled the company to offer to each of the fortunate surviving members a continuance of his insurance as an ordinary profit policy, receiving annual dividends, at the annual premium originally charged to him, if he left the reserve value of his policy in the hands of the company; and to give him his share of the surplus in cash, which he could re-invest with the company either for a reversionary addition to his policy, *if he should then be in good health*, or in reduction of the future premiums. If the subscriber did not wish to keep up the original insurance, he might, with his full share of the fund, comprising both reserve value and surplus, purchase a paid-up policy (with a proviso that the sum assured thereby should not exceed the original sum if he be not in good health), or a life annuity.

The scheme, thus stripped of all the verbiage with which it has been surrounded and obscured, bears a remarkable resemblance to the original plan of Tonti; and in starting this form of policy a direct appeal was made to the gambling instincts of the human race. A fund was to be formed by the annual subscriptions of those who joined a certain class, and at the end of a fixed term the survivors alone, who had rigidly conformed to every regulation, were to share in its distribution, the only exception being that if a subscriber who had abided by the regulations happened to die during the period, then his relatives should receive the insurance provided in his policy. The insurance thus provided was, therefore, a temporary one for the tontine period, and such as could be secured by half, or less than half, the subscriptions paid; and the fund would, consequently, under ordinary conditions, consist simply of the subscriptions, less the expenses and the cost of a temporary assurance, accumulated during the term at the rate of interest actually realized on the investments. Such a fund, however, if divided amongst the whole of the survivors, would not present very grand results, especially if the expenses amounted to 20 or 25 per-cent of the subscriptions; and so the policies were made to carry the most arbitrary and onerous conditions, having the effect of limiting the numbers who should share in the final distribution. Thus, if the premium be not paid on or before the very day it is due, then the insurance and all the premiums paid shall be forfeited*; and from this arbitrary rule there is no redress; for if a man, through unforeseen circumstances, happen to be even one day over the time, he forfeits everything. It is very easy to conceive that a man might be called away suddenly on urgent business, perhaps to a death-bed, and forget his premium in the excitement, or go for a holiday and omit to provide for the due payment of his premium; and in either case all benefit would be lost. Again, all companies transacting real insurance business recognize the fact that the ordinary annual payments are in excess of the value of the current risk, and therefore give the assured, who is unable or unwilling to keep up his payments, a surrender-value on his retirement, or will advance him a loan to the extent of the surrender-value, to enable him to keep his policy in force; but it was a natural feature of the original tontine scheme, that no surrender-value was given, and no assistance rendered to enable the subscriber to meet his payments. The only consolation, therefore, that could be offered to a member who failed in his payments was, that he had paid a very heavy premium for a temporary assurance. Other conditions and regulations of a most arbitrary and onerous character respecting the future life of the individual—his occupation, movements, habits, and even the nature of his death—were introduced into the original policies, the like of which were never heard of in this country, and an attempt was even made to prevent the assignment of the policies, the obvious tendency of which was to increase the funds and limit the number who should participate in the final distribution. The policy was made absolutely valueless either as a saleable commodity or as a security for an advance; for no sensible person would ever accept such a document with no surrender-value attaching to it, or which might be vitiated at any moment by the act or habits of the life assured; and as not even a premium could be borrowed on it from anyone, unless perhaps the life happened to be on his death-bed, the assured, if he had not the means himself of keeping his policy in force, had to forfeit all his contributions to the funds.

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* In order to transact business in this country the companies found it necessary to allow the usual thirty days' grace; but one of them has a provision that if advantage is taken of the days of grace "a fine at the rate of 10 per-cent per annum shall be paid to the company for the time deferred."

The growing intelligence of the public in this country with respect to the real nature of tontine policies, has induced the companies to withdraw most of the objectionable conditions in future policies, and they have gone so far as to make the policies "incontestable", with certain reasonable reservations as to residence during the first year or two. They even promise to give, as a "surrender-value", after three years, *a right to a paid-up policy,* if applied for within six months from the date of lapse. This last feature is put forth as constituting the policy a "non-forfeiting tontine" one, when no assistance whatever, by way of loan or otherwise, is given to prevent the forfeiture, and when every benefit connected with the tontine fund is forfeited!

From what I have said, it is evident that the character of the tontine policy has changed greatly from the original design; and even the peculiar feature of separate funds being kept, of the contributions of the subscribers to each class formed every year, has long since vanished. The funds are inextricably mixed; and the mode of dealing with them is wrapt up in great mystery. One office does venture upon some kind of explanation in its returns to the Board of Trade, but it takes refuge for its action in the statement that "the charter of the society provides only that an equitable share of the surplus found at an investigation shall be apportioned to each policy. The mode of apportionment is left to the discretion of the directors." The other office vouchsafes no information whatever.

I do not intend to follow the tontine scheme through all its recent changes, but I may be allowed to express the opinion that a modern tontine policy is nothing more than an ordinary kind of assurance, having deferred bonuses accruing at the end of

10, 15 or 20 years, as the case may be; with the advantage of a little extra surrender-value if the policyholder wishes to discontinue his assurance *at the end of the tontine period,* and the disadvantage that all benefits will be forfeited (except a right to a paid-up policy) if the payments are in arrear a single day, coupled with the absence of any help whatever being given to keep the original tontine policy in force.

It would not be fitting for me, in this place, to criticize the management and operations of the tontine-granting offices; but I do not like to leave the subject without giving some figures of the business transacted in this class of policy. At the outset, I must say, that I have found it practically impossible to obtain any satisfactory statistics from official records. That the two offices (for the business is practically confined to two) which I will call A and B, transact an enormous business is undoubted. They are "big" concerns, in the American sense of the word "big." From the New York Insurance Reports I extract the following figures relating to all classes of policies:

	A		B	
	No.	Amount	No.	Amount
		£		£
Policies in force, commencement of year 1885	78,047	45,876,517*	83,699	61,881,834*
New policies issued during the year	18,496	13,106,303†	23,255	18,232,009†
Old policies revived during the year	70	55,237	713	628,498
Old policies increased in amount	...	10,951	...	...
Additions by dividends during the year	...	531,800	...	341,769
	96,613	59,580,808	107,667	81,084,110
Deduct policies decreased and ceased to be in force . .	10,195	7,645,908	11,795	9,616,461
Total policies in force at end of year	86,418	51,934,900	95,872	71,467,649
Total Assets 31 Dec. 1885 . .	...	14,388,998	...	13,109,519

* I have calculated the dollar as worth 4*s*.

† Including not taken up, 2,760 for £2,517,123, and 3,000 for £3,127,373, respectively.

Business in New York State during 1885.

	A		B	
	No.	Amount	No.	Amount
		£		£
Policies in force 31 Dec. 1884 .	10,409	6,796,249	17,257	13,506,839
Policies issued during 1885 .	2,005	1,565,476	2,672	2,989,603
	12,414	8,361,725	19,929	16,496,442
Deduct policies ceased to be in force during 1885 . . .	1,260	985,854	1,525	1,556,634
Policies in force in New York, 31 Dec. 1885	11,154	7,375,871	18,404	14,939,808

These are certainly gigantic figures; but it must be borne in mind that the business is drawn from the whole world, and that seven-eighths of it is obtained outside the State of New York. How much of the business is done on the tontine scheme, and where the tontine business principally comes from, I have no means of ascertaining. It would be exceedingly interesting to have an analysis of that business, showing how much comes from each country where the offices have agencies or carry on their operations. On referring to the returns to the Board of Trade in this country, I find that only one office (B) gives particulars of its tontine policies; and as the only means of comparison, for the purpose in hand, afforded by the New York Insurance Reports, are the total sums assured and the surplus belonging to the two different classes of policyholders, I extract from the Board of Trade Returns the only available figures for making an estimate:

Date	TOTAL POLICIES		TONTINE POLICIES		Total Surplus	Surplus belonging to Tontine Policyholders
	No.	Amount	No.	Amount		
		£		£	£	£
31 Dec. 1880	52,209	35,519,541	16,683	13,279,059	1,825,658	856,646
31 Dec. 1883	74,980	57,298,593	32,205	29,446,184	2,502,032	1,185,256

These figures do not offer much assistance towards the determination of the ratio of tontine policies to the total number of all classes; but we shall probably be erring on the safe side if we assume that it is in the ratio of the tontine surplus to the total surplus. And now, turning back to the New York Insurance report, I find the following statistics relating to the business in force as at 31 December 1885;

Office	Total Surplus	Tontine Surplus	Total Assurances	
			No.	Amount
	£	£		£
A	2,643,009	624,748	86,418	51,934,900
B	3,294,636	1,926,600	95,872	71,467,649

and from these facts I estimate that the total tontine policies in existence throughout the world on 31 December 1885 numbered about 76,500, for a total sum assured of £54,000,000. These again are large figures, but when we consider that the representatives of these companies are to be found in the remotest corners of the earth, and that the expenses of management are over 20 per-cent of their enormous premium income, I do not think that the result is so marvellous after all. It would probably be a very liberal estimate to assume that one-tenth of the total tontine business has been obtained in this country; and if we also assume, as we are fairly entitled to do, from the total records, that one-fifth is new business, we arrive at the conclusion that 1,530 of these policies assuring £1,080,000 are annually issued here, and that half that number, or about 765 policies for £540,000 annually cease.

In concluding this part of my subject, allow me to say that I do not believe we are the effete race that some of our American cousins would make us out to be. I believe there is enough public spirit, and enough energy left in us, to appropriate and adopt any scheme which is intrinsically good, and for which there is a natural demand; but it is because the actuaries in this country, from the first, appreciated the defects of the scheme; because we have applied our energies to improving the benefits to the assured, rather than harking back to the dark ages of insurance; and because we felt sure that the tontine scheme in its original form was a plant of a forced growth, raised and nurtured on extravagant anticipations, that no British office has ever adopted it. The offices in the United States seem to be of very much the same opinion, for, notwithstanding the boast that "This plan has been imitated far and wide by those companies who have enterprize", I find from the "Report of the Special Committee of the Legislature of "the State of New York appointed to investigate Tontine

"Insurance," that only six other offices in the whole Union endeavoured to obtain business on that plan; and on searching the New York Insurance Reports I find that altogether those offices have a surplus fund belonging to the tontine policyholders of only £11,569. The two offices which obtain that class of business here have, therefore, as I have before intimated, practically the monopoly of the whole world.

I am inclined to think that the craze for tontine policies is dying out in America; and my principal reason for thinking so is that as one extreme always in time produces a revulsion to the opposite extreme, so we find that in America the pendulum of popular craze has already swung from tontine policies offering a little insurance with a large amount of speculative investment, to mutual assessment certificates which offer all insurance with no investment.

MUTUAL ASSESSMENT SCHEME.

Just as the tontine scheme was founded on a very old financial plan for raising money; so, likewise, is the mutual assessment scheme founded upon an old but benevolent plan for alleviating the sorrows and sufferings of humanity. The practice to which I refer is the one vulgarly, but expressively, known as that of "passing the hat round", for the purpose of making a collection for a family that has fallen into distress through the death of the husband and father. I do not think I shall err in stating that that practice dates from the earliest antiquity. The subscriptions being voluntary, each man would subscribe according to his means and inclination; but I have no doubt it was very early reduced to a system; and where small bodies of men were bound together by a common employment, or mutual interests or affection, a custom grew up by which each man bound himself to pay a certain sum on the death of a fellow workman, companion or friend, towards defraying the expenses of his burial and providing a small sum for his family. This arrangement is common in many works where a number of men are employed, and goes by the name, I believe, of the "yard fund": being a fund collected in the yard of the works. When the ages of the men employed in these works are limited according to the time that they are able to do rough work, the contributions remain fairly steady, and there is no excessive grumbling on the part of the young men; but where the work is light and can be performed by youths and old men, the plan seldom answers.

Numerous attempts have been made, from time to time, to apply the system to a general body of subscribers, the plan being known as the "levy system", but they have all failed after a few years, by reason that as the members grew old the calls became more frequent; and, there being no common interest to bind the members together, the young men fell out and the old men found no benefit in subscribing for each other; and so the society, like a rope of sand, having no cohesion, fell to pieces. The ingenuity of the Americans has, however, led them to the discovery of a method of converting the crude ore into something which has the appearance of real metal, but whether the product has any intrinsic value or lasting qualities time, the great arbiter of all things, will certainly prove. It will be seen, however, that it will not bear the test which we should naturally apply in such a case.

It is quite evident that, short of a strong mutual interest or affection, there is no inducement for a young man to subscribe at a common rate to a fund from which the families of his elders draw the largest benefits. This difficulty is surmounted in the American assessment societies by making each member subscribe, at periodic intervals, towards the payment of the accrued death claims, according to his age at the date of the levy; the ratio of his subscription being in proportion to the probability of dying (q_x) according to the mortality table selected, which appears, in most cases, to be the American Experience Table (Homans, 1868). The expenses of management are provided, or supposed to be provided, by a fixed annual or semi-annual levy in proportion to the nominal sums assured. By this plan each man would, if the deaths occurred according to the mortality table, be practically subscribing for the deaths happening amongst the members of his own age at the date of levy. In fact, he would be subscribing for a term policy for one year, renewable at the end of each year for another term of twelve months, at a premium, increasing each year according to his increased age, and payable by instalments at short intervals, generally every two months. By this arrangement another difficulty is surmounted, namely, the question of amount which the family of a deceased member is to receive. In the old yard fund the subscription is limited, and therefore the amount to be received is limited; or it may be, as in the levying Societies, that the amount to be received is fixed, and the subscriptions variable; but by the new scheme a man may subscribe for any amount he chooses, and, so long as his subscription is sufficient to pay for the current risk and his share of the current expenses, no injury is done to the other members. That is the system upon which the Mutual Assessment Society pure and simple is based; and the working of it will be best illustrated by the figures of the largest of the class in the United States, namely, the Knights of Honor, Supreme Lodge, which, from its title, I presume is a kind of large affiliated friendly society, similar in composition to our Odd Fellows and Foresters, although constituted for a very different object.

The ledger assets at the commencement of the year 1885 were	£18,772
INCOME DURING 1885.	
Semi-annual dues	6,424
Assessments	599,812
Other receipts	4,340
	£629,348
DISBURSEMENTS DURING 1885.	
Losses and claims (including assessments returned £161)	£615,961
Expenses	12,427
Balance 31 December 1885	960
	£629,348

This does not exhibit a very flourishing state of affairs; and as no capital is necessary to start a concern of this kind, it is not surprising to learn that these companies spring up like mushrooms, and die as quickly. Short of a common interest (which I suppose does exist in some form in the society I have selected), there is nothing whatever to induce a healthy man to keep up his subscriptions after he has got tired of paying assessments on the deaths of others, and the first flush of excitement at the thought of providing a fancied benefit for his family at a cheap rate has passed away. Innumerable occasions would arise when he would find it inconvenient to pay, and he would drop out, intending perhaps to enter again at some more convenient time.

It would appear self-evident, that without creating some future benefit for "persistent" members, and also levying a kind of tax on new entrants, there is no chance whatever of a society of this kind having more than a fleeting existence. It occurred, therefore, to one or two of the cleverer men connected with these societies, that a future benefit could be provided by increasing the "mortuary calls", as they term the periodic assessments, by a fixed percentage (generally 33⅓ per-cent) of the amount required to meet the accrued claims, so as to afford the means of creating a fund to reduce the contributions of "persistent" members after a certain number of years, and to compel "healthy outgoing members to leave some money behind to protect the company against adverse selection"; and that to obviate the temptation to a subscriber to cease his payments for a time and join again at a future date, a tax should be levied in the shape of an entrance fee and the payment of the fee for medical examination. To make certain that the applicant does pay the medical fee, the doctor, in his report, has to make a declaration that his fee has been paid.

This modification of the assessment scheme is known in America as "The Harper Plan", and appears to have been put into practice about the beginning of the year 1881. It had the effect of giving a greatly increased vitality to the assessment principle, and new societies based upon this plan rapidly sprang up all over the United States.

Having thus far analyzed the principles upon which the latest development of mutual assessment schemes is based, I propose to direct your attention to the method by which it is carried into practical effect; and in doing so I shall quote from the documents and literature of the largest society of this class. I am unfortunately compelled to confine my observations to the publications of one society, because I do not possess a complete set of any other; but the one I have selected may, I think, be safely accepted as a type of all. The document which affords the most accurate information is the policy, or "Certificate of Membership" as it is termed. This document recites, that in consideration of the application form containing full and true statements, of the payment of the admission fee, and of the dues for expenses, "and of the further "payment of all mortuary assessments, payable at the home office "of the association in the city of New York, within thirty days "from the first week-day of the months of February, April, June, "August, October and December, of each and every year during "the continuance of this certificate (or from such other periods as "the board of Directors may from time to time determine), and "within thirty days from the day of the date that each assessment "is ordered", the association receive A. B. as a member. Within thirty days after satisfactory proof of death of the member during the continuance of the certificate, subject to the conditions that follow, there shall be payable to C. D., if living at the time of said death, otherwise to the legal representatives of the said member, the sum of ———— "from the DEATH FUND of the association, at "the time of the said death, or from any moneys that shall be "realized to the said FUND, from the *next* assessment to be made "as hereinafter set forth, and no claim shall be otherwise due or "payable, except from the reserve fund as hereinafter provided."

The following is an abstract of the conditions:

I. When the death fund is insufficient to meet existing claims, an assessment shall be made on every member at such rate, according to the age of each member, as may be established by the board, and the net amount received (less 25 per-cent to be set apart for the reserve fund) shall go into the death fund.

II. The net earnings, together with the 25 per-cent of the assessments, shall constitute a "Reserve Fund," which shall be deposited with a trust company for the exclusive benefit of the members of the association. The reserve fund, above $100,000 (£20,000), and in excess of sums represented by outstanding bonds, may be applied to the payment of claims in excess of the American experience of mortality, and to making up any deficiency in the death fund after an assessment has been made. The interest on the reserve fund is to be placed to the credit of the death fund (not the reserve fund).

III. After the expiration of each period of five years during the continuance of the certificate, a bond shall be issued for an equitable proportion of the reserve fund, and the *principal* shall be available ten years afterwards towards paying future dues and assessments, but if membership cease from any cause, the bond shall be null and void, and the principal applied to increase the Bonds at the next quinquennial apportionment of members ***holding certificates issued during the same year as this certificate.***

IV. No agent has authority to alter contracts.

V. The contract is not binding till the certificate has been delivered to the member, and is subject to the conditions contained in the constitution and bye-laws of the association and the amendments that may be made thereto.

VI. An assessment note addressed to a member at his last address in the books shall be sufficient.

VII. Notices of assignment must be delivered to the head office.

VIII. Proofs of death shall include full and true answers, ***under oath,*** to all questions relating to the life, health, and death of the member.

IX. The contract does not include death of member by his own hand, but a sum equal to the assessments paid by him with 6 per-cent interest may be paid by the board.

X. The entire contract is subject to the constitution, bye-laws, and regulations of the association, and the laws of New York. No proceedings shall be brought after lapse of one year from date of death.

XI. If payments stipulated shall not be paid, or if member remain more than twenty-four hours in any place where authorities have declared yellow fever to be epidemic, or shall engage in blasting, mining, submarine, or aeronautical operations, work or manage a steam engine, &c., &c., or if any statement in application be untrue, then certificate shall be null and void.

To these must be added a condition which is to be found in the prospectus:

A member who lapses his certificate may re-instate the same at any time within one year for good cause shown, and upon satisfactory evidence of good health, and upon payment of all delinquent dues and assessments—(whatever delinquent dues may mean).

Printed in red ink on the top of the policy is the following announcement:

After five years from the date of this certificate it is incontestable for any cause, except non-payment of dues or mortuary assessment at the times and place, and in the manner herein provided—the age of the member being correctly stated in the application for this certificate.

The following "Table of Rates" is endorsed on the policy:

TABLE OF RATES.

ADMISSION FEE.

$1,000 ... $8.00 $2,000 ... $12.00 $3,000 ... $15.00 $5,000 ... $20.00
$10,000 ... $30.00 $15,000 ... $50.00 $20,000 ... $70.00.

DUES.

The dues for expenses are $2.00 on each $1,000, payable annually in advance.*

ASSESSMENT RATE TABLE.

No assessments will be made while there remains in the death fund a sum sufficient to pay the existing claims in full.

The basis of the assessment rate for each member, according to the age taken at the nearest birthday, on each 1,000, is as follows:—

Age	Rate	Age	Rate	Age	Rate
15 to 25	1.00	39	1.40	53	2.75
26	1.02	40	1.44	54	3.00
27	1.04	41	1.48	55	3.25
28	1.06	42	1.52	56	3.50
29	1.08	43	1.56	57	3.75
30	1.10	44	1.60	58	4.00
31	1.12	45	1.64	59	4.25
32	1.14	46	1.68	60	4.50
33	1.16	47	1.72	61	5.00
34	1.20	48	1.76	62	5.50
35	1.24	49	1.80	63	6.00
36	1.28	50	2.00	64	6.50
37	1.32	51	2.25	65	7.00
38	1.36	52	2.50		

* I have recently learnt that the annual dues have been increased 100 per cent.

I cannot quite make out how the above assessment rates are calculated. The MAXIMUM SUM, according to the prospectus, which may be collected from a member for death losses in any one year, before the reserve fund can be used, is the rate of mortality per-cent according to the American experience tables with the addition of 33⅓ per-cent, which gives the following amounts for every 1,000 insured; age 25 10.76, age 35 11.93, age 45 14.80, age 55 24.76, age 65 53.51, from which it would appear that the company can make a greater number of calls on the young men than upon the old;—for a man under 30 could be called upon 10 times in the year, while a man aged 55 could only be called upon 8 times.

Here let us pause a little while to consider the nature of the contract and the objects aimed at in the regulations. A man is to pay a current premium for the current risk and subscribe to the current expenses; and to prevent the premium growing too high when the risk becomes heavy, he is made to subscribe to a fund raised for the object of reducing his assessments at some distant date. It is all insurance and no investment; and is in perfect contrast to the original tontine scheme in which the idea was to give a little insurance with the promise of a large investment. A great feature of the new scheme is that the member is not called upon to pay his whole premium for a year's risk in advance, but to pay it by instalments at the end of every two months, according to the actual losses which have to be provided for. This gives an appearance of extreme cheapness, and has a charm for the needy which it is difficult to withstand. The frequency of the calls may not be felt in the early days, especially when, owing to recent selection, the claims are light; but after a time they must necessarily become irksome and annoying. The society I refer to has only been in existence about five years, and as the new entrants have increased very rapidly, the average age of a policy cannot exceed 1½ years, and during that time the company, it is stated, has made only 23 calls, aggregating 24½ single assessments, against 30, which could have been made. That is a result which might have been expected by any one acquainted with the effect of selection, but it proves, *according to the prospectus*, that persons who join assessment companies live longer than those who join level premium companies! It is, however, essential to the life of a scheme of this kind that the payments for claims should be as few as possible, and yet that every inducement should be given to the members to pay their assessments while they live; and it will be seen that just as the conditions inserted in the tontine policies had a tendency to increase the fund by forfeitures and limit the members who should participate in the final distribution, so here the regulations will tend to a limitation of the death claims. The certificate requires all mortuary assessments to be paid at the home office in New York—(I suppose when they establish themselves here, the companies will have some place in this country where they may be paid)—within thirty days from the first weekday in every other month in the year, or from such other days as the board may from time to time determine; and an assessment note addressed to a member at his last address in the books shall be sufficient. A member has therefore at least six chances of forfeiting his certificate during the year, and very serious chances too, because as he does not know beforehand what he will have to pay he cannot remit in anticipation of the call, nor can he always make suitable provision for its payment. Illness, removal, absence from home, miscarriage of the post, may all cause a failure of the payment of a call; and although the company will be glad to receive the over-due payments within a year if the member is in good health, the company will not pay the assurance if the member dies after the days of grace allowed, or permit him to revive his membership if in bad health. It is also provided that proof of death must include answers *under oath* to all questions relating to the life, health and death of the member; a condition which might be serious, considering that the applicant has to answer about 150 questions; but against that must be set the provision that after five years the policy is "incontestable."

That you may not consider the risk I have described an imaginary one, I reproduce a cutting from a New York journal called *Insurance*, stated to be the result of an examination of the claims of one of these companies, conducted under the direction of the Superintendent of Insurance of the State of New York:

> "In June last there were outstanding against the association 91 death claims, amounting to £89,400. Of these only 43, amounting to £37,400, had then been recognized as valid; 23 claims for £21,200 were classed as 'undergoing investigation and not yet approved'; 5 claims for £4,500 were awaiting proofs; 20 claims, aggregating £21,700, were resisted; 12 on the ground of fraud and 8 because of lapse."

The question of lapse when a claim occurs, must always, from the nature of the business, be a serious one, and be continually arising.

Another feature which demands notice, is the method of providing for the expenses. According to the "certificate", and the prospectus, we are led to believe that the initial cost is provided out of the admission fees, and the other expenses out of the annual dues. The prospectus distinctly states:

> "The executive officers are required by the constitution, at the expiration of every sixty days * * * to call upon our living members for a sum applicable to the death fund *equal to the approved death claims*";

and: "The expenses of management are limited, as follows", reciting the figures which I have given above as endorsed on the policy.

Taking the average policy as £600, and the average age 30, the admission fee would be 22 per-cent of the average annual mortuary assessment; and the annual dues of 2 per 1,000, payable in advance, would be 18 per-cent. This would seem to be a liberal estimate, seeing that the member has to pay for his own medical examination, that the agent gets a single payment for the introduction (which is generally the whole of the admission fee and no more), and that there is no expense for collection beyond postage, as the calls have to be remitted to the head office direct. If we dissect the accounts, however, we shall find that the expenses are not always limited to these sources of revenue.

Mortuary Account for the year 1885.

Receipts.		Disbursements.	
Assessments	£240,914	Death Claims	£167,735
Interest	1,391	Carried to Reserve Fund	60,229
		Surplus	14,341
	£242,305		£242,305

Expenses Account for the year 1885.

Receipts.		Disbursements.	
Admission fees received at Home Office	£563	Dues returned to Members (upon refusal)	£41
Admission fees returned by Agents (estimated)	24,000	Admission fees returned by Agents (per contra)	24,000
Annual Dues	43,399	Commissions to Agents	6,929
Medical Examiners' fees paid by Members (estimated)	5,459	Medical Examinations paid by Home Office	503
Deficiency	9,961	Medical fees paid by Members direct (per contra)	5,459
		Cost of levying and collecting the Assessments	9,586
		Cost of investigating, auditing, and settlement of death claims	5,342
		General Expenses of Management	30,797
		Furniture and Fixtures	725
	£83,382		£83,382

I cannot proceed further with the analysis of the accounts, as the society has made up the reserve fund to a date different to what they have their general fund; but, to make the statement complete, I must add the following details:

ASSETS.	
Invested Assets	£127,976
Other Assets, consisting of interest due and accrued, and Assessments and Annual Dues *called but not paid*	87,655
	£215,631
The Reserve Fund on 21 January 1886 was . .	£111,640

The business done in 1885 is thus set forth:

	No.	Amount
Certificates in force 31 Dec. 1884. .	20,779	£17,090,400
Certificates written during 1885 . .	13,748	10,200,300
Total	34,527	27,290,700
Certificates ceased during 1885 . .	3,263	2,620,000
Total in force 31 Dec. 1885 .	31,264	£24,670,700

I have inserted the above figures more with the view of showing the exact nature of the business than of trespassing out of my legitimate path by offering any criticisms on them; but I hope I may be pardoned if I show how the expenses are to be measured when comparisons are made with other companies. The usual test of expenditure, when applied to ordinary companies, is to compare the total annual expenditure with the total office premium income; and to arrive at the figures in an assessment society which would correspond to the office premiums in a level premium company, the assessments, admission fees, annual dues and medical fees, must be added together. Applying the proper test to the above accounts, it will be found that the expenditure amounts to 26·6 per-cent of the total subscriptions; and that, even if we omit the initial expenditure from both sides (that is the admission fees retained by the agents and the medical fees paid by the members), the ratio amounts to 18·7 per-cent. As the annual dues only admit of an average expenditure of 15·4 per-cent of the gross collections, omitting the admission and medical fees, we find, as a consequence, that the expenses account shows a deficiency of £9,961. To cover this deficiency the management claim to deduct £9,586 from the mortuary assessments "for the cost of levying and collecting the same", so that they tacitly admit that they have already assessed the members for more than the amount required to pay the death claims, and raise the reserve fund. The existence of such a power would admit of any amount of extravagance; and its exercise would, if not checked, possibly lead to heavy demands on the members in future, and large drafts on the reserve fund.

The constitution and application of the reserve fund is peculiar. It is, in the first place, to consist of 25 per-cent of the assessments, and the *net earnings*; but if the assessments are limited to the death claims, I do not see where the net earnings are to come from, unless it be the excess of the annual dues over expenses, which may be, as appears above, a *minus* quantity. The interest of the reserve fund is to be placed to the credit of the death fund, so that the reserve fund is not to accumulate at interest; and the whole fund in excess of £20,000, and the sums represented by outstanding bonds, may be applied to the payment of claims in excess of the American experience of mortality. There is a possibility, therefore, of the reserve fund being greatly reduced, if the mortality experience eventually proves similar to that of preceding companies of a like nature. As to the application of the fund, very little information of a definite character is given. All that we are informed is, that an apportionment of the reserve fund is to be made every year, and that bonds are to be issued for an equitable proportion of the fund each year to those members only whose certificates have been in force for exactly 5, 10, 15, &c., years, and that the principal of the bonds is to be available 10 years afterwards towards paying future dues and assessments. The method of distribution, as above foreshadowed, was not, however, sufficiently complicated for the promoters; for, according to the certificate, "should membership hereunder cease from any cause, said bond shall at once become null and void, and any portion of said principal not then used shall be applied to increase the bonds issued at the next quinquennial apportionment *to other members of the association holding certificates issued during the same year as this certificate*"; so that a separate fund of lapsed bonds must be kept for every year of the existence of the company.

The prospectus states that "by the issuing of bonds every five years, a constant application of the reserve fund tends to diminish, and finally to extinguish, the payment of premiums"; but how! remains a dead secret. From the interesting table which I have appended to this paper, it will be seen that, assuming the reserve fund is accumulated at 4½ per-cent interest, and the limit of increase in the annual assessment is reached in 17 years, a person entering at the age of 40 would, after the 17 years, have to pay a level premium 30 per-cent higher than the ordinary level premium; and if that is the case where the fund is accumulated at 4½ per-cent compound interest, what will be the result where no interest whatever is credited to the fund? I have made an estimate which will be shown later on in a diagram.

Having reviewed the principles and practice of these companies, let us now consider what essential difference there is between the payments of a person who joins an assessment company, such as I have described, and of one who joins a level premium company: and in making this comparison, let us be careful to compare things of the same kind. In the first place, the contract on the part of the mutual assessment company is to pay a definite sum and no more, and in that respect it corresponds to a non-profit policy of the level premium companies. In the second place the premium charged by the level premium companies combines the cost of the risk for the whole of life, and the contribution towards the expenses. In comparing, therefore, the assessment rates (exclusive of the dues for expenses) with the level premiums, we must throw off the loading (which is the contribution towards expenses), and make the comparison with the net or prime-cost portion of the premiums. Next, as this scheme is brought forward as a rival to the "old-line" companies, and as a new and improved method of providing a sum for the benefit of a member's family at his death, and not as a temporary expedient, to cover the possibility of death during a short period, we must compare the cost up to the extreme limit of life, and not stop at the age when the "natural premiums" or mortuary assessments begin to grow heavy. The misleading nature of the prospectus of only giving the cost up to age 60 is at once exposed by exhibiting the fact that, by the table used (the American experience) out of 1,000 persons living at age 20, 625 will be alive at age 60; out of 1,000 living at age 30, 678 will be alive at 60; and out of 1,000 living at 40, 741 will be alive at 60. The pure or mathematical annual premiums charged by the level premium companies are simply the present value of all the mortuary payments that would be made to a mutual assessment company pure and simple, distributed into equal annual payments for the whole of life. The small additional payment which the policyholder in an ordinary assurance company makes during the first few years of his insurance is sufficient, when accumulated at compound interest, to reduce to a low level the heavy payments which he would otherwise have to make during the latter period of his existence. To illustrate this, I have made a diagram—following p. 208—showing in a graphic form the rates of premium for 100 of assurance which would have to be paid according to the different methods. The straight black line is the level premium payable according to Meech's table of mortality and 4½ per-cent interest, —the data adopted by Mr. Fouse in the tables appended to this paper. [I use Mr. Fouse's tables because they are ready to hand; and in order to compare things of a like kind, it is necessary to adopt the same data throughout.] The dotted line represents the "natural premiums" ($100q_x$) according to the American table of mortality. The line formed of strokes and stars represents the same premiums increased 33⅓ per-cent; and the broken line is a representation of Mr. Fouse's table No. 4 in the appendix, after travelling along the line of strokes and stars to the point where it breaks off, and shows the effect when the reserve fund, formed of 33⅓ per-cent of the natural premiums, and the profit from lapses, is accumulated at 4½ per-cent compound interest, and applied to reduce the assessments to a dead level after 17 years. When, however, the interest on the reserve fund is applied to reduce the assessments of the whole body of members, new and old, or, worse still, is swallowed up by additional expenditure, the broken line will approach towards the line of strokes and stars; and the thin continuous line shows the estimated scale of assessments based upon the application of the reserve fund, without interest, to the reduction of the payments after 15 years, as far as I can make out the description in the certificate and prospectus of the Harper plan. I cannot imagine that any man who really wanted to provide a sum at his death, and having such a diagram placed before him, would prefer to start from the slightly lower ground where the line represented by the strokes and stars starts from, and climb such a steep hill as there represented, even if he did get upon some table land after a time, to starting at the commencement of the straight black line and keeping on the level for the whole of his journey. I fancy when the hill began to grow steep that most of the healthy members would drop out, and only the diseased and decrepit would remain; and the natural result would be, that the death rate would rise above the average, that the reserve fund would have heavy calls made upon it, that the younger members would find no advantage in contributing to the excessive mortality with no prospect of a fund to pay their own death-claims, and that then would follow the final collapse. The worst and most lamentable part of the feature is, that that time may not be reached for 20 or even 30 years.

That these companies have been obtaining a large amount of support in America is a fact which cannot be denied. The following figures are taken from the report of the New York insurance superintendent, and have reference to the "co-operative organizations", as they are termed, operating within his province. The returns are made up to 31 Dec. 1885.

Certificates in force 31 Dec. 1884	584,804
Issued during the year 1885	181,983
	766,787
Terminated during the year	113,701
In force 31 Dec. 1885	653,086
Number of Associations	138
Invested Assets	£563,092
Other Assets	594,330
Total Assets	£1,157,422
Total liabilities (claims not paid)	£496,446
INCOME.	
Received from members	£3,269,024
Other receipts (interest and fines)	57,146
Total	£3,326,170
DISBURSEMENTS.	
Claims	£2,692,354
Expenses (14·3 per-cent of total contributions)	467,518
	£3,159,872

No information is given of the amount of nominal insurance carried; but from the report of the executive committee to the eleventh annual convention of the mutual benefit life associations of America, I obtain the following "statement of the assessment business of the *reputable* companies of the United States in 1885."

Number of Companies reported	415
New Members admitted in 1885	366,585
Number of Members at the close of the year	1,838,130
Insurance in force at the close of the year	£821,661,757
Assessments collected during the year	£5,421,094
Death claims paid	£5,013,184
Expenses of conducting the business of the year	£920,285
Assets at the close of the year*	£2,009,659
Losses paid since organization	£29,605,604
Deaths during the year	13,098

* Consisting largely of assessments called but not paid.

These figures are rather startling; and although they include the returns of a large number of trade organizations and local friendly societies, the statement, nevertheless, shows that the average nominal insurance carried is £447, and the average claim paid on each death was £383.

If this class of so called insurance is pushed in this country with the same vigour as the tontine scheme has been, I should not be at all surprised to find that a very large business is obtained. There are plenty of people here who will buy a thing because it is cheap, even though they know or suspect it to be rubbish. One reason given why these associations have increased their business so largely in recent years is, that nearly every agent for the level premium companies—(I must use the term coined by these assessment companies as being the most distinctive)—is agent for an assessment company, for he finds that very often when he cannot induce a man to effect a real insurance, he can persuade him to subscribe for an assessment certificate; and thus he benefits where he otherwise would not. The agent only gets the admission fee, and has no interest in the continuance of the certificate; but he nevertheless is generally enabled to secure a double advantage for himself, because the man having become familiar with the benefits of life assurance, can be more easily persuaded after a time to drop the assessment policy and take out a real insurance in an old-line company.

There are other matters connected with this business which I should very much have liked to have brought before your notice, particularly the legal aspect of the question; but I have already occupied too much of your time. It matters, however, very little perhaps, for it may possibly happen that much of what I have said already may be inapplicable in a few years; because these companies may, like the tontine companies, alter entirely the character of their policies and the nature of their contracts as time goes on. There is sufficient evidence of the unsettled condition of these assessment companies to be found in the report prepared by Mr. L. G. Fouse, which I have appended to this paper. That report is very clear, and gives an intelligible table of the progress of a class calculated on certain bases; but it will be seen that the conditions differ very materially from those I have been considering. For instance, it presupposes that no one shall benefit in the reserve fund until he is 50 years of age if he enter at 25, or till after 60 if he enter at 45; and further that the reserve fund will be accumulated at 4½ per-cent interest and preserved intact for the purposes to which he applies it. The assumption that the lapse rate will be the same as in Meech's table of lapses in the thirty American offices is unreasonable. At the same time it must be confessed that a greater rate of lapse would benefit the persistent members, always supposing that the new entrants would be sufficient to fill up the gaps made by the lapses, and that the expenses did not exceed the annual dues. On the other hand the selection must always be great against an assessment company, and the reserve fund would suffer in consequence of the excessive mortality.

It is very remarkable that these companies have been able to evade all the stringent laws passed by the several States in America for regulating assurance companies; and it gives additional force to O'Connell's remark, that he could drive a coach and six through any Act of Parliament that the legislature chose to pass. It is quite evident that associations of this kind, when subject to no legislative regulations, are open to great abuse. They are often started for the benefit of the promoters, and they undoubtedly offer abundant opportunities for mismanagement and fraud. You will not therefore be surprised to learn that the State of Massachusetts has passed a law requiring each assessment company to accumulate an emergency fund, the amount of which fund must, at all times, be not less than, but not necessarily the specific proceeds of, one full assessment, and may be indefinitely more. This fund is to be deposited with the state treasurer in exclusive trust for the payment of death and disability claims of the company, and can be withdrawn for that purpose only upon a requisition endorsed by the insurance commissioner, or, in the event of the insolvency or dissolution of the corporation, by the receiver under order of the court in furtherance of the trust.

This is an exceedingly moderate and reasonable law; but it is stated to have had the effect of dissolving several of the companies, and preventing others doing business in the States.

Conclusion.

Both the schemes which I have passed under review are each, in their way, peculiarly fitted to the conditions of life in a new country. Where there is a wide field for energetic labour; where the struggle for existence is hardly felt; where there are vast fields uncultivated as yet by man, and capable of absorbing not only the increasing population of the country, but emigrants from all the rest of the world; where the accumulation of wealth for keeping the present and future generations in idleness, or beyond the reach of want, is small, compared with the population; and where nearly everyone sets out with the intention of being the architect and builder of his own fortune; a man starts with very different ideas as regards the future to what an individual does in an old crowded-up country like this, with its ancient accumulations of wealth. A man in the new country may, for instance, start on an undertaking where his brains and personal supervision and energy tend more to the creation of a fortune than a large capital. He resolves to work as hard as possible for the next 10 or 15 years, with the view of amassing a fortune and retiring at a time when he is able to enjoy the results of his own labours. He does not want to be bothered with the difficulties and anxieties of investing his earnings, and he would like to leave his family provided for if he should die in the meantime. What better could he do with his money than place it in the hands of a safe insurance company, promising a return of his money at the end of a fixed period, with large accumulations. For his purpose the tontine scheme, as blazoned forth in the prospectuses, exactly meets his wants. If his undertaking prospers he can keep up his payments punctually, and at the end of 10 or 15 years, if the promises of the company are realized, he can draw out his money with the promised additions. If he dies, his family gets a good round sum, larger than he could have saved for them; and if he fails—well! what does it matter to him? If he had invested the money himself, or had effected an ordinary policy, his creditors would claim it, and that would be no benefit to him; while, as it is, his payments fall into the lottery for the benefit of those who started like himself and have been more successful. But even in his failure such a policy might be more valuable to him than any other form of investment, because, as the policy is practically valueless to his creditors, they may agree to leave him that, take a composition, and start him again in business. Such a condition of affairs hardly exists in our country, where the population is so crowded, the competition so keen, and the difficulty of starting again in life so great. The people in this country, for whom that class of policy appears to be most suited, are those who have an established fortune and do not know what to do with their surplus earnings. Their primary object is to seek a good investment, and the insurance part of the scheme is quite a secondary consideration. They are persons who would not effect an ordinary insurance; and, so far as they are concerned, the British offices do not suffer in the least. If they were the only people who were induced to assure under this scheme the prospects at the end of the tontine period, considering the heavy expenditure of the offices, would be very poor indeed; but, unfortunately, others are tempted to take out tontine policies, for whom the scheme is not in the least suited, and they, when hard times come, have to lose the benefit of all their payments.. It often happens, however, that (having learnt the folly of joining a lottery scheme in which they are heavily handicapped, but yet, having learnt to appreciate the benefit of making a provision for their families) these persons are more easily induced afterwards to invest in a more perfect form of assurance; and thereby I believe that the British offices are benefited by the competition of the American tontine offices.

Again, in the new world, a man will start a business with next to no capital, and will want to put every penny he can spare out of his earnings, into the business. He resolves to amass a fortune by creating a large capital in a business; and all the insurance that he wants is such as will be sufficient to provide for his family should he die early, and he wants it at the cheapest possible rate. The mutual assessment scheme exactly suits him. His contributions are the lowest possible, and are collected at short intervals. He pays them the same as he pays his wages. It matters very little to him whether the society breaks up in a dozen years or not. It suits his immediate purpose, and if he continues in good health he can always assure in another society at current rates. There are very few people in our country who can afford to play fast and loose with insurance like that; but it unfortunately happens with this scheme as with the tontine, that it is so exceedingly tempting that numbers are induced to join for whom it is quite unsuited. Low rates and easy payments have great attractions. But what is the end of it all? If the man is not saving in any other way, or building up a business which will last after his death, he will find in his old age that he is paying a heavy and increasing premium for that which he could have obtained very much cheaper on the level premium system; and, even worse, he may find that the company has dissolved, and that he has not a penny to leave behind him.

Reviewing the whole subject, I have come to the conclusion that we have nothing whatever to fear from the competition of either of these American schemes. If it were not for the ultimate disappointment and suffering that would inevitably ensue, I am inclined to think we ought to welcome the assessment companies to our shores, because they would familiarize thousands of people

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l. Increasing rates, according to American table, loaded for old object of reducing the cost to members at an advanced age, and of compelling healthy outgoing members to leave some money behind to protect the company against adverse selection.

"3rd. Increasing rates, without any loading, except an emergency fund, equal to one natural annual premium, to be collected in instalments.

"4th. Compromise rates, or rates increasing in part only, and only partially loaded, or just enough to make up the deficiency in the increase of the rate.

"Messrs. Phelps and Litchfield selected the first, and Messrs. Stevens and Taylor, the second. The vote being a tie, and the question of difference between the two principles being of more practical than mathematical importance, I deemed it best not to express my own preference, but to construct two systems of rates, governed by the two principles respectively. I shall first present rates constructed according to the second principle, namely: increasing rates each year of insurance, according to the American mortality table, loaded 33⅓ per-cent. The rates represent the yearly cost of insurance, and may be divided into three, four, six or more assessments. I advise making not more than four assessments a year. The greater the number of assessments the greater the expense for postage, printing, clerical work, and the greater the annoyance to policyholders. In Table No. 1, hereto subjoined, the first column represents the age at entry, the second column the yearly rate at age of entry, the third column the highest rate or limit of increase, to be kept at that rate through life, by the 33⅓ per-cent loading, and the fourth column is the age at which the maximum increase is attained and at which the first dividend is to be declared.

"TABLE No. 1.—*Showing Rate for each Age, the Limit of Increase from each Age of Entry and the Age when such Limit is Attained.*

Age at Entry	Yearly Rate at each Age	Highest Rate or Limit of Increase	Age when Limit of Increase in Rate is Attained	Age at Entry	Yearly Rate at each Age	Highest Rate or Limit of Increase	Age when Limit of Increase in Rate is Attained
25	10·75	18·00	49½	43	14·01	32·04	58½
26	10·84	18·47	50	44	14·42	33·21	59
27	10·92	19·02	50½	45	14·88	35·58	60
28	11·02	19·56	51	46	15·41	36·04	60½
29	11·12	20·11	52	47	16·00	37·50	61
30	11·23	20·66	52½	48	16·66	39·23	61½
31	11·35	21·30	53	49	17·46	41·95	62
32	11·47	21·93	53	50	18·37	43·77	62½
33	11·61	22·57	53½	51	19·38	44·78	63
34	11·77	23·30	54	52	20·50	46·87	63½
35	11·92	24·11	54½	53	21·77	49·05	64
36	12·10	24·94	55	54	23·18	51·42	64½
37	12·31	25·75	55½	55	24·68	54·06	65
38	12·54	26·66	56	56	26·50	56·79	65½
39	12·77	27·66	56½	57	28·44	59·76	66
40	13·05	28·50	57	58	30·57	62·83	66½
41	13·33	29·70	57½	59	32·96	65·52	67
42	13·66	30·85	58	60	35·58	69·16	68

"The above rates are constructed so as to embody and give effect to all the important factors that enter into the science of life insurance, namely: selection, mortality, lapse and interest. Of these, selection and lapse are the least understood, and have given rise to the numerous errors so prevalent in contemporaneous assessment insurance. Selection, when exercised by the company in the admission of members, is a decided benefit to the company, but when exercised by the retiring members, it is a decided loss to the company. To determine the effect of selection on a company with any degree of accuracy, recourse must necessarily be had to the experience and statistics of life insurance companies. In Meech's table of mortality, published in System and Tables of Life Insurance, Table XXVI., and constructed from data furnished by thirty American offices, covering a period of thirty years, selection, both in favor of and against the insurer, was taken into account, and its benefits were incorporated because it represents the actual mortality among selected lives. For this reason, therefore, this table should be accepted and applied in preference to any other, in the construction of any table of rates, and since it is the table I used to test the rates of Table No. 1, it is herewith given from age twenty-five.

TABLE No 2.—*Meech's Table of Mortality.*

Age	Deaths	Age	Deaths	Age	Deaths	Age	Deaths
25	7·03	44	10·76	63	33·10	82	158·70
26	7·11	45	11·20	64	35·74	83	172·45
27	7·19	46	11·68	65	38·64	84	187·52
28	7·27	47	12·22	66	41·78	85	203·63
29	7·38	48	12·81	67	45·28	86	220·84
30	7·48	49	13·45	68	49·04	87	239·88
31	7·60	50	14·17	69	53·24	88	259·55
32	7·72	51	14·95	70	57·77	89	292·60
33	7·87	52	15·81	71	62·77	90	328·15
34	8·02	53	16·75	72	68·21	91	358·54
35	8·21	54	17·77	73	74·14	92	389·73
36	8·39	55	18·93	74	80·70	93	425·00
37	8·59	56	20·17	75	87·79	94	462·45
38	8·82	57	21·56	76	95·50	95	500·00
39	9·07	58	23·06	77	103·99	96	558·82
40	9·36	59	24·70	78	113·18	97	600·00
41	9·64	60	26·52	79	123·18	98	666·66
42	9·99	61	28·52	80	134·06	99	1,000·00
43	10·35	62	30·70	81	145·83		

The lapse rate applied was also taken from System and Tables of Life Insurance, p. 100, at age forty, up to and including the twenty-fourth year of insurance. Beyond the twenty-fourth year of insurance the experience of the friendly societies of Great Britain, or a lapse rate of one per-cent, was applied. Meech's tables does not extend beyond thirty years at age forty, and, indeed, above the twenty-fourth year of insurance his data were so meagre and limited as to be of little value, and for this reason the experience of the friendly societies was adopted beyond the twenty-fourth year of insurance.

TABLE NO. 3.—*Meech's Table of Lapses. Age of Entry, 40 Years.*

Years of Insurance	Per-cent of Lapses	Years of Insurance	Per-cent of Lapses	Years of Insurance	Per-cent of Lapses
1	17·4	9	2·5	17	1·0
2	9·6	10	2·9	18	0·3
3	7·8	11	2·5	19	0·9
4	6·8	12	1·6	20	1·0
5	5·7	13	1·6	21	0·9
6	4·6	14	2·0	22	0·5
7	4·9	15	0·9	23	0·6
8	3·6	16	1·0	24	1·0

The rate of interest was fixed at 4½ per-cent, this being deemed a fair rate and likely to be earned by the company, and being the rate established by the laws of nearly all the States for computing reserves—the net rate of interest earned by life companies in 1883 was 5·19 per-cent, and in 1884 it was 4·53 per-cent.

The rates set forth in the second column of Table No. 1 represent the actual cost indicated by the American Mortality Table, loaded 33⅓ per-cent. The effect of such loading, of the lapse rate, and of the mortality, taking into account the factor of selection, as indicated by Meech's Table, was ascertained by a calculation hereto subjoined. In this calculation I have taken but one age, or age 40, and assumed in Table No. 1 that a similar calculation for other ages, applying the same factors or tables, would proportionately produce the same result. To be strictly accurate, a calculation like the following should be made for each age. However, if this were done, the change, if any, in the rates given in Table No. 1 would be very slight.

Table No. 4.—*Showing the natural process of disposing of* 1,000 *persons, each* *years old at entry, representing* 1,000,000 *of assurance, by applying the Mort* *and Lapse Experience and Loading Natural Premium at* 33⅓ *per-cent.*

Years of Insurance	Number of Members at beginning of year	Number of Lapses	Number of Deaths	Number that pay Assessments	Gross amount paid by each	Total amount paid each year	Amount of losses each year	Amount of surplus at 4½ per-cent	Amount of Dividend for each	Total Dividend each year
1	1,000·00	174·00	8·54	909·00	13.05	11,862.45	8,540	3,386.97	...	...
2	817·45	78·47	7·50	774·46	13.33	10,323.55	7,500	6,416.23	...	..
3	731·47	57·05	7·02	699·43	13.66	9,554.21	7,020	9,285.86	...	
4	667·38	45·38	6·67	641·36	14.01	8,985.45	6,670	12,071.25	...	..
5	615·33	35·07	6·43	594·58	14.42	8,573.84	6,430	14,806.53	...	..
6	573·82	26·39	6·27	557·49	14.88	8,295.45	6,270	17,533.62	...	...
7	541·15	26·51	6·17	524·81	15.41	8,087.32	6,170	20,283.08	...	...
8	508·46	18·30	6·10	496·26	16.00	7,940.16	6,100	23,156.27	...	.
9	484·05	12·10	6·12	474·94	16.66	7,912.50	6,120	26,031.63	...	...
10	465·83	13·50	6·17	455·98	17.46	7,961.58	6,170	29,024.70	...	...
11	446·14	11·15	6·24	437·44	18.37	8,035.77	6,240	32,167.47	...	..
12	428·74	6·76	6·38	422·17	19.38	8,181.84	6,380	35,457.38	..	..
13	415·60	6·65	6·51	409·02	20.50	8,384.91	6,510	38,970.05	...	..
14	402·44	8·04	6·65	395·09	21.77	8,601.10	6,650	42,723.70	...	..
15	387·74	3·48	6·86	382·56	23.18	8,867.74	6,860	46,699.22	...	...
16	377·38	3·77	7·10	371·94	24.68	9,179.72	7,100	50,915.96	...	..
17	366·50	3·66	7·36	360·94	26.50	9,564.91	7,360	55,451.46	...	..
18	355·47	1·06	7·65	351·11	28.44	9,985.85	7,650	60,335.23	...	
19	346·75	3·11	7·95	341·22	30.57	10,431.09	7,950	64,887.11	2.07	693
20	335·69	3·35	8·25	330·38	32.96	10,889.65	8,250	69,058.53	4.46	1,445
21	324·08	2·91	8·55	319·24	35.58	11,358.91	8,550	72,816.71	7.08	2,213
22	312·60	1·56	8·89	307·37	38.50	11,834.90	8,890	76,093.46	10.00	3,021
23	302·14	1·81	9·24	297·11	41.72	12,395.43	9,240	78,887.69	13.22	3,848
24	291·08	2·91	9·58	283·38	45.25	12,822.95	9,580	81.082.00	16.75	4,664
25	278·59	2·78	9·89	272·65	49.16	13,403.47	9,890	82,823.58	20.66	5,493
26	265·90	2·65	10·21	259·47	53.49	13,879.05	10,210	83,977.71	24.99	6,323
27	253·03	2·53	10·52	246·51	58.26	14,361.67	10,520	84,538.91	29.76	7,141
28	239·98	2·40	10·81	233·37	63.52	14,824.30	10,810	84,499.10	35.02	7,941
29	226·76	2·26	11·05	220·10	69.33	15,260.23	11,050	83,885.64	40.83	8,714
30	213·44	2·13	11·25	206·74	75.68	15,646.84	11,250	82,714.11	47.18	9,436
31	200·05	2·00	11·54	193·28	82.65	15,974.59	11,540	80,865.97	54.15	10,099
32	186·50	1·86	11·69	179·73	90.21	16,213.44	11,690	78,456.38	61.71	10,672
33	172·95	1·73	11·70	166·23	98.30	16,341.92	11,700	75,596.72	69.80	11,134
34	159·52	1·59	11·74	152·85	106.89	16,338.14	11,740	72,236.00	78.39	11,459
35	146·18	1·46	11·78	139·55	116.02	16,188.96	11,780	68,358.85	87.52	11,634
36	132·93	1·32	11·63	126·95	125.82	15,972.82	11,630	64,190.87	97.32	11,675
37	119·97	1·19	11·46	113·54	136.41	15,474.63	11,460	59,604.58	107.91	11,579
38	107·31	1·07	11·10	101·22	148.08	14,988.66	11,100	54,882.07	119.58	11,375
39	95·13	0·95	10·70	89·30	161.09	14,385.34	10,700	50,051.41	132.59	11,068
40	83·48	0·83	10·21	77·95	175.94	13,692.89	10,210	44,906.18	147.14	10,951
41	74·43	0·74	9·92	69·09	192.61	13,309.35	9,920	39,923.81	164.11	10.463
42	63·76	0·63	9·26	58·81	211.46	12,435.96	9,260	35,111.52	182.96	9,854
43	53·86	0·53	8·52	49·33	232.38	11,463.30	8,520	30,870.04	203.88	8,929
44	43·80	0·43	7·49	39·83	255.41	10,175.53	7,490	26,285.79	226.91	8,139
45	35·86	0·35	6·70	32·83	281.80	9,251.49	6,700	23,373.16	253.30	7,295
46	28·80	0·28	5·84	25·73	314.06	8,080.76	5,840	20,236.34	285.56	6,473
47	22·66	0·22	4·98	20·06	...			...	...	
48	17·45	0·17	4·16	15·28	...	538,702.85	401,210	...	...	223,746
49	13·11	0·13	3·39	11·35	...	...	21,850	...	...	...
50	9·59	0·09	2·78	8·15	...	...		...	...	...
51	6·71	0·06	2·21	5·57	...	...	423,060	...	...	...
52	4·43	0·04	1·59	3·61	...	...	...	...	...	...
53	2·79	0·02	1·08	2·23	...	...	...	...	...	...
54	1·67	0·01	0·72	1·30	...	...	...	...	...	...
55	0·94	0·00	0·94	0·00	...	...	...	...	...	...

Note.—In making the calculation three decimals were used, but to economize space only two are given At the end of the 46th year the company has only 22·66 members, and after deducting lapses it had only 21,850 The persisting members were then 85 years old, and there being 20,236.34 in the reserve fund, no further were made, because the reserve fund, with interest, was more than enough to pay all future losses—it left a bal about 1,500. To sum up, we find that to dispose of 1,000,000 insurance we collected 538,702.85, returned in 223,746.57, making the direct tax on the members only 314,956.28 to pay death losses amounting in the 428,060

It will be observed from the foregoing table that at the beginning of the first year there were 1,000 members, each 40 years old. From Table No. 3 we find that during the first year of insurance 174 lapsed, reducing the average number of risks exposed during the year to 909, and by applying to these the death rate in Table No. 2, we find the number of deaths to be 8·54, representing 8,540 insurance. The 909 members are each assessed 13.05, being the American Table loaded 33⅓ per-cent, from which the company realizes 11,862.45. The death loss being only 8,540, I placed the balance, plus six months' interest, or 3,386.97, in the reserve fund. The second year of insurance the table starts with 817·46 members, and the table of lapse and mortality is applied to this number the same as before, and the calculation is continued in this manner until the cost of insurance has reached the sum of 28.50, which, after adding the average expense dues of assessment companies, would equal the level premium of an old-line company at age 40. Notwithstanding the accumulation of the 33⅓ reserve, the members aged 40, under this system, will be in the company eighteen years before the cost of insurance will equal the average level premium. Thereafter, by the application of the reserve, the cost of insurance is kept at the level rate of 28.50. This will be seen by subtracting each member's dividend from his gross premium. In this calculation it is assumed that each member is insured for 1,000, making the insurance liability at the beginning of the company 1,000,000. By adding together the column of losses paid each year, it shows that only 423,060 of the 1,000,000 matured by death, and of this sum 314,956.28 were paid by a direct tax on the members, and the balance, or 108,103.72, was made up by interest and lapse profit. In brief, the foregoing calculation shows that a man entering at age 40 pays 13.05, and this rate increases a little each year, according to the American Table, until the member is 57 years old. His rate then, after he has been in the company 18 years, after adding the average expense dues of assessment companies, will be what it is in a level-premium legal reserve company at age 40, age of entry, and through the 33⅓ per-cent loading the rate is kept at that until the last man in the company has been retired. *A sum equal to the loading must be collected and improved annually regardless of the actual mortality experienced by the company.* While this calculation makes a favorable showing as compared with the cost of level-premium insurance, it also demonstrates the recklessness and fallacy of the statements made by some few representatives of assessment insurance, to the effect that a reserve of 25 per-cent would render policies self-sustaining at the expiration of fifteen years. There is no experience or data that will warrant such a claim, and it is therefore unjustifiable and inexcusable.

DISCUSSION.

The **PRESIDENT** (Mr. A. Day), said he quite agreed that British offices had nothing to fear from the competition of American schemes, either tontine or assessment. British offices were fully capable of assimilating all that was good in the systems of the American companies, and they would thereby probably do better for the British public than the American offices could do. There was room for all. While the assurance offices on this side had been seeking to afford increased freedom for the assured and a diminution of the risk of non-forfeiture, the conditions of the American offices appeared to have been growing not only more stringent but even onerous. He was amazed to hear that the American companies should have tontine assurances to the extent of 54 millions of sums assured, and he was equally surprised at Mr. Manly's statement that 821 millions was the amount assured in American offices under the assessment system. That was nearly twice the amount assured in all the British offices put together. But there was one consolation left. British companies had something at the back of their 450 millions of nominal liability. There was about 150 millions of reserves in respect of the 450 millions, whereas it appeared that there was only about two millions of assets against the American 821 millions of assessment assurances.

Mr. NEWBATT remarked that novelty did not necessarily mean heresy, and he fully recognized the great ingenuity which the Americans had shown in the devices and schemes now under consideration. He had no objection to the tontine scheme on principle—that is, the tontine scheme as practised in modern times, and not as carried to the extremes of Tonti, its founder. They were all more or less practisers of the tontine principle, at all events, in regard to bonuses—some offices to a large extent, and some to a small extent, but all more or less. One company had lately introduced a bonus system, which it distinctly called the tontine bonus system. His objection to the tontine system and also to the assessment system of the Americans was that they contained so much that was fallacious and even false. Mr. Manly had been able to show that the tontine system as originally put forward by the American companies had gradually been stripped of much that was objectionable, and that it had now almost come down to ordinary endowment assurance in which bonuses were accumulated for the benefit of survivors. The assessment scheme was full of the very evils of which the tontine system was being denuded; but he thought that before very long many of the objectionable features would be removed, and that the assessment scheme would in its simple form be little more than a system of short term assurances. Mr. Manly said that in his opinion the tontine schemes were better suited to capitalists than any other. It was undoubtedly the fact that in England capitalists had largely adopted the tontine schemes, but why they should be particularly suited to capitalists was not so easy to understand. He was surprised, too, to hear Mr. Manly's opinion that these schemes were adapted to the conditions of American life. Both systems addressed themselves to popular greed, which was probably a very strong characteristic of young races. As regards the assessment scheme, a very remarkable pronouncement was made some years ago, quite contrary to Mr. Manly's idea that these schemes were adapted to American necessities. The Mutual Life Insurance Company of New York had issued a pamphlet, in which they gave utterance to the opinion that endowment assurance and not short-term assurance was the system best adapted to the wants of the American people. That seemed to him to be a sober and intelligible opinion, and commended itself to his reason far more than those fleeting will-o'-the-wisp schemes which were good for a few years for the young and healthy, but which were apt to prove a delusion and a snare and a source of great disappointment to the old, the infirm, and the decrepid. Formerly American companies could realize 7 per-cent on their investments; but Mr. Manly showed that the interest for 1884 had fallen to 4½ per-cent. If assurance companies in this country valuing at 3 per-cent could earn 4 per-cent, they were in a better position for giving bonuses and benefits to their policyholders than offices which valued at 4 and 4½ per-cent, and which made barely more than 4½, or perhaps 5. The whole tendency of the assessment schemes was opposed to British ideas; for the desire of British offices was to give value for their premiums, and they had set themselves to work to strip life assurance of all its drawbacks and disadvantages, with the object of enabling the assured to retain throughout the whole of their lives the benefit which they desired to secure.

Mr. M. N. **Adler** observed that, clever and inventive as the American people were, they could not claim originality in regard to the mutual assessment scheme. The identical system was adopted some fifty years ago by a mutual society in Hanover. That office charged a very moderate rate for young lives when they entered. Contributions in subsequent years were levied in proportion to the claims that had become due in each year, and according to the then ages of the members. Each member had, therefore, to pay a higher tax as his age advanced. If a member attained a very old age, the amount of his annual contribution would be very alarming, and to obviate this they instituted a reserve fund, so that after the age of 75 one uniform rate of contribution was levied. A connection of his, a contributor to the fund, had now reached the age of 84, and although he had paid the sum assured over and over again, he still had the pleasure of paying a premium at a rate exceeding 8 per-cent. His hearers would probably not be surprised to learn that the society was far from popular. So far as he could see, the assessment scheme started in America must eventually collapse.

Mr. E. **Justican** remarked that tontine policies were of two classes—namely, the "Pure Tontine", with its onerous and vexatious conditions, and the modified form known as the "Semi-Tontine", the latter being non-forfeitable, and carrying the right to a paid-up policy after three premiums had been paid. One great objection to the tontine scheme was, that the amount of paid-up policy granted at the end of the term depended on the *then* state of health of the assured; in other words, the cash bonus was treated (in effect) as a single premium at the then age, the life practically being again selected.

Mr. F. B. **Wyatt** urged the importance of accumulating a substantial reserve, without which an assurance business could have no stability. One of the largest and oldest of the American offices had a system of renewable term policies. The premiums were higher than would be charged in Great Britain. The idea was this—at the expiration of his term a member had the option of renewing his assurance for another ten years without any medical examination at the higher premium. The surplus, or accumulated profits, were applied only to the levelling of premiums. With seventeen years' experience they had never had any need to charge the higher premiums. Unless there was some large modification in the present assessment system, companies carrying on such business must eventually die out. The large reserves held by British offices had been taken exception to by certain assessment societies. The accumulated or aggregate reserve of about 140 millions, they alleged, was simply a tax on the assured, and served no good purpose. Yet they were themselves compelled to resort to the establishment of reserves. He felt sure that before many years had elapsed they would have to come back to the old system of level premiums.

Mr. G. **King** entirely concurred with the hearty welcome everyone appeared to accord to their American rivals. British offices need only take care to let it be clearly understood what were their leading features, and how firm and solid was the basis upon which the British companies stood. If they taught the principles of life assurance to their customers it would soon be apparent that they were not doing the foolish thing that some of the American companies accused them of doing. The more life assurance was discussed the better would it be for all. He believed that British offices had already reaped benefit from the competition of the American companies, and hoped they would continue to do so.

Mr. A. H. **Bailey** said he could date back the mutual assessment scheme to the year 1721, when charters were granted to the Royal Exchange and London Assurance Corporations. The assurances originally granted under those charters bore some resemblance to the mutual assessment scheme, having been undertaken for no period longer than one year. The only difference was that, instead of raising the premiums from year to year as the age increased, there was a uniform rate up to about 50 of 5 guineas per-cent. The business was small, but extremely profitable. That was a mutual assessment scheme in its essence, and nothing else. Something of the same description used to be more in vogue than it was now, as assurances on an increasing scale of premiums were at one time common. He saw no objection to such systems as the latter if they were really in request. Life assurance was not an investment, but as the public demanded something in the nature of an investment this explained the great increase of endowment assurances. He agreed with Mr. Manly, that the tontine scheme appealed to the gambling element in human nature, which was very strong in all ages, and which had developed itself very considerably in the demand for bonuses in this country. It was notorious that the old Equitable Office, which first gave bonuses to the assured, thereby brought to itself a large amount of new business—another instance of the rush of the public to participate in a fund to which they had not contributed. The tontine scheme appealed to the very same tendency on the part of human nature.

Mr. G. H. **Ryan** thought Mr. Manly's paper had done much to clear their minds of some of the numerous misconceptions which were prevalent concerning the American schemes. The tontine scheme was, as Mr. Manly had clearly shown, practically nothing more than assurances at non-profit rates, or a little higher rates, with deferred bonus. Such modes of assurance had lately become very popular with

British assurance companies. Mr. Manly had called attention, as being a remarkable point, to the fact that the American assurance companies had succeeded in evading the penal statutes regulating life assurance companies in the United States. Instead of that being a noteworthy result in the working of the system, he (Mr. Ryan) submitted that it was the very fundamental cause of their origination. Had it not been for the fact that the promoters of these companies saw that a mutual assessment scheme could be devised which could evade the existing statutes, he strongly believed they would never have come to light. Further than that, it appeared to him to be almost a necessary result of such a compulsory system of State control as was in operation in America. It stood to reason, that where a net-premium valuation was enjoined on all companies, it must, in the prevailing conditions, be practically impossible for any new assurance companies to be started on the ordinary basis. That being so, the energy and enterprize of persons acquainted with life assurance matters, who might be attracted to that field of business by the growing popularity of life assurance, were driven perforce into channels which they must all regard as both illegitimate and undesirable. That such a system of State intervention would ever be tolerated in this country was a matter which need not be seriously contemplated for a single moment. But he firmly believed, that were a statute upon the lines of the American laws to be passed by the Legislature of Great Britain, they would find that the very results now decried would follow as a direct outcome of such legislation.

Mr. T. J. Searle pointed out that there were sometimes funds held by employers of labour resulting from the compulsory assurance of the men employed, and when the employé left he lost his share in the reserve. Such cases involved considerable injustice upon those who left the employment, and he submitted that a system which provided in fact short-term assurances for uncertain terms would effectually meet the requirements. In Liverpool there existed such a fund in connection with the police force. A mutual assessment fund created under such circumstances would obviously be conducted on the most favourable conditions, for, by the nature of their occupation, the men would neither be too young nor too old, and would have been medically examined on admission to the force, and they would have all the practical advantages of a splendid organization. He explained the working of this organization, and showed that an assessment of about 24*s.* per-cent per annum would provide for the death benefits and the expenses. Only one other thing was needed to make the system complete, and that was that the assurance of members of the force should be compulsory.

Mr. F. A. Straker considered that the one merit of the mutual assessment scheme was, that it enabled a man to get as much assurance as possible with what money he could spare. With regard to the objection that the payments became intolerably heavy in the latter years, there were two ways of making a level premium—one was to increase the earlier payments at the expense of the later, and the other to decrease the sum assured. Many people commenced life with much need for assurance, and with little money to spare for the purpose, but with the expectation of making a larger income, saving more, and so needing less assurance as time went on. He considered that a scheme by which the sum assured would be gradually diminished, instead of the premium being increased, in later years might be acceptable to a large class of the community.

On the motion of the PRESIDENT, a hearty vote of thanks was passed to Mr. Manly for his paper.

Mr. H. W. MANLY, in reply, said he was inclined to agree with the President of the Mutual Life Office of New York, that endowment assurances were better suited for the American people than either of the schemes which they had been discussing; but the fact remained, that the people themselves did not think so. There could be no doubt that the Americans believed in assurance far more than the English. They took a livelier interest in all that concerned assurance business; and hence these fancy schemes obtained more notice, and got subscribed to mostly as a species of speculation. Mr. Ryan had undoubtedly given the correct explanation why the assessment companies had succeeded in evading the penal statutes regulating life assurance companies in the United States. As to yard funds, he believed that within certain limits, and under certain conditions, they were beneficial both for the masters and the men. He had been instrumental in framing one which had worked most satisfactorily.

Death of Mr. J. Hill Williams.

IT is with more than the usual expressions of regret that we have to announce the death of the much-esteemed and respected Actuary to the English and Scottish Law Life Assurance Office, and the termination of a long, successful and unblemished professional career. For nearly forty years the name of Mr. John Hill Williams has been known and honoured in the actuarial world.

In the year 1846 Mr. Hill Williams was appointed to succeed his brother as Actuary and Secretary to the Assurance Society, the affairs of which he continued to direct until the very day of his death. Previously to holding that appointment he had served in the Honourable East India Company's Navy, from which he retired when that Company first began making reductions and changes, and then spent some time both in this country and in Ireland engaged in engineering and other scientific pursuits. In 1856 he also became Actuary to the Law Reversionary Society, succeeding in that post the distinguished mathematician, Professor Sylvester.

Mr. Hill Williams was one of the original members of the

Institute of Actuaries, which was founded in 1848. He was for a long time a member of the Council, filling also in succession the offices of Honorary Secretary and Vice-President. Upon the death of the late Mr. Tucker, in 1875, he attained the important position of President, and during his tenure of this office he ably maintained the authority of the chair, and fulfilled the responsible and often difficult duties of the post in a way that secured for him the regard, at the same time that it commanded the respect, of the members of the Institute. In the words of the present President, when referring at the sessional meeting in November last to the lamented death of Mr. Hill Williams, "He was wise in council. "He occupied the presidential chair with singular dignity, "discretion and courtesy."

Although he was not a frequent contributor to the literary transactions of the Institute, his valuable translations from the works of contemporary German authors on professional and cognate subjects will obtain for him a grateful remembrance from the student of insurance subjects, and will remain as a permanent evidence of his desire to promote the primary aim of the Institute—the advancement of actuarial learning. He also devoted considerable attention to the question of the assessment of the property and income tax, and was called upon to give evidence before the Parliamentary Committee on Income Tax in 1851.

For some years past, Mr. Hill Williams, on account of failing health, was prevented from taking so active and prominent a part as previously in the proceedings of the Institute, but nevertheless, his sudden death, on the 16th November last, at the age of 72, has caused in the front rank of the profession a loss which will be long and greatly felt. Endowed as he was with scientific aptitude of no mean order, and thoroughly conversant with the multifarious duties of the profession and the complicated business of which the actuary undertakes the direction, he was ever ready with unfailing kindness to impart to others in all departments of the profession, the advice and assistance which his sound knowledge, wide experience, and many attainments so eminently qualified him to render.

The Council of the Institute has placed upon record its estimation of the high personal qualities of Mr. Hill Williams, its appreciation of his valuable services, and its sense of the loss which by his death has fallen on his friends and the profession in a resolution, from which we have been permitted to make the following extract:

> "Entering the profession later in life than usual "* * * * he adorned it by the variety of his attain-"ments, by the dignity and courtesy of his demeanour, and "the integrity of his life."

Phthisis Pulmonalis.

THE subject of consumption is of so great importance in relation to life assurance, that I asked Dr. Bristowe, whose authority is well known, if he would favour the readers of the *Journal* by drawing up a short paper explaining the present scientific conception of this disease. He has accordingly, with great kindness, prepared the following very interesting and valuable memorandum.

It will be observed, as a matter of great interest and possibly susceptible hereafter of some approximate estimate, that a person descended from a consumptive stock who presents himself for assurance possesses two chances of escaping the disease, namely, (1), the pabulum or susceptibility necessary for the development of the bacillus, may not have been inherited; and (2), even though such pabulum or susceptibility has been transmitted, and provision thus created for the reception of the bacillus, the bacillus itself may never enter the system.

It will also be noticed how locality and the particular circumstances of life affect the prospect of receiving the disease. To take an example: a man who ceases to reside with consumptive relatives, and migrates to and settles in New Zealand or Australia, thereby immensely increases his chances of immunity.

T. E. YOUNG.

A BRIEF STATEMENT OF THE MODERN VIEWS OF THE CAUSE, NATURE, AND PREVENTION OF PULMONARY CONSUMPTION, AND OF TUBERCULOSIS (OR SCROFULA).

The discovery made and announced two or three years ago by Dr. Koch, and since confirmed by numerous observers in this and in other countries, that by special means of staining, and by the aid of very high powers of the microscope, characteristic minute vegetable organisms, known as bacilli, may always be found in the specific morbid growths of tubercle, no matter where they occur, and consequently in the diseased parts of the lungs, and in the sputa, of persons suffering from pulmonary phthisis or consumption; and that these bacilli may be cultivated outside the body, and when thus cultivated and inoculated into healthy animals cause in them fatal tuberculosis; has thrown a new and important light on all so-called "scrofulous" or tubercular diseases, and is leading all thoughtful physicians to reconsider the opinions which had hitherto been generally accepted with regard to the nature, behaviour, and treatment of these affections.

It was formerly universally held, and is still held by many, that tuberculosis is a disease which, though predisposed to by heredity, by climatic conditions, by hygienic errors, or by various states of impaired health, originates within the system, and consists simply in a kind of unhealthy inflammation or degraded nutritive process which, having once commenced, has a tendency to diffuse itself throughout the organism. But the universal presence of bacilli in tubercular growths, and the fact that tubercle may be imparted to hitherto healthy animals by implanting a few of these bacilli into their bodies, show conclusively that tuberculosis is not a so-called diathetic disease, is not a disease of spontaneous growth, but that it owes its origin to the entrance into the system of foreign living organisms, and that it is allied therefore to the specific and generally infectious febrile disorders, such as small-pox, scarlet-fever, hooping-cough, cholera, hydrophobia, ague, and syphilis, which are all now recognized as due to specific organisms or contagia.

It need scarcely, perhaps, be said, that the diseases just named, though grouped together, differ widely from one another in their symptoms, in their methods of attack, in their progress, and in their results; that each one has a tendency to affect particular organs or particular parts of the system; that some are directly and immediately contagious, while in others the contagiousness is not strikingly apparent, and is by many persons denied; that in some infection is acquired mainly through the air and by the breath, in others mainly by the food, in others only by direct inoculation, and in many cases probably indirectly and by routes which have not yet been discovered; and that some of them have a limited duration and are more or less protective against subsequent attacks, some are invariably fatal, and some lurk in the system and are liable to break out from time to time throughout the remainder of the sufferer's lifetime. These differences depend, doubtless, on the different habits and qualities of the different species of micro-organisms which constitute the contagia of the different diseases. And it may be pointed out in this connection that, although we know that in the case of the most virulently contagious diseases the contagium passes directly from those already sick to those whom they infect, we know very little as yet with regard to where contagia lurk outside the human body. We have reason, however, to believe, that some of them are nursed in the lower animals; that some of them flourish in water, in sewage, in the soil; and that some of them in their generation pass through alternate phases of virulent and harmless existence. We know further, that persons are not equally liable to contract disease on exposure to its contagious influence, and that the susceptibilities in this respect of the same individual vary from time to time; that the condition of health of the patient at the time of exposure has much to do with determining whether he shall take the disease to which he is exposed or not; and that certain constitutional conditions favour, and certain others resist, both the attacks of the disease-bearing intruders and their subsequent colonization or ravaging of the invalid territory.

Accepting the belief, then, that tubercular growths depend on the presence of specific living organisms which cannot arise spontaneously in the body, and must necessarily be introduced from outside, and that the spread of tubercle throughout the body is due to the breeding and diffusion of such organisms, how can we explain the prevalence of tuberculosis in certain localities, and its recognized tendency to play havoc in certain families? Tuberculosis is undoubtedly a disease of temperate climates, and it has been proved to occur mainly in low-lying damp localities. It may be observed that these facts are no more compatible with the disease being one that arises *de novo,* in the bodies of persons whom temperate climates and low moist districts have affected injuriously in some mysterious manner, than with its being a disease of a malarial or endemic character, like ague or typhoid fever, diseases imparted by specific organisms which under certain conditions infect the soil, the drinking-water, and the atmosphere. Again, it must be fully admitted, that tuberculosis does tend to prevail in families, that not only do consumptive parents procreate children who will have a tendency to consumption, but that in such families child after child not unfrequently succumbs, until all have fallen victims to the disease. But it is at least as easy to suppose that in such cases the disease spreads from one to another by contagion, as that it is due merely to an inherited vice of constitution. It may, of course, be assumed, in accordance with views already discussed, and is in no sense incompatible with a full belief in the dependence of tubercle on a virus or contagium, that there may be something in climatic and hygienic conditions, and something also derived by inheritance, which render persons, not, as has been hitherto largely supposed, liable to develop tubercle, but a more suitable soil than others for the growth and spread of the tubercle bacillus after this has been introduced accidentally into the system.

If the view with regard to the nature of tubercle here propounded be correct, the extreme importance of removing phthisical patients from the localities, or from the houses in which they have contracted the disease, to more healthy neighbourhoods, or to more wholesome houses, becomes apparent. But it becomes still more important that the hitherto healthy children of families in which consumption has been rife should be thus treated early; and especially it becomes important for the health of the others, that, when once a member of a household shows signs of consumption, every precaution should be taken, as in the case of any other infectious disease, to prevent its spread.

It should be added, in confirmation of what has been stated above, that abundant clinical evidence has now accumulated to confirm the truth of the belief that consumption is directly infectious, and that by the early adoption of suitable precautionary measures its spread may be arrested.

J. S. BRISTOWE, M.D., F.R.S.

NOTICES OF NEW BOOKS.

Algebra: an Elementary Text Book. Part I (pp. xx and 542). By G. Chrystal, Professor of Mathematics in the University of Edinburgh. A. & C. Black. 1886.

I have pleasure in introducing to the notice of students of the Institute this admirable Text Book.

A sounder guide in the discussion of principles could not be obtained than Prof. Chrystal, who, in addition to a distinguished career as a mathematician at the University of Cambridge, possesses high teaching qualifications acquired in his official position in the University of Edinburgh. The student who aims at a grasp of *principles*, and not mere dexterity in applying *rules*, will find this book a stimulating discipline.

Before mentioning some of the points discussed and elucidated in this volume, it should be noticed with satisfaction that the conception of Limits is introduced. The algebra of Prof. De Morgan was, I believe, the first elementary book of importance which had the merit of attempting to familiarize the student's mind from the outset of his mathematical training with this idea; and no greater service could be performed in the interests of students, than the re-editing of his Treatise by a competent hand to the present development of the science, avoiding that author's diffuseness while retaining his fresh and vigorous mode of presenting principles, and his clear and strictly logical exposition and establishment of scientific methods. The algebra of Prof. Todhunter omitted this conception of Limits; and the volume now submitted rightly recurs to De Morgan's practice. As a means of educational discipline, no less than with the object of giving the student a steady and graduated mastery for future applications in geometry and mechanics, the conception of Limits should be impressed upon him from the outset of his mathematical career, so that when its practical applications occur at a later stage he may be fully possessed of the notion, instead of having to study the conception as a new subject when the mind possibly has become less plastic. For, even in arithmetic and Euclid, the idea from the first confronts us. What in the former is the statement that $\frac{1}{3} = \cdot 3333 \ldots$ but that the fraction is the limit of an infinite convergent series represented by the decimal? What, again, is the statement that the surd $\sqrt{2}$ is equal to another interminable decimal but an assertion involving the idea of a limit? In Euclid, too, the definitions of a point and a straight line are but limiting cases of physical representations; and in algebra, to take one example, the equation $a^0 = 1$, and the operation of equating indeterminate coefficients, are cases of limits.

The author again, as a matter of principle, commences with the notion of algebra as a gradual generalization of arithmetic, for as Prof. Kelland explains (in the introduction to his work on Quaternions) the science started with all the restrictions of arithmetic, and gradually developed by discarding one after the other.

Prof. Chrystal begins with a discussion of the fundamental laws and processes of algebra, introducing and applying the Law of Association, the Law of Commutation, and the Law of Distribution.

Among other subjects, I would call attention to his discussion of the Theory of Rational Integral Functions; the Principles of Homogeneity and Symmetry; the Factorization of Integral Functions; a discussion of Real and Imaginary Quantities; the Interpretation of $x^{p/q}$, x^0, x^{-m}; Propositions respecting 0; the Graphical Representation of a Function, and the Graphical Discussion of various Cases; the Solution of Equations by means of Determinants; Maxima and Minima; the System of Graphs; an interesting and instructive chapter on Complex Numbers; Conditional Equations in general, with a discussion of the General Notion of an Analytic Function; the Variation of a Function, with a discussion of Limiting Cases; and Propositions relating to Discontinuity and Continuity.

It is very gratifying to observe, that in his chapter on the Theory of Interest and Annuities-Certain, Part I of the Institute Text-Book, by Mr. Sutton, is twice referred to as the authority upon the subject.

Various historical notes of great interest are interspersed; and the whole subject, as far as can be criticized by a learner like myself, is treated with originality, freshness, and force.

It may be considered whether the *stylus* method of representing fractions (p/q) which is adopted here might not be usefully introduced, as a great saving of space, into the *Journal of the Institute* generally.

T. E. YOUNG.

The Actuarial Certification of Annuity Tables, under "The Friendly Societies Act, 1875."

As announced by the President at the Sessional Meeting of the Institute held on the 17th ultimo, an important communication has been addressed by Mr. W. Sutton, the Actuary to the Chief Registry of Friendly Societies, to the Council of the Institute, on the subject of the actuarial certification of annuity tables under "The Friendly Societies Act, 1875." This communication is appended, and from it our readers will see that any Fellows of the Institute of five or more years' standing may make application to the Chief Registrar to be added to the list of actuaries approved by the Treasury for the above purpose. The clause of the Act referred to in Mr. Sutton's letter is as follows:

> "No society assuring to any member a certain annuity shall "be entitled to registry, unless the tables of contributions "for such assurance, certified by the Actuary to the "Commissioners for the Reduction of the National "Debt, or by some actuary approved by the Treasury, "who has exercised the profession of actuary for at "least five years, be sent to the Registrar with the "application for registry."

We need only add that it must be a matter of general gratification that the Institute should receive so early an official recognition of its altered status under the Charter.

REGISTRY OF FRIENDLY SOCIETIES, CENTRAL OFFICE,
28, ABINGDON STREET, S.W.,
LONDON, 14 *December* 1886.

GENTLEMEN,—I am desired by the Chief Registrar to ask you to inform the Council of the Institute of Actuaries, that he has been authorized to receive applications from Fellows of the Institute of Actuaries, to be approved as Actuaries to certify annuity tables for

Friendly Societies, under Section 11 (5) of "The Friendly Societies Act, 1875."

Such applications must be made in writing, and be addressed to the Chief Registrar, and must show:

(1) That the applicant has been a Fellow of the Institute for at least five years after passing the necessary examinations; and

(2) That the applicant has, from the time of becoming a Fellow, exercised the profession of Actuary.

I am, Gentlemen,
Your obedient servant,
WM. SUTTON.

The Honorary Secretaries
of the
Institute of Actuaries, London.

SYLLABUS OF THE MESSENGER PRIZES, 1886.

SUBJECT—FRIENDLY SOCIETIES.

The Council of the Institute of Actuaries have resolved to offer a Prize of £50 to the author of the best Essay upon this subject, and a further sum of £25 will be awarded to the author of the second best Essay, or be divided between two or more Essayists, if and as the Adjudicators may recommend.

Without restricting candidates in their treatment of the subject, the Council think it desirable to suggest that the Essays should include:

1. A brief historical sketch of the Friendly Society system in this country, including a short account of the salient features of the legislation on the subject, particularly that of recent years. Reference is invited also to the Friendly Societies, and to the legislation relating thereto, in other countries.

2. A description of the various classes of Friendly Societies at the present time, including the affiliated orders, such as the Manchester Unity of Oddfellows, the Ancient Order of Foresters, &c. It is desirable that the description should include a classification of societies in respect of the nature of the benefits assured to the members, and the manner in which those benefits are paid for—whether by fixed periodic contributions, by levies to meet current claims, by levies of fixed amount on the happening of certain events, &c.—including also the cases where societies are subsidized from extraneous sources.

3. An enquiry into the applicability of actuarial principles and data to the various classes of societies, both in respect to the benefits assured and the methods in use with regard to payments for those benefits.

In this section should also be included: (*a*) an investigation into the proper methods of keeping accurate records of the experience of societies, and of obtaining therefrom the necessary data for actuarial purposes; (*b*) a critical account of well-known existing data, and an enquiry as to how far they may be made use of for the purposes of particular societies; (*c*) an exact enquiry into the effect of secessions in the various classes of societies, and the manner in which, and extent to which, secessions should be introduced into valuations of their contracts—and in dealing with this question, the treatment of negative values should receive careful consideration; (*d*) an examination of the relative effect of rates of mortality and rates of sick pay in determining the values of sick-pay benefits—in other words, how far the rates of sick-pay benefit will be interfered with by a light or heavy mortality; (*e*) an enquiry as to the number under risk in various classes of societies necessary to render probable a steady experience, particular attention being given to the difference (if any) in this respect between societies granting death benefits only and those granting sick-pay or accident benefits, with or without death benefits. And, where practicable, statistics should be furnished bearing upon the various points under discussion.

The Essays may, *inter alia*, indicate such changes in the existing practice of societies, or in the existing legislation and accompanying executive arrangements, as seem to the writer desirable—and in dealing with this part of the subject a comparison is invited between the legislation relating to Friendly Societies and that affecting ordinary life insurance companies.

CONDITIONS OF THE COMPETITION.

1. That the competition be open to all members of the Institute, with the exception of past or present members of Council.
2. That the Essays be sent in to the President of the Institute before or on 31 December 1887.

(*If Essays are written in a foreign language, they should be accompanied by a literal English translation.*)

3. That the names of the competitors be sent in under seal, with a Motto corresponding to one to be prefixed to the Essay; the Motto and Essay not to be in the handwriting of the competitor.
4. That the Essay or Essays to which a Prize may be awarded be the property of the Institute.
5. That a Prize or Prizes be only awarded if the Adjudicators shall consider an Essay or Essays to be worthy of the distinction.
6. That unsuccessful Essays be returned, on application at the Institute, with the corresponding envelopes unopened.

The Position of the Profession under the Charter.

THE Council of the Institute of Actuaries, in appending a copy of the Bye-Laws, as allowed by the Lords of the Privy Council, think it desirable to make some observations in connection with the important changes which have lately taken place in the constitution of the Institute.

By the grant of a Royal Charter of Incorporation* the Institute has become a Public Body, and its Members have acquired a higher professional status than they formerly could claim. They will, as Members of that Body, be called upon to discharge many important professional duties, both of a private and public nature, and it will be incumbent upon them at all times to bear in mind that they are acting not merely as private individuals, but as Members of a publicly recognized Profession, whose reputation is more or less in their hands. Particularly will this be necessary when Members of the Institute are consulted in reference to the affairs of public companies transacting assurance business, and other institutions of a like character, where there may arise a conflict of interests as between the company or institution and the assured, or a possibility that actuarial reports, statements, opinions, valuation balance-sheets, and the like may be publicly advertized or circulated, or otherwise made use of, in a manner likely to mislead or confuse the public mind.

Under its old constitution, the Institute did not interfere with its Members in the honourable exercise of their profession, nor has it the power to do so under its new constitution, which is in this respect similar to that of other chartered professional bodies.

The Council, therefore, confidently rely upon every Member so exercising his profession as to maintain and, if possible, add to the public reputation of the Institute as a professional body whose Members are jealous both for their own honour and for the reputation of the Institute.

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* The text of the Charter will be found in the *Journal* (xxv, 5).

AT THE

COUNCIL CHAMBER, WHITEHALL,

The 26th day of February, 1886.

BY A COMMITTEE OF THE LORDS OF HER MAJESTY'S MOST HONOURABLE PRIVY COUNCIL.

Present:

LORD PRESIDENT,
SIR LYON PLAYFAIR,
MR. TREVELYAN.

WHEREAS there was read this day at the Board a letter dated 12th February, 1886, from Messrs. Markby, Stewart & Co., submitting certain Bye-Laws of the Institute of Actuaries;

AND WHEREAS, by the 45th Article of the Charter for the Incorporation of the Institute, it is provided that Bye-Laws made by the Institute shall not commence or have effect until they have been submitted to and allowed by the Lords of the Council;

NOW THEREFORE, their Lordships having taken the said Bye-Laws into consideration (a copy of which is hereunto annexed) are pleased to allow the same.

C. L. PEEL.

BYE-LAWS.

OBJECTS.

The Institute of Actuaries was established, and has since been incorporated by Royal Charter, for the objects following—that is to say:

(*a*) For the purpose of elevating the attainments and status, and promoting the general efficiency, of all who are engaged in occupations connected with the pursuits of an Actuary.

(*b*) For the extension and improvement of the data and methods of the science which has its origin in the application of the doctrine of probabilities to the affairs of life, and from which life assurance, annuity, reversionary-interest, and other analogous institutions, including friendly societies, derive their principles of operation.

(*c*) For the consideration of all monetary questions involving, separately or in combination, the mathematical doctrine of probabilities and the principles of interest.

I.—PRELIMINARY.

1. These Bye-Laws, being the first Bye-Laws made under the above-mentioned Charter, in these Bye-Laws referred to as the Charter, shall come into operation as soon as they are allowed by the Lords of Her Majesty's Most Honourable Privy Council, which time is in these Bye-Laws referred to as the commencement of these Bye-Laws.

2. Words in these Bye-Laws have the same meaning as in the Charter; and expressions in these Bye-Laws referring to Members, Fellows, Associates, Students, Honorary Members, and Corresponding Members, are to be construed as having reference to Members, Fellows, Associates, Students, Honorary Members, and Corresponding Members, of the Institute.

II.—THE COUNCIL.

3. The first election of members of the Council shall take place at the Annual General Meeting held next after the commencement of these Bye-Laws. The number of members of the Council shall be thirty, including a President, four Vice-Presidents, a Treasurer, and at least two Honorary Secretaries.

4. The Council shall, at least fourteen days previous to every Annual General Meeting, send to each Fellow and Associate, at his usual or last known address, a list containing the names of those Fellows whom they recommend for election as President, Vice-Presidents, Treasurer, Honorary Secretaries, and other members of Council, for the ensuing year; and such list shall be the balloting list for such election.

5. At the Annual General Meeting, each Fellow and Associate may erase any name or names from the balloting list, and may

substitute the name or names of any other Fellow or Fellows eligible for each respective office; but the number of names on the list after such erasure and substitution must not exceed the number to be elected, otherwise such balloting list shall be rejected by the Scrutineers, two or more in number, to be chosen from those present by the Meeting previous to the balloting for election, who shall receive the votes and report the result of their scrutiny to the Chairman.

6. Subject to the provisions of the Charter and of these Bye-Laws, the number of members of the Council present and acting, requisite for the transaction of business, shall be five.

7. Any vacancy occurring in the Presidentship, or among the Vice-Presidents, or in the offices of Treasurer or Honorary Secretary, or among other members of the Council for the time being, shall be filled up by the Council at some meeting of the Council specially summoned for that purpose; and the appointment or appointments so made shall continue in force until the next Annual General Meeting.

8. The members of the Council going out of office at an Annual General Meeting shall be deemed to be in office until the members of the Council for the ensuing year have been elected.

III.—POWERS AND DUTIES OF THE COUNCIL.

9. The Ordinary Meetings of the Council shall be held at such times and places as the Council may from time to time determine.

10. Special Meetings of Council shall at any time be called by order of the President, or of any two Vice-Presidents, or of the Honorary Secretaries, or at the request of any five members of the Council.

11. Notice of Ordinary and Special Meetings of Council shall be sent to each member of the Council, at his usual or last known address. It shall not be necessary in any case to prove that such notice has been delivered or sent, but the same shall be taken as delivered or duly sent, unless the contrary be shown; and the non-receipt of any notice by any member or members of the Council shall not invalidate the proceedings of any Meeting of Council. Such notice shall be as ample as the circumstances will permit, but shall in no case be of less than twenty-four hours' duration.

12. At all Meetings of Council, the President, and in his absence, the senior Vice-President present, shall be Chairman;

and in the absence of the President and the Vice-Presidents, the Chairman shall be elected from among those present.

13. At all Meetings of the Council, in the event of difference of opinion, the majority shall rule the minority, except where otherwise required by the Charter or by Bye-Laws, but a ballot may in all cases be demanded by any Member; and in case of an equality of votes, the Chairman shall have a casting vote in addition to his original vote.

14. Subject to the provisions of the foregoing Bye-Laws, the Chairman of any Meeting of the Council may, with the consent of the Meeting, adjourn the Meeting from time to time, and from place to place; but no business shall be transacted at any adjourned Meeting other than the business left unfinished at the Meeting from which the adjournment took place. No notice need be given of any adjourned Meeting, unless it is so directed in the resolution for adjournment.

15. Subject to the provisions of the Charter and of Bye-Laws, the Council may appoint Committees from among members of the Council or other Members of the Institute, with such powers as may be necessary or convenient for the purposes for which the Committee is appointed, and may fix the quorum thereof, and may lay down rules for regulating their proceedings.

16. Minutes shall be made in proper books, to be provided for the purpose, of all resolutions and proceedings of Meetings of the Council, and of the Committees thereof; and every Minute signed by the Chairman of the Meeting to which it relates, or by the Chairman of a subsequent Meeting, shall be sufficient evidence of the facts therein stated.

17. The Council shall, as soon as practicable after the Annual General Meeting in each year, publish, or cause to be published, a List of the Members of the Institute, together with such other particulars in respect to the Institute or its objects as it may deem advisable, and shall distinguish, in such a way as shall to the Council seem fit, the various classes of Members, namely, Fellows, Associates, Students, Honorary Members and Corresponding Members, and the examinations respectively passed by the individual Members of the various classes; and such list shall be supplied to Members and others gratuitously or at such prices as the Council shall from time to time determine.

18. For the purpose of publishing such list, each Member shall in each year, at and within a time to be fixed by the Council, send

in to the Honorary Secretaries his name and address, failing which the list shall be drawn up on such information as the Council may possess.

19. The Council shall have the management and control of the funds of the Institute, and may invest all funds not needed immediately for the ordinary purposes of the Institute, in the name of the Institute, or in cases where the Council may think fit in the names of Trustees to be appointed by the Council, in such securities as the Council may determine, with power to sell or vary such securities in such manner as the Council shall determine; but if invested in the name of the Institute, the same shall not be sold, varied, or dealt with, except according to instructions issued under the Common Seal of the Institute.

20. Notwithstanding anything herein contained, the Council may from time to time borrow money and pay interest thereon.

21. The Council may make such provision for pensioning retired officers and servants of the Institute as to the Council may seem just.

22. The Council shall from time to time make regulations with regard to the use of the Library of the Institute by the Members of the Institute and such other persons as the Council may think fit to admit.

23. The Council shall have the entire management and control of the publication of the Transactions of the Institute, to be published under the name of the *Journal of the Institute of Actuaries*, or such other appropriate name as the Council may, from time to time, think fit; and may appoint an Editor or Editors, with or without an Assistant Editor or Assistant Editors, with such powers necessary or convenient for the management of the said *Journal* as it may from time to time think fit, and shall determine their duties, salaries, and remuneration, and may make such arrangements and enter into such agreements with them as it may think fit. The said *Journal* shall be published at such times and under such conditions as to circulation and charges in connection therewith as to the Council may seem fit.

24. All papers, memoirs, letters, or other communications proposed to be read at the Ordinary Meetings of the Institute, shall be first submitted to the Council, which shall decide on their fitness to be read, and at the same time, or subsequently, shall decide on their fitness to be inserted in the *Journal of the Institute of Actuaries*.

25. All papers, memoirs, letters, or other communications, submitted to the Council to be read at the Ordinary General Meetings of the Institute, shall, in the absence of any special agreement to the contrary, become the absolute property of the Institute upon their being so read, and in the event, but not otherwise, of their publication in the *Journal of the Institute of Actuaries* within a year of their being so read.

All communications submitted only for publication in the *Journal of the Institute of Actuaries* shall become the absolute property of the Institute in the event, but not otherwise, of their being so published within a year from the time of submission.

26. The Council shall every year prepare, or cause to be prepared, a statement of the receipts and expenditure of the Institute for the year ending the 31st of March, and of its liabilities and assets at that date; and such statement, after having been verified and signed by the Auditors, shall be laid before the next Annual General Meeting, together with any report thereon made by the Auditors.

27. The Council shall draw up a report on the affairs of the Institute and the past year's transactions, which shall be submitted to the next Annual General Meeting.

28. The Council shall have the custody of the Common Seal.

29. The Common Seal shall not be affixed to any instrument, deed, or other document, except by order of the Council and in the presence of at least two members of the Council, and in accordance with such other regulations as the Council shall from time to time prescribe.

IV.—OFFICERS OF THE INSTITUTE.

The President.

30. The President shall be elected every year at the Annual General Meeting from the Fellows of the Institute. He shall preside by right at all General Meetings, and at all Meetings of the Council at which he is present. In case of an equality of votes, either at a General Meeting or a Meeting of Council, he shall have a second or casting vote. The President for any year shall be eligible for re-election for the ensuing year: provided always that no Fellow shall be elected President more than two years in succession after the commencement of these Bye-Laws, and that any Fellow who has been President for two years in succession after the commencement of these Bye-Laws shall not be eligible for that office again for a period of two years.

31. The first President to be elected under these Bye-Laws shall be elected at the Annual General Meeting held next after the commencement of these Bye-Laws.

The Vice-Presidents.

32. The Vice-Presidents shall be four in number, and shall be elected every year at the Annual General Meeting from the Fellows of the Institute. In the absence of the President from any General Meeting, the senior Vice-President present shall preside, and be for the time being invested with the President's powers. Not more than three of the Vice-Presidents for any year shall be re-elected for the ensuing year: provided always that no Fellow shall be elected Vice-President more than four years in succession after the commencement of Bye-Laws.

33. The first Vice-Presidents to be elected under these Bye-Laws shall be elected at the Annual General Meeting held next after the commencement of these Bye-Laws.

The Treasurer.

34. The Treasurer shall be elected every year at the Annual General Meeting, from the Fellows of the Institute. He shall receive all moneys due to the Institute, and pay in the same to the Institute's bankers, and sign all cheques ordered by the Council to be signed by him, and keep an account of his receipts and disbursements. The Treasurer for any year shall be eligible for re-election for the ensuing year: provided always that no Fellow shall be elected Treasurer more than four years in succession after the commencement of these Bye-Laws.

The Treasurer holding office at the commencement of these Bye-Laws shall continue in office until the Annual General Meeting first held under these Bye-Laws.

The Honorary Secretaries.

35. The Honorary Secretaries, two at least in number, shall be elected every year, at the Annual General Meeting, from the Fellows of the Institute. They shall, under the direction of the Council, conduct the ordinary correspondence and other ordinary business of the Institute; and one at least of them shall attend the General Meetings and Meetings of the Council, and prepare minutes of proceedings in connection therewith.

The Honorary Secretaries for any year shall be eligible for re-election for the ensuing year: provided always that no Fellow shall be elected Honorary Secretary more than four years in succession after the commencement of these Bye-Laws.

The Honorary Secretaries holding office at the commencement of these Bye-Laws shall continue in office until the first Annual General Meeting held under these Bye-Laws.

The Auditors.

36. The Auditors, three at least in number, shall be elected every year, at the Annual General Meeting, from the Associates of the Institute. The Auditors elected at an Annual General Meeting shall, prior to the next Annual General Meeting, audit the annual statement of receipts and expenditure and of liabilities and assets prepared by the Council in pursuance of these Bye-Laws, and shall verify and sign the same; and present a special report thereon to the Annual General Meeting, whenever it may appear to them proper and desirable to do so.

Not more than two of the Auditors for any year shall be eligible for re-election for the ensuing year: provided always that no Associate shall be elected Auditor more than three years in succession after the commencement of these Bye-Laws.

The Auditors holding office at the commencement of these Bye-Laws shall continue in office until the first Annual General Meeting held under these Bye-Laws.

37. The Officers of the Institute going out of office at an Annual General Meeting shall be deemed to be in office until the Officers of the Institute for the ensuing year have been elected in their respective places.

V.—CLASSES OF MEMBERS, THEIR RIGHTS AND PRIVILEGES.

Fellows.

38. No person shall be eligible as a Fellow who has not attained the age of twenty-one years.

39. From the date of the Charter and until the expiration of five years from the date thereof, a person shall be eligible as a Fellow on any of the following grounds, and in respect of the first three, whether he is already an Associate or Student or not, namely:—

(*a*) That he has practised as an Actuary for upwards of ten years in the United Kingdom of Great Britain and Ireland, or in one or other of its Colonies or Dependencies.

(*b*) That he is the Actuary or Assistant-Actuary, or the principal officer, if responsible for the actuarial work, of some Life Assurance, Annuity, or Reversionary Interest Society, or other analogous institution, having its Head-Office in the United Kingdom of Great Britain and Ireland, or in one or other of its Colonies or Dependencies.

(*c*) That he is in the service of the Crown in the capacity of Actuary or Assistant-Actuary of some department of the Government.

(*d*) That he has passed the Examinations declared by the Rules of Examination to appertain to the class of Fellows.

40. After the expiration of five years from the date of the Charter, no person, whether already an Associate or Student or not, shall be eligible as a Fellow unless he shall have passed the Examination or Examinations declared by the Rules of Examination to appertain to the Class of Fellows. Nevertheless, it shall be in the power of the Council to dispense with the aforesaid Examination or Examinations in the case of a candidate possessing one or other of the qualifications respectively set out in paragraphs (*a*), (*b*), and (*c*) of the previous Section, provided that not less than one-half of the whole Council shall vote, in person or by proxy, in favour of such dispensation.

41. Every Fellow shall be eligible as a member of the Council, and for appointment to all offices of the Institute, and entitled to all its rights and privileges, subject to the provisions of the Charter and of these Bye-Laws.

Associates.

42. No person shall, after the commencement of these Bye-Laws, be eligible as an Associate, who has not attained the age of twenty-one years.

43. From the date of the Charter, and until the expiration of three years from the date thereof, a person shall be eligible as an Associate on either of the following grounds, and, in respect of the first mentioned, whether he is already a Student or not, namely:—

(*a*) That his experience in matters relating to the profession of Actuary is such as to render him, in the opinion of the Council, worthy of election to the class of Associates.

(*b*) That he has passed the Examination or Examinations declared by the Rules of Examination to appertain to the class of Associates.

44. After the expiration of three years from the date of the Charter, no person, whether already a Student or not, shall be eligible as an Associate, unless he shall have passed the Examination or Examinations declared by the Rules of Examination to appertain to the class of Associates. Nevertheless, it shall be in the power of the Council to dispense with the aforesaid Examination or Examinations in the case of any person, whether a Student or not, of experience in matters relating to the profession of Actuary, provided that not less than one-half of the whole Council shall vote, in person or by proxy, in favour of such dispensation.

45. Every Associate shall be eligible for appointment to such offices of the Institute as Bye-Laws prescribe, and entitled to all its rights and privileges, subject to the provisions of the Charter and of these Bye-Laws.

Students.

46. No person shall be eligible as a Student who has not attained the age of sixteen years.

47. No person shall be deemed eligible as a Student unless he shall have given such evidence of his educational fitness as the Council shall from time to time prescribe; and it shall be in the power of the Council to require, in such cases as it may think fit, that a person applying to become a Student shall pass the Examination or Examinations declared by the Rules of Examination to appertain to such cases.

48. All Students shall have the privilege of attending Ordinary General Meetings of the Institute, but shall not be qualified to vote thereat, or in connection therewith. Students attending an Ordinary General Meeting may, on the invitation of the President or other Chairman for the time being, take part in the discussions following the reading of papers, memoirs, or other communications. Students may not attend any Annual or Special General Meetings, or have any right of voting in connection therewith. Subject always to such conditions and limitations as the Council may from time to time think fit to prescribe, they shall be entitled to the use of the Library, and of the accommodation afforded by the rooms or offices of the Institute.

Honorary Members.

49. A person who, either on account of his position, or of his eminence in science and his experience in matters relating to the profession of an Actuary, appears to the Council to be able to render assistance in promoting the objects of the Institute, and who is not professionally engaged in practice as an Actuary, shall be eligible as an Honorary Member. No such person shall be recommended by the Council for election as an Honorary Member unless at least one-half of the whole Council shall vote, in person or by proxy, in favour of such recommendation.

50. All Honorary Members shall have the privilege of attending Ordinary General Meetings, and may take part thereat in the discussions following the reading of papers, memoirs, or other contributions, but shall not be qualified to vote. They shall be entitled to make use of the Library, and of the accommodation afforded by the rooms or offices of the Institute.

Corresponding Members.

51. A person who is resident in any foreign country, and who is professionally engaged in practice as an Actuary, and appears to the Council to be able to render assistance in promoting the objects of the Institute, shall be eligible as a Corresponding Member. No such person shall be recommended by the Council for election as a Corresponding Member, unless at least one-half of the whole Council shall vote, in person or by proxy, in favour of such recommendation.

52. Corresponding Members shall have, subject to the provisions of the Charter and of these Bye-Laws, the same rights and privileges as Honorary Members.

VI.

A.—ELECTION AND ADMISSION OF FELLOWS AND ASSOCIATES.

53. Except in the cases hereafter mentioned in Sections 58 and 59, Fellows and Associates shall be elected by ballot at Ordinary General Meetings, and the proportion of votes required for election shall be at least three-fourths of the numbers voting, provided always that only Fellows shall vote in the election of Fellows.

54. A person desirous of being admitted as a Fellow or Associate of the Institute, shall be proposed and recommended, according to a form to be prescribed by the Council, in which form the full name, place of business (if any), usual residence and qualifications of the candidate shall be distinctly specified, and also the class to which admission is sought. The form shall be signed by the candidate and, in the case of proposal for admission to the class of Fellow, by at least four Fellows; and, in the case of proposal for admission to the class of Associate, by two Fellows and by at least two other Members who must be either Fellows or Associates; and in all cases of proposal for admission to either the class of Fellow or Associate, all the Members signing the form shall certify to their personal acquaintance or general knowledge of the candidate.

55. The proposal so made shall be delivered to the Honorary Secretaries of the Institute, and submitted by them to the next Meeting of Council; and if the Council shall find the candidate qualified according to Bye-Laws, and the proposal for admission be approved by the Council, the Chairman of the Council shall sign the proposal, which shall be read at the first following Ordinary General Meeting, and then be suspended in some conspicuous place in the building used by the Institute, until the next Ordinary General Meeting, at which latter the candidate shall be balloted for.

56. The signatures to the form of proposal may be dispensed with, provided that at least one-half of the whole Council shall vote, in person or by proxy, in favour of such dispensation; and in the event of the signatures being thus dispensed with, the form of proposal shall state that the candidate is recommended for election by the Council.

57. In the event of the candidate being elected, he shall be required to sign the form of obligation to be prescribed by the Council, and at the same time to pay the entrance-fee and subscription prescribed by these Bye-Laws.

58. In the event of a Student having passed the Examination or Examinations declared by the Rules of Examination to appertain to the class of Associate, the Council shall, upon his making application in a form to be prescribed by the Council, and upon payment of the additional subscription prescribed by these Bye-Laws, cause him to be transferred to the class of Associate, subject to the provisions of the Charter and of these Bye-Laws.

59. In the event of an Associate or Student having passed the Examination or Examinations declared by the Rules of Examination to appertain to the class of Fellows, the Council shall, upon his making application in a form to be prescribed by the Council, and upon payment of the additional subscription prescribed by these Bye-Laws, cause him to be transferred to the class of Fellow, subject to the provisions of the Charter and of these Bye-Laws.

B.—ELECTION OF HONORARY AND CORRESPONDING MEMBERS.

60. A person recommended by the Council for election as Honorary or Corresponding Member, shall be proposed in a form to be prescribed by the Council, and the proposal shall be read at an Ordinary General Meeting, and then be suspended in some conspicuous place in the building used by the Institute, until the next Ordinary General Meeting, at which latter the person recommended shall be balloted for, and the proportion of votes required for election shall be at least three-fourths of the numbers voting.

C.—ADMISSION OF STUDENTS.

61. A person desirous of being admitted into the Institute as a Student shall make application according to a form to be prescribed by the Council; and such form shall be signed by the applicant and two Members of the Institute, who must be either Fellows or Associates.

62. The application shall be delivered to the Honorary Secretaries of the Institute, and submitted by them to the next Meeting of the Council, and if approved by the Council, the applicant shall be considered to have been admitted a Student after he has signed the form of obligation to be prescribed by the Council and paid the fee and subscription prescribed by these Bye-Laws; provided always that in such cases as the Council may think fit, the applicant shall, before signing the said form, be required to pass the Examination or Examinations declared by the Rules of Examination to appertain to the admission of Students.

D.—REINSTATEMENT OF MEMBERS.

63. In the event of a person who has been a Member, and who has from some cause other than expulsion ceased to be a Member, making a written application to the Council expressing his desire to be reinstated a Member of the class to which he formerly belonged, the Council may, if it think fit, cause him to be so reinstated, upon such terms and conditions as it may think fit.

VII.—EXAMINATIONS.

64. Subject to the provisions of the Charter and these Bye-Laws, and for the purpose of promoting the objects of the Institute, the Council shall cause such Examinations to be held as it may think fit, and shall prepare and publish Rules to regulate such Examinations, and to define the cases and circumstances under which the said Examinations shall severally apply, the periods at which they shall be held, the subjects which they shall respectively comprise, the fees, if any, which shall be paid or deposited by candidates in respect of such Examinations, and the nature of the certificates, if any, to be granted to successful candidates; and the Council shall from time to time vary or rescind any of the said Rules of Examination, or add thereto, in any such manner as it may from time to time think fit.

VIII.—FEES AND SUBSCRIPTIONS.

65. Every person making application to be admitted into the Institute as a Student shall, at the time of making such application, pay a fee of one guinea, or such other sum as the Council may from time to time prescribe, not exceeding the entrance fee of Associates.

Every Student shall pay an annual subscription of one guinea for the year current of admission and subsequent years, or such other sum as the Council may from time to time prescribe, not exceeding that for the class of Associates.

66. Every Associate elected after the commencement of these Bye-Laws shall pay an entrance-fee of two guineas, or such other sum as the Council may from time to time prescribe.

Every Associate transferred from the class of Student under Section 58 shall pay no entrance-fee upon such transfer.

Subject to the provisions of the Charter, every Associate at the date of the Charter, or elected subsequent thereto but prior to the commencement of these Bye-Laws, shall pay an annual subscription of one or two guineas respectively, according as he does or does not reside more than thirty miles from the building for the time being used by the Institute.

Every Associate elected after the commencement of these Bye-Laws, or transferred from the class of Student under Section 58, shall pay an annual subscription of two guineas for the year current of election or transfer and subsequent years.

67. Every Fellow elected after the commencement of these Bye-Laws shall pay an entrance-fee of five guineas, or such other sum as the Council may from time to time prescribe.

Every Fellow transferred from the class of Students or Associates under Section 59 shall pay no entrance-fee.

Subject to the provisions of the Charter, every Fellow who has become a Fellow prior to the commencement of these Bye-Laws shall pay an annual subscription of two or three guineas respectively, according as he does or does not reside more than thirty miles from the building for the time being used by the Institute.

Every Fellow elected after the commencement of these Bye-Laws, or transferred from the class of Students or Associates under Section 59, shall pay an annual subscription of three guineas for the year current of election or transfer and subsequent years.

68. The terms, if any, upon which Fellows, Associates, and Students may compound for payment of their future subscriptions to the Institute shall be such as the Council may, if it think fit, from time to time prescribe.

69. Subscriptions shall be payable yearly in advance, and shall be due and payable on the first day of October in each year; and in the event of a Member failing to pay his subscription within two months of that date, he shall by such failure be suspended from all rights and privileges of membership appertaining to the class to which he belongs; and if such subscription be not paid before the 31st day of March of the following year he may be declared by the Council a defaulter, and if so declared shall cease to be a Member of the Institute.

70. Until a Member shall have paid up all his arrears, and given notice of resignation in writing to one or other of the Honorary Secretaries, he shall be considered liable for all subscriptions due from him under these Bye-Laws.

IX.—EXPULSION OF MEMBERS.

71. In the event of a charge of improper conduct on the part of a Member being made in writing, signed by at least ten Fellows in the case of the said Member being a Fellow or Honorary Member or Corresponding Member, and by at least ten Members who are either Fellows or Associates in the case of the said Member being an Associate or Student, and delivered to one or other of the Honorary Secretaries, the Honorary Secretaries shall summon a Special Meeting of the Council to consider the said charge; and if the Council shall be of opinion that the same has sufficient foundation, and is of such a nature as to require investigation, it shall cause the Honorary Secretaries, or one of them, to send to the Member so charged a statement in writing of the charge against him, and shall summon another Special Meeting of the Council to consider his explanation. At least twenty-one days' notice in writing of such Meeting shall be given to the said Member in order that he may be present at such Meeting and be heard if he think fit, and if the Council at such Meeting or any adjournment thereof shall decide that the charge is, in its opinion, established, and that it would be discreditable to the Institute that the Member so charged should continue a Member of the Institute, it shall call a Special General Meeting, at which a ballot shall be taken among the Fellows present only, or among the Fellows and Associates present, according as the Member so charged is in the first case a Fellow or Honorary Member or Corresponding Member, and in the second case an Associate or Student, and in the event of not less than thirty Fellows in the first case, or thirty Fellows and Associates in the second case, being present and voting, and three-fourths of the votes being given for the said Member's expulsion, the Chairman of the Meeting shall declare that the Member charged is no longer a Member of the Institute, and his name shall be forthwith erased from the List of Members.

In the event of a Member's expulsion the Council shall be at liberty to cause notice thereof to be published in such newspapers or journals as it may select.

X.—GENERAL MEETINGS.

Annual General Meetings.

72. The Annual General Meeting shall be held on the first Saturday in June in each year, or on such other day in June, and at such time and place, as the Council may determine, subject to a fortnight's notice being sent to each Fellow and Associate. At the Annual General Meeting the Council shall submit their report on the affairs of the Institute and the past year's transactions, together with the statement of receipts and expenditure of the Institute for the year ending the previous 31st of March, and of its liabilities and assets at that date, duly verified and signed by the Auditors, and the Auditors' Report, if any.

73. At the Annual General Meeting shall be elected, in manner provided by these Bye-Laws, the President, Vice-Presidents, Treasurer, Honorary Secretaries, and other Members of Council for the ensuing year, and the Auditors.

74. At the Annual General Meeting may be considered questions relating to the direction and management of the affairs of the Institute, provided at least fourteen days' notice, specifying the nature of the questions to be considered, has been given by the Council; and the Council shall be bound to give such notice upon receiving, at least one month before the Annual General Meeting, a requisition to do so signed by at least five Members, either Fellows or Associates; but no Annual General Meeting shall have power to make, alter, revoke or dispense with, any Bye-Law whatever.

Ordinary General Meetings.

75. Ordinary General Meetings shall be held at such times and places as the Council may from time to time appoint.

76. The business of the Ordinary General Meetings shall be the election and admission of Members in manner prescribed by these Bye-Laws. Reports, Letters, and Papers, on subjects interesting to the Institute may be read, and points of theoretical or practical interest to the profession discussed, with the sanction of the Chairman of the Meeting; but no question as to the direction and management of the affairs of the Institute shall be considered, except such as may arise out of the confirmation of the Minutes of the Annual General Meeting, or of any Special General Meeting; nor shall any question be discussed or motion made having reference to the making, altering, revoking, or dispensing with any Bye-Law.

77. Fellows and Associates shall have the privilege of introducing strangers at the Ordinary General Meetings, under such regulations as the Council shall from time to time prescribe.

Special General Meetings.

78. The Council may at any time call a Special General Meeting, and shall, subject to the provisions of these Bye-Laws, be bound to do so upon a requisition in writing, signed by at least ten Members who are either Fellows or Associates, specifying the purpose for which the Special General Meeting is to be called. Subject to the provisions of these Bye-Laws, no business shall be transacted at a Special General Meeting unless at least twenty Members be present, and entitled to vote; and in the event of this number not being present within half an hour of the time appointed for the meeting, the meeting shall be thereupon and thereby dissolved.

79. Subject to the provisions of these Bye-Laws, a fortnight's notice shall be given by the Council to each Fellow and Associate of the time and place at which the Special General Meeting is to be held, and of the business to be dealt with thereat; and no business except that named in the notice shall be considered at such Special General Meeting.

General Meetings, whether Annual, Ordinary, or Special.

80. At all General Meetings of the Institute, whether Annual, Ordinary, or Special, the President of the Institute for the time being shall be Chairman; and in his absence, the senior Vice-President present; and in the absence of the President and Vice-

Presidents, a Member of the Council chosen by the Meeting; and in the absence of all the Members of the Council, a Fellow of the Institute chosen by the Meeting.

81. The non-receipt by any Member or Members of notice of any General Meeting shall not invalidate the proceedings of the meeting to which such notice relates.

82. Subject to the provisions of these Bye-Laws the Chairman of any General Meeting may, with the consent of the meeting, adjourn the meeting from time to time, and from place to place; but no business shall be transacted at any adjourned meeting other than the business left unfinished at the meeting from which the adjournment took place. Unless otherwise directed in the resolution for adjournment, no notice need be given of an adjourned meeting.

83. Subject to the provisions of the Charter and of these Bye-Laws, every resolution or amendment proposed and seconded at a General Meeting shall be put to the meeting by the Chairman, and decided on by show of hands; and the declaration of the Chairman as to the majority shall be final. On such declaration being made, a poll may be demanded in writing by at least ten Members present and entitled to vote at the meeting; but a poll shall not be taken as to the election of a Chairman, the appointment of Scrutineers, or the adjournment of a meeting.

84. On a poll being so demanded, the Chairman shall forthwith reduce the resolutions or amendments, on which the poll is to be taken, into the form of alternative propositions, or otherwise state the same in such a way as best to take the sense of the Members upon the substantial question or questions contained in such resolutions or amendments. Voting papers containing such propositions shall be then issued by the Council, within seven days after the meeting, and shall be returnable so as to be received by the Council within fourteen days after the meeting.

85. The demand of a poll shall not prevent the continuance of a meeting for the transaction of any business other than that on which the poll was demanded.

86. Subject to the provisions of the Charter and of these Bye-Laws, at every General Meeting at which a poll is demanded, the Meeting shall appoint three of their number as Scrutineers, any two of whom shall be competent to act. The Scrutineers shall meet not less than fifteen nor more than eighteen days after the Meeting; and they, or any two of them, shall draw up a report of the result of the voting, stating (among other things) what voting papers have been rejected wholly or in part and for what reason. Such report shall without delay be forwarded to the Council, and the voting shall take effect from the date of the report of the Scrutineers, except in the case of a meeting adjourned to receive the report of the Scrutineers, in which case the voting shall take effect from the date of the adjourned meeting, and such report shall be final and conclusive as to the result of the voting.

87. Minutes shall be made in proper books, to be provided for the purpose, of all resolutions and proceedings of General Meetings; and every Minute signed by the Chairman of the Meeting to which it relates, or by the Chairman of a subsequent Meeting, shall be sufficient evidence of the facts therein stated.

XI.—ENACTMENT OR ALTERATION OF BYE-LAWS.

88. The Council, when it may consider it expedient to propose the enactment of any new Bye-Law, or the alteration of any existing Bye-Law, shall call a Special General Meeting of Fellows and Associates to decide on the same; and the Council shall be at all times bound to call such Special General Meeting on a requisition in writing signed by at least ten Members who are either Fellows or Associates. The notice calling such Special General Meeting shall give particulars of the new Bye-Laws proposed and of the proposed alterations of existing Bye-Laws, and the voting at such Special General Meeting shall be by ballot.

XII.—INDEMNITY OF COUNCIL AND OFFICERS OF THE INSTITUTE.

89. The Members of the Council, Treasurer, Honorary Secretaries, Auditors, and other officers mentioned in these Bye-Laws, shall be indemnified by the Institute from all losses and expenses incurred by them in or about the discharge of their respective duties, except such as happen from their own respective wilful default or neglect.

90. No Member of the Council, Treasurer, Honorary Secretary, Auditor, or other officer mentioned in these Bye-Laws, shall be liable for any other Member of the Council, Treasurer, Honorary Secretary, Auditor, or other officer, or for joining in any receipt or document, or for any act of conformity, or for any loss or expense happening to the Institute, unless the same happen from his own wilful default.

The Library of the Institute.

THE Council of the Institute are desirous of making the Library as complete and representative as is practicable, and in this object they invite the cordial co-operation of the members. They will be glad if members will take an interest in the subject, and will communicate upon the following points from time to time with the Honorary Secretaries, namely:

i. Offering suggestions — having regard to reasonable limits of cost—for effecting the object desired by the purchase of books not at present in the Library;

ii. Sending, for preservation in the Library, extracts of a valuable character from papers (furnishing the full name and date) relating to life and accident assurance and to friendly societies;

iii. Mentioning (and, if possible, presenting) papers, pamphlets and books, with necessary particulars, of a valuable nature, which deal with the subjects just specified;

iv. Mentioning—and this suggestion especially applies to corresponding members, and foreign and colonial members—

  (*a*) What Government or State papers relating to subjects cognate to those pursued by the Institute are issued; when they are issued; how they can be obtained; and at what cost:

  (*b*) What local societies exist of a similar or cognate character to that of the Institute; whether they publish transactions, and when; and whether it would be desirable to enter into friendly relations with them by the interchange of Journals.

v. Donations to the Library of suitable books, papers and pamphlets, will be gladly received and acknowledged in the *Journal.*

It is proposed in future issues to keep the subject before the attention of members by a special print on the cover of each number of the *Journal.*

ED. *J.I.A.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Method of estimating Expected Deaths and Expected Claims; with observations upon certain modes of deducing the number exposed to the risk of death. By Gerald Hemmington Ryan, *Fellow of the Institute of Actuaries, and of the Statistical Society; Actuary of the Marine and General Mutual Life Assurance Society.*

[Read before the Institute, 31 January 1887.]

I.—On Expected Deaths and Expected Claims.

It is a common thing to find in the reports of life assurance companies a statement to the effect that the actual deaths and claim payments have been under the expectation, or have represented a certain percentage of the expectation. The records of life assurance literature would appear to show that little attention has been given to the methods on which these comparisons are based, the only contribution to the *Journal* bearing directly on the point being a paper by Mr. G. M. Low (*J.I.A.*, xviii, 195). But as the matter possesses a considerable and growing importance, the following note has been prepared with the object of suggesting a method by which such estimates may be computed according to a plan which seems reasonable and defensible on scientific grounds.

In this country, it has been the invariable custom to construct tables of experience based upon the observations of lives or policies, and no investigation appears to have been made in which the experience is deduced on the basis of the amounts insured. In America, however, greater attention has been given to the rates of loss, and the elaborate tables giving the experience of 30 American offices, which were published some two or three years ago, were entirely based upon the rates of loss. If we consider the assumptions which underlie these two methods respectively, it is seen that many arguments may be brought forward in support of the adoption of tables based upon rates of loss. In the first place, it is patent that, as insurance companies are financial bodies, the rate of loss is the pre-eminently significant factor. It is not, that is to say, the rate of mortality in itself which determines the prosperity of a life office, but it is rather the extent and incidence of the annual death claims. Another hypothesis, for which there seems abundant reason, is that persons assured for large sums are subject to a different rate of mortality from those on whose lives only small sums have been assured. In spite of the crucial test of medical examination, it has been found by the American companies that among the larger policies a much heavier rate of loss prevails than among the smaller ones. This affords a strong indication of the fact that persons assuring their lives are able to exercise a material selection against the company. Many men, whose health and constitution are sufficiently good to enable them to pass a satisfactory medical examination, may yet have good reason for believing that their chances of longevity are below the average; and in these cases the incentive clearly is to effect larger policies of assurance than might otherwise be taken out. Against the plan of using the amounts assured in framing tables, it may be urged that, in employing the rates of loss as the basis of our tables of experience, we are using a factor which may fluctuate considerably, and the exact reproduction of which in the future may be somewhat doubtful. It may, further, be contended that there is nothing by which the actual results of the experience can be tested; whereas, in regard to the rates of mortality, we have always the fixed standard of the life tables, exhibiting the general mortality of the country, to serve as a guide. The objection on the score of larger probable fluctuations in the loss rates is likely to be urged with greater effect by companies in Great Britain than by kindred institutions in America, seeing that the latter, as a general rule, contain a broader mass of observations. But the advocates of the system in America forcibly contend that the experience in regard to death claims is just as likely to recur in the future as that in which account only is taken of the rate of mortality; also, that the rates of loss supply as safe and trustworthy a basis for future estimates as the rates of mortality.

Without entering into a discussion on these general questions, I may, perhaps, be allowed to suggest that the rates of loss might be employed with great advantage in comparisons of the actual with the expected claims. Hitherto, I believe, two systems have been variously followed in making such comparisons. The first, to take the expected deaths (computed in the ordinary way), multiplying the resulting number by the average amount of a policy; and the second, to take the amounts at risk at each age and multiply them by the corresponding rate of mortality. As conclusively demonstrated by Mr. G. M. Low in his paper already referred to, the first of these expedients is a very rough approximation; while, with regard to the second, which Mr. Low recommends for adoption, the alternative objections hold (1) that we are multiplying together functions of a heterogenous nature, or (2) that the assumption is involved that the amounts are subject to the same decremental force as the lives assured. The suggestion, then, that I would here make, is that while the expected deaths should be derived from the ordinary rates of mortality, the expected claims should be obtained in a similar way by employing the rates of loss. I am fully aware that this would necessitate the construction of tables of experience based upon amounts assured; but, in my humble opinion, the utility and value of such tables would be so great that I cannot but think that, in any future stock-taking of the experience of British offices, an attempt should be made to calculate them, in addition to the ordinary results.

A consideration of the practical bearing of the question under discussion induces the belief that the comparisons of actual and expected deaths and claims are, generally speaking, obtained in a somewhat rough manner. It is a matter of quite usual, indeed almost universal, occurrence, that the rates of mortality and death-claim experienced by a life assurance company should fall considerably short of the expectation. How this expectation is arrived at is not generally stated, and it may easily happen that the favourable result shown would sometimes disappear if a more exact standard were employed. For example, in a company transacting a very large new business, it is apparent that the H^M Table does not with sufficient accuracy represent its probable experience in regard to mortality. If we took such a company and computed the expected deaths, according to this table, we should virtually be exaggerating that quantity, since, owing to the recentness of the medical examination upon a considerable proportion of the observations, the H^M mortality would not really be that which such a company should anticipate. If, therefore, the actual deaths fairly coincided with the expected when computed on this basis, so far from the comparison being favourable to the company, as appearances would seem to indicate, in reality the experience could not be considered at all satisfactory. The most thorough means of deducing the true expected deaths would doubtless be to employ select tables of experience, such as those which Mr. Sprague has published in the pages of the *Journal* (*J.I.A.*, xxii, 394). But the employment of such a perfect instrument is attended with much labour; and on that account it may, not unreasonably, fail to pass into general adoption. I would accordingly suggest for the consideration of the members of the Institute, whether the results, which the more complete and detailed analysis would bring out, may not be approximately obtained in a far simpler way by the following process:

First, the lives at risk should be separated into two classes: (1) those that have been under observation for five years and upwards; and (2) those under observation for shorter periods. The probable deaths among the former body should then be computed by multiplying the numbers at risk by the rate of mortality according to the $H^{M(5)}$ Table, while those referable to the latter class should be ascertained by multiplying the lives exposed by the rate of mortality applicable solely to the first five years of assurance. The means of making the last-named calculation we already possess in the table which Mr. R. Teece contributed to the *Journal* (*J.I.A.*, xxii, 256) and which Mr. W. T. Gray afterwards independently calculated (*J.I.A.*, xxiv, 283). By following such a plan we should arrive at the number of deaths which might fairly be expected to arise, taking into account, in a broad and general way, the influence of selection which, presumably, would still be current on a large proportion of the data.

A further suggestion which I would, with deference, submit is that tables exactly similar to the $H^{M(5)}$ and $H^{M(0\text{-}4)}$ should be constructed on the basis of amounts assured, and that the rates of loss should be multiplied into the amounts at risk, duly classified in the manner already set out, in order to show the expected claims. I append a sketch of a form which will fully explain the process referred to. It will be seen that I have used the symbols $q_x^{(0-4)}$ and $q_x^{(5)}$ to denote the rate of mortality during the first five years of insurance, and after that period respectively; and that $r_x^{(0-4)}$ and $r_x^{(5)}$ have been employed to represent similar functions in regard to the rate of loss, q_x and r_x standing for the average rates.

Specimen Schedule for Calculation of Expected Deaths and Expected Claims.

Expected No. of Deaths						Expected Amount of Claims						
Lives at Risk (less than 5 years insured)	Rate of Mortality $q_x^{(0-4)}$	Expected Deaths (Entrants in last 5 years)	Lives at Risk (more than 5 years insured)	Rate of Mortality $q_x^{(5)}$	Expected Deaths (Entrants before last 5 years)	Amounts at Risk (less than 5 years' duration)	Rate of Loss $r_x^{(0-4)}$	Expected Claims (less than 5 years' duration)	Amounts at Risk (more than 5 years' duration)	Rate of Loss $r_x^{(5)}$	Expected Claims (more than 5 years' duration)	Age
												20
												25
												30
												35
												40
												45
												50
												&c.
												&c.
												Total

NOTE.—The ages might either be taken as the central ages of a quinquennial group, or weight could, if desirable, be given to the incidence of the observations.

In evidence of the fact that a marked difference has been found to exist in the experience of a life assurance company in regard, severally, to the rates of mortality and loss, I am able to produce the following figures bearing on the point. These results have been calculated from data supplied in the Mortality Experience of the Connecticut Mutual Life Insurance Company from its establishment up to the end of the year 1878—a valuable and instructive contribution to insurance statistics, which furnishes us with an additional measure of the mortality of assured lives in America. The facts have been arranged in decennial groups of ages, and the rates of mortality and loss have been computed for these periods of life.

TABLE *showing the Rates of Mortality and Loss calculated from the observations of the Connecticut Mutual Life Insurance Company.*

Ages at Exposure	RATES OF MORTALITY PER-CENT			RATES OF LOSS PER-CENT			Ages at Exposure
	$100q_x$	$100q_x^{(0-4)}$	$100q_x^{(5)}$	$100r_x$	$100r_x^{(0-4)}$	$100r_x^{(5)}$	
21–30	·680	·634	·940	·667	·603	1·067	21–30
31–40	·757	·687	·868	·757	·715	·874	31–40
41–50	·952	·853	1·024	·977	·930	1·086	41–50
51–60	1·634	1·531	1·672	1·717	1·579	1·784	51–60
61–70	3·281	2·319	3·420	3·532	2·126	3·802	61–70
71–80	7·000	5·128	7·018	7·103	3·108	7·138	71–80

The above figures appear important as giving an unmistakable indication of the difference which, under certain conditions, exists between the death rates and loss rates. Other results, further, could be quoted from the same source in corroboration of the fact, were there any necessity for confirmatory evidence. It will also be noticed that the variations between the several rates are not insignificant; and if the expected deaths and claims were first computed by applying the average values of q_x as given above, to the data of any life office, and then recalculated according to the method explained, which involves the employment of $q_x^{(0-4)}$, $q_x^{(5)}$, $r_x^{(0-4)}$ and $r_x^{(5)}$, there is every probability that the aggregate results would be found to vary considerably. The figures contained in the foregoing table give every facility for such an experiment being made, the outcome of which would, unquestionably, be full of interest.

I may be allowed to add that there is an inherent ambiguity lurking in the phrases "expected deaths" and "expected claims", since these quantities may sometimes be computed according to the mortality table adopted for the purposes of valuation, and sometimes, according to the truest available exponent of the mortality of assured lives. When it is found that a company has realized a certain amount of profit in an inter-valuation period, it is customary to trace out the various sources from which this surplus has arisen; and according to analysis it may appear that a considerable portion has arisen by reason of the prevalence of a favourable rate of mortality. In other words, the company may have incurred a smaller amount of claim disbursements than the valuation basis provided for. In such a case the saving on claims would be represented by the difference between the actual and the expected "strain by claims"—that is the amount of the claims less the reserves held against them—the expected strain being computed on the basis of the mortality table used in the valuation, according to the methods expounded by Mr. C. D. Higham (*J.I.A.*, xx, 153). There is, however, no conclusive reason why the true expected deaths and expected claims should depend upon the valuation table; on the contrary, they should, I am disposed to believe, be measured by a more trustworthy and exact instrument, whenever obtainable.

The existing methods, when applied to old companies, progressing at a steady and uniform rate, would probably be found to yield results sufficiently exact for practical purposes, since the distribution of the observations would not perhaps differ very widely from that of the H^M Table. In the case of companies whose development is taking place on a larger scale, it seems however a matter well open to discussion, whether more accurate processes should not be employed. Indeed, it is important that companies whose new annual business bears a high proportion to their total contracts should not, in this respect, employ the results of an average table, like the H^M. For their probable deaths would not only thereby be computed on a false basis, but the consequent exaggeration of such expectation—which would manifestly place their actual experience in an unduly favourable light—might have the dangerous result of inducing them to relax their strictness in admitting new members.

As a final remark, it is worthy of notice that in America a greater weight and prominence appear to be given to the results of a comparison of actual and expected deaths or claims than is the case in this country. Of this I lately met with a pertinent illustration in an advertisement of a certain company, appearing in a transatlantic insurance journal. The wording of the announcement was as follows: “A low rate of mortality “makes cheap insurance. Mortality experience from 1866 “to 1884 inclusive. Probable death losses according to the “American Experience Table of Mortality, the standard of the

“ ——— insurance department	(1,293 lives)	$3,952,293
“ Actual experience during period	(869 „)	2,773,695
“ Difference . . .	(424 lives)	$1,178,598.”

I need hardly point out, after what has been said in the foregoing pages, that if the contentions there adduced are entitled to support, the above comparison may be entirely fallacious. The new business of most American companies has, in recent years, shown a tendency to rapid expansion; and under these circumstances the use of an average table of experience for the measurement of their expected claims cannot be justified upon scientific grounds.

II.—ON CERTAIN METHODS OF DEDUCING THE NUMBER EXPOSED TO RISK.

My reference to Galloway's method of deducing the exposed to risk, in the discussion following Mr. Gray's paper which appears in the last volume of the *Journal* (p. 383), elicited the invitation from Mr. Manly that I should prepare a paper embodying my views on the subject. This step I felt a difficulty in taking for several reasons, not the least important being that the theme would be rather too slender a subject for treating in the ambitious form of a paper. But I now gladly take the opportunity of adding to the preceding contribution—dealing with a closely-allied subject—such observations on the question as may be necessary to illustrate the views I put forward, in a crude and incomplete form, on that occasion.

The methods employed respectively by Galloway, the Institute and Mr. Meikle (in his manipulation of the Scotch observations) may be expressed in the following formulas, the symbols used having the same significance as in Mr. Gray's paper already referred to.

1. *Galloway's Method.*

$$\begin{aligned} E_{x-\frac{1}{2}} &= \Sigma^{x+\frac{1}{2}}(n) - \Sigma^{x-\frac{1}{2}}(d+w+e) - \tfrac{1}{2}(w+e)_{x+\frac{1}{2}} \\ &= \Sigma^{x+\frac{1}{2}}(n) - \Sigma^{x-\frac{1}{2}}(f) - \tfrac{1}{2}(w+e)_{x+\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \qquad (1) \end{aligned}$$

writing (f) for $(d+w+e)$

This formula assumes that the limiting date of the experience is a single point of time, and that therefore the discontinuants and existing will have been under observation for half a year in the final, incomplete policy-year.

2. *Institute Method.* (*Mortality Experience*, p. 18.)

$$E_x = E_{x-1} + \tfrac{1}{2}(n_x + n_{x+1}) - \tfrac{1}{2}(w_x + w_{x+1}) - (d_x + e_x) \quad . \quad . \qquad (2)$$

This formula assumes that the observations are carried up to the close of a calendar year, and therefore that in the last calendar year the entrants and discontinuants are only under observation for half a year.

3. *Scotch Method.* (*J.I.A.*, xiii, 261.)

$$E_x=\Sigma^x(n)-\Sigma^x(f)+\tfrac{1}{2}(n_{x+1}-w_{x+1}) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This formula involves the same principles and assumptions as No. 2.

Leaving Galloway's method out of view for the moment, and considering the two other systems, the superiority of the Scotch over the English system will be obvious, and a practical application of each to selected figures will entirely strengthen the opinion thus formed. But by an apparent algebraical device we can reduce the Scotch formula to a simpler form, and at the same time indicate how the arithmetical process in applying the method can be materially abridged. For we can write in place of equation No. 3

$$E_x=\Sigma^x(n-f)+\tfrac{1}{2}(n_{x+1}-w_{x+1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By this formula we can arrive at the results obtained by Mr. Meikle, but with the saving of one step in the operation. An illustration will best show the advantage of the modified form of the expression.

Meikle's Formula (*J.I.A.*, xiii, 268).

Age x	n_x	f_x	$\Sigma^{x-1}(f)$	$\Sigma^{x-1}(n)$	Δ	$\frac{n_x-w_x}{2}$	E_{x-1}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
26	710	353	1,970	3,529	1,559	276·	1835·
27	878	470	2,323	4,239	1,916	345·	2261·
28	910	526	2,793	5,117	2,324	349·	2673·

It may be explained that Δ in col. 6 represents $\Sigma(n)-\Sigma(f)$; and $E_{x-1}=\Delta_x+\dfrac{n_x-w_x}{2}$.

Modification of Meikle's Formula (same figures).

Age x	n_x	f_x	$(n-f)_x$	$\Sigma^{x-1}(n-f)$	$\frac{n_x-w_x}{2}$	E_{x-1}
(1)	(2)	(3)	(4)	(5)	(6)	(7)
26	710	353	357	1,559	276·	1835·
27	878	470	408	1,916	345·	2261·
28	910	526	384	2,324	349·	2673·

It will be noticed that not only is one column saved in the modified process, but also that the large figures in cols. 4 and 5 of the original arrangement are not required.

By this shorter form of Mr. Meikle's method I have computed many tables of experience, and after testing other plans I am prepared to say that I have not found any one possessing the same advantages. The great flaws in the Institute scheme are its cumbrousness, want of clearness of comprehension, and the absence of facilities for verification. These shortcomings are not present in the same degree in the simplified form of Mr. Meikle's process; and I commend it to the thoughtful consideration of students of our science.

The Institute and Mr. Meikle's methods, however, presuppose an arrangement of the observations to have been made according to calendar years. The distinguishing feature of Galloway's method, on the other hand, is that the policies are followed through complete policy-years or years of insurance. This latter arrangement carries with it several material advantages, especially in that it enables the rate of mortality to be traced through the true years of insurance. Great confusion has, I believe, been occasioned by the invaluable statistics collected by the Institute of Actuaries being so prepared that the nominal first year of insurance —or year 0 of insurance, as it has been termed, conveniently but not perspicuously—embraced a period of six months only. The awkwardness cannot, however, be avoided when the facts are regarded in relation to calendar years of experience, and the only way out of the dilemma, as far as I can see, is to abandon the principle of calendar years in favour of that of policy years. It may be worth considering what steps should be taken, assuming it to be granted that Galloway's system may, in its main features, be followed. In the first instance, if the great facilities afforded by the use of cards in the preliminary classification of the facts are to be preserved, the form of card should accord with that used by the Scotch rather than that adopted by the English companies. The dates of entry and exit, the duration of the policy, and the age at exit, should be stated in years and months, as on the Scotch cards. The observations could then either be carried up to a common date, or each policy followed to its anniversary in any chosen calendar year. Under some circumstances the first course may be found preferable, but the latter, on the whole, appears more convenient and serviceable, as it secures the completion of so many full years of insurance upon all the policies existing at the close of the observations. A suitable variation would of course be required for the formula for deducing the number exposed to risk, according as the first or second arrangement was eventually determined upon. Thus the appropriate expression in each alternative case would be as follows:

Galloway's Method.

(1) Assuming observations to terminate on a fixed date, as for instance 31 December, in the final year,

$$E_{x-\frac{1}{2}}=\Sigma^{x+\frac{1}{2}}(n)-\Sigma^{x-\frac{1}{2}}(f)-\tfrac{1}{2}(w+e)_{x+\frac{1}{2}}$$

(*see* Formula 1, p. 256.)

(2) Assuming the observations to terminate on the completion of an exact number of policy-years in the limiting calendar year,

$$E_{x-\frac{1}{2}}=\Sigma^{x+\frac{1}{2}}(n)-\Sigma^{x-\frac{1}{2}}(f) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

on the supposition that *all the lapses and surrenders occur at the end of the policy-years.*

That is, $$E_{x-\frac{1}{2}}=\Sigma^{x-\frac{1}{2}}(n-f)+n_{x+\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The condition as to the incidence of the withdrawals made in the last given formula cannot, however, be endorsed or defended. The lapses occurring among policies effected at annual premiums would take place on the completion of the policy-years, and should be so represented on the experience cards. But not so the lapses among half-yearly, quarterly, and, I may now add, monthly assurances, which would obviously occur at odd times throughout the policy-year. The case of surrenders has also to be considered; and although the withdrawals from this cause would have a tendency to take place towards the end of the policy-year, and shortly before a further premium became payable, in annual premium assurances, the other classes of policies would again introduce a disturbing element; while financial necessities and personal circumstances of a variety of kinds would also lead to policies being surrendered at irregular dates. It would be essential, consequently, to amend the last formula by introducing a correction for the withdrawals. This should, I think, take the form of a deduction of the aggregate number of years—the unit of reference being a year of life—by which the actual number of years completed in the year of secession falls short of the total number that would be observed if withdrawals occurred at the close of an exact policy-year. Calling this correction Cw, the revised expression becomes,

$$E_{x-\frac{1}{2}}=\Sigma^{x-\frac{1}{2}}(n-f)+n_{x+\frac{1}{2}}-Cw_{x+\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (7)$$

In the course of time it might be found from experience that sufficient accuracy could be obtained by treating the correction as equal to one-third, or one-fourth, or some other fraction, of the withdrawals in the final year of observation. But it would be far more satisfactory, and would involve no further trouble of any consequence, to compute its precise value. A slight addition to the particulars contained on the experience cards would be necessary, the "withdrawal" card being as under:

No.______ **Amount £**______

Life Assured______

Character of Risk______

Date of Entry______ $1878^5/-$

" Exit by L. or S.______ $1886^6/-$

Complete Duration ... ... $8^1/-$ $(-\frac{11}{12})$

Age at Entry* 33

" Exit 41

Cause of Death______

Remarks______

* If ages completed were used in lieu of ages next birthday, as afterwards suggested, the entry should here be 32½, and the age at exit, 40½, the full number of years of duration being alone added to save confusion.

Here it is seen that the plan of stating the dates of entry and exit in years and months, as adopted in the Scotch form of card, is suggested, the convenience of which will be obvious on consideration. For the purpose of this note, it is sufficient to point out that by this means the correction for the withdrawals can be easily ascertained. Against the number denoting the "complete duration" of the policy, a negative quantity has been placed in the above specimen card, representing the number of months not completed by the policy in its final year of experience. Now since the unamended formula 6 (p. 259) assumes that each lapsed or surrendered assurance endures for the whole of its year of exit, the total must be taken of all the negative quantities appearing on each withdrawal card. The cards being assumed to have been sorted according to age at exit, this becomes a simple mental operation, and the value of the correction C*w* is easily obtained. In the instance given above, the complete duration is $8\frac{1}{12}$ and the original formula (No. 6) assumes that nine complete assurance years have been experienced. This is incorrect, and overstates the true duration by $\frac{11}{12}$ths, which fraction is therefore included with the negative sign prefixed, in parentheses, against the duration.

The method now suggested would, I believe, be more satisfactory and convenient than those usually employed in this country. To be placed to its credit is the important fact that the rate of mortality in the first and all successive years of insurance, would be an exact determinable function; while it may be doubted whether, in point of simplicity of application or facility of comprehension, the prevailing systems can claim any superiority.

The only other suggestion I would make on this occasion is that all the observations should be scheduled according to completed ages instead of current ages. I believe that students encounter a great obstacle in endeavouring to master this branch of their work, in the diverse arrangements made in tabulating the observations against each year of age. Taking that most valuable production, the Mortality Experience prepared by the Institute, itself, we find in one table, the data are classified according to the age next birthday, in another to the age last birthday, while the very same facts appear in different parts of the volume opposite different ages—compare, for example, the total "died" for ages 22, 23, and 24, on page 15 with the totals on page 146. In other tables, outside the Institute volume, the greater anomaly exists that the observations in some of the columns of a single table relate to the age next birthday, whereas those in other columns refer to the completed age. There is surely no reason for arrangements of this kind which create a confusion so easily avoided. Outside the prospectuses and office registers of life assurance companies, it would certainly, in my opinion, be greatly preferable to regard completed ages only; and the simplest alteration of the age-particulars, in copying them from the registers on to the statistical cards, would alone be necessary to enable the facts to be arranged in the way suggested with complete facility.

The suggestion last advanced would effect a slight modification in formula 7 given on page 259, which would take the following form:

$$E_{x+\frac{1}{2}}=\Sigma^{x-\frac{1}{2}}(n-f)+n_{x+\frac{1}{2}}-Cw_{x+\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (8)$$

or as x may be just as legitimately assumed to represent the successive "half-ages" as integral ages, we might write the expression:

$$E_x=\Sigma^{x-1}(n-f)+n_x-Cw_x \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

By the use of this formula, I venture to think that great advantages and facilities would be gained. In order to avoid any misconception of the process I recommend for trial and consideration, a table is appended in which the figures given in Galloway's Report on the Experience of the Amicable Society (Tables 1 and 2) are rearranged according to the suggested plan. It should be stated that Galloway does not give the number entering at each age in a separate column; but the series can be obtained directly from his first table.*

A glance at this table will show at once the extreme simplicity and clearness of the method now advocated. With a single exception, every column is checked by one of the preceding steps in the process—in fact, it may be termed a kind of "block system", which pulls us up at each column until the line has been cleared by the removal of any inaccuracies that may have crept into the results.

It remains for me only to add that, having obtained E_x for all half-age values of x, the values of the function at integral ages could be arrived at, either by Galloway's expedient of taking the arithmetical mean of each two adjacent terms of the series, or, as would under many circumstances be preferable, by employing some more accurate method of interpolation. In this event, the series, d_x, would have to be similarly and independently operated upon; but it would probably be found more expeditious and equally convenient to deal with the function q_x, derived direct from E_x and d_x. These matters are, however, somewhat beyond the scope of this note, which merely purports to call attention to a means by which the number exposed to the risk of mortality may appropriately be computed from rough observations.

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* For the purpose of completeness, the correction for the discontinued is introduced in this table, the value of one-third of the discontinued at each age being arbitrarily assigned to it.

EXPERIENCE OF THE AMICABLE SOCIETY.

Figures given by Galloway (Report of 1841: *Tables* 1 *and* 2*), re-arranged according to Formula* 9 *on page* 262.

Completed Age	Entered	Died	Discontinued	Existing	Total "Left" (Died, Discond. and Existing)	Excess of Entered over Left	Sum of Preceding Column to $x-1$	Total Number entering under observation at Age x	Correction for Discontinued	Exposed to Risk	Completed Age
x	n_x	d_x	w_x	e_x	f_x	$(n-f)_x$ or g_x	$\overset{x-1}{\Sigma}(g)$	$\overset{x-1}{\Sigma}(g)+n_x$	Cw_x	E_x	x
9½	1	...	...	...	...	+ 1	...	1	...	1·	9½
10½	4	...	1	...	1	3	1	5	·3	4·7	10½
11½	...	...	...	...	...	...	4	4	...	4·	11½
12½	4	...	...	1	1	3	4	8	...	8·	12½
13½	3	...	...	...	...	3	7	10	...	10·	13½
14½	1	...	...	...	...	1	10	11	...	11·	14½
15½	5	...	...	...	...	5	11	16	...	16·	15½
16½	4	...	...	2	2	2	16	20	...	20·	16½
17½	6	...	...	...	...	6	18	24	...	24·	17½
18½	4	...	2	1	3	1	24	28	·7	27·3	18½
19½	5	...	...	...	...	5	25	30	...	30·	19½
20½	10	...	1	3	4	6	30	40	·3	39·7	20½
21½	22	1	3	1	5	17	36	58	1·	57·	21½
22½	30	1	2	4	7	23	53	83	·7	82·3	22½
23½	46	...	5	6	11	35	76	122	1·7	120·3	23½
24½	62	1	5	4	10	52	111	173	1·7	171·3	24½
25½	64	1	7	8	16	48	163	227	2·3	224·7	25½
26½	83	...	18	5	23	60	211	294	6·	288·	26½
27½	75	3	14	9	26	49	271	346	4·7	341·3	27½
28½	103	2	11	17	30	73	320	423	3·7	419·3	28½
29½	102	2	13	19	34	68	393	495	4·3	490·7	29½
30½	124	3	16	24	43	81	461	585	5·3	579·7	30½
31½	124	6	20	29	55	69	542	666	6·7	659·3	31½
32½	134	5	13	23	41	93	611	745	4·3	740·7	32½
33½	123	2	19	37	58	65	704	827	6·3	820·7	33½
34½	126	8	21	30	59	67	769	895	7·	888·	34½
35½	136	7	18	45	70	66	836	972	6·	966·	35½
36½	129	8	19	51	78	51	902	1,031	6·3	1,024·7	36½
37½	119	12	18	47	77	42	953	1,072	6·	1,066·	37½
38½	117	8	11	49	68	49	995	1,112	3·7	1,108·3	38½
39½	129	10	18	58	86	43	1,044	1,173	6·	1,167·	39½
40½	136	12	15	73	100	36	1,087	1,223	5·	1,218·	40½
41½	114	15	14	48	77	37	1,123	1,237	4·7	1,232·3	41½
42½	93	10	9	62	81	12	1,160	1,253	3·	1,250·	42½
43½	103	16	21	73	110	− 7	1,172	1,275	7·	1,268·	43½
44½	107	15	17	65	97	+10	1,165	1,272	5·7	1,266·3	44½
45½	83	11	16	74	101	−18	1,175	1,258	5·3	1,252·7	45½
46½	90	14	16	70	100	10	1,157	1,247	5·3	1,241·7	46½
47½	68	21	8	56	85	17	1,147	1,215	2·7	1,212·3	47½
48½	74	20	17	71	108	34	1,130	1,204	5·7	1,198·3	48½
49½	77	15	8	63	86	9	1,096	1,173	2·7	1,170·3	49½
50½	61	12	16	87	115	54	1,087	1,148	5·3	1,142·7	50½
51½	69	24	9	66	99	30	1,033	1,102	3·	1,099·	51½
52½	52	16	6	57	79	27	1,003	1,055	2·	1,053·	52½
53½	65	14	9	68	91	26	976	1,041	3·	1,038·	53½
54½	57	24	13	67	104	47	950	1,007	4·3	1,002·7	54½
55½	49	25	13	64	102	53	903	952	4·3	947·7	55½

EXPERIENCE OF THE AMICABLE SOCIETY.

Figures given by Galloway (*Report of* 1841: *Tables* 1 *and* 2), *re-arranged according to Formula* 9 *on page* 262—(*continued*).

Completed Age	Entered	Died	Discontinued	Existing	Total "Left" (Died Discond. and Existing)	Excess of Entered over Left	Sum of Preceding Column to $x-1$	Total Number entering under observation at Age x	Correction for Discontinued	Exposed to Risk	Completed Age
x	n_x	d_x	w_x	e_x	f_x	$(n-f)_x$ or g_x	$\overset{x-1}{\Sigma}(g)$	$\overset{x-1}{\Sigma}(g)+n_x$	Cw_x	E_x	x
56½	43	22	4	46	72	29	850	893	1·3	891·7	56½
57½	27	19	5	66	90	63	821	848	1·7	846·3	57½
58½	40	20	8	45	73	33	758	798	2·7	795·3	58½
59½	35	20	4	41	65	30	725	760	1·3	758·7	59½
60½	31	29	6	55	90	59	695	726	2·	724·	60½
61½	35	23	2	37	62	27	636	671	·7	670·3	61½
62½	23	16	3	44	63	40	609	632	1·	631·	62½
63½	18	24	1	48	73	55	569	587	·3	586·7	63½
64½	15	27	...	40	67	52	514	529	...	529·	64½
65½	17	23	5	29	57	40	462	479	1·7	477·3	65½
66½	18	16	1	35	52	34	422	440	·3	439·7	66½
67½	10	17	2	27	46	36	388	398	·7	397·3	67½
68½	9	24	...	25	49	40	352	361	...	361·	68½
69½	4	23	...	32	55	51	312	316	...	316·	69½
70½	5	29	...	14	43	38	261	266	...	266·	70½
71½	7	11	...	12	23	16	223	230	...	230·	71½
72½	...	16	1	18	35	35	207	207	·3	206·7	72½
73½	...	11	...	15	26	26	172	172	...	172·	73½
74½	...	14	...	11	25	25	146	146	...	146·	74½
75½	...	18	...	9	27	27	121	121	...	121·	75½
76½	...	9	...	7	16	16	94	94	...	94·	76½
77½	...	9	1	9	19	19	78	78	·3	77·7	77½
78½	...	8	...	6	14	14	59	59	...	59·	78½
79½	...	5	...	4	9	9	45	45	...	45·	79½
80½	...	8	...	5	13	13	36	36	...	36·	80½
81½	...	3	...	1	4	4	23	23	...	23·	81½
82½	...	1	...	1	2	2	19	19	...	19·	82½
83½	...	1	...	3	4	4	17	17	...	17·	83½
84½	...	3	...	3	6	6	13	13	...	13·	84½
85½	...	4	...	...	4	4	7	7	...	7·	85½
86½	...	...	...	1	1	1	3	3	...	3·	86½
87½	...	...	...	1	1	1	2	2	...	2·	87½
88½	...	1	...	...	1	1	1	1	...	1·	88½
Total	3,530	798	505	2,227	3,530	...	36,605	40,135	168·3	39,966·7	Total

DISCUSSION.

The PRESIDENT (Mr. A. Day), in inviting discussion, said, that the modes of calculating the expectation of claims seemed to be extremely various, and were only to be compared with the many different ways in which, in the 6th Schedule of the return to the Board of Trade, the subject of the average rate of interest of insurance funds was treated. Directors seemed to have very curious ideas about the expectation of claims. If they were told the claims expected were so much, and the actual claims so much less, they imagined that all the difference was profit; but when, as sometimes happened, the actual claims were more than the expected, they did not so often see any mention of that fact in the annual reports. Mr. Ryan had criticized, with perfect fairness, the manner in which the Institute had prepared the first volume of its Tables of Mortality Experience, but there would come a time when a new investigation would have to be undertaken, and those who conducted the new investigation might have the pleasure of being themselves criticized by a future audience.

Mr. C. D. Higham said Mr. Ryan truly remarked on the different rates of claim among policies for large and small amounts, but he had startled them by saying that the large are the worse. Excepting large policies in connection with loan transactions, they would all have said off-hand that the contrary was the case, though it would not be so easy to give grounds for this opinion. Only one company had taken out the mortality in respect of policies for £5,000 and upwards, and this was found very much more favourable than the general rate. The results were not published, and, after all, they related to one office only. In America the results appeared to be different, and this question of death rates and claim rates became a very interesting one. They would certainly agree with Mr. Ryan that it would be very satisfactory when next the Institute investigated mortality experience for sums assured to be brought into review. He (Mr. Higham) also hoped that rates derived exclusively from reversionary bonuses might be computed, for selection would then be evident to its fullest extent. It would not trouble him that there were no similar records with which they could be compared, for they would, no doubt, have a running connection with the death percentages. The figures Mr. Ryan had given from the Connecticut Mutual were very interesting, but he was surprised he had not added those derived from the American Thirty Offices' Experience, in which the characteristics he spoke of might be more readily traced, and were free from the stigma of being only from one company's books. There was much to be learned on this subject. For instance, in the case of entries at the middle ages this selection did not appear to wear itself out, but the claim rates were higher than the death rates to the end. Confining attention solely to policy-years 0 to 4, and comparing the two percentages for those entering under age 30, the claim rates were the lower, while for the next few ages both rates were about the same; then the claim rates were higher till about 70, after which, speaking generally, they were less again in the Thirty Offices, and not very dissimilar to the death rates in the Connecticut Mutual. He thought less weight was to be given to the consciously bad life effecting a larger policy than would otherwise be the case, than to the general selection of the assured against the office. In spite of Mr. Ryan's remark as to the actual claims being generally reported less than the expected, he would, no doubt, recollect that when the claims had been bad, the subject would often be omitted from the report; nor had he stated that, as he believed was the case, the $H^{M(5)}$ was not generally used in preparing claim estimates. He (the speaker) thought that an all-round H^{M} estimate was sufficiently near the mark in most cases, as, indeed, the paper practically admitted. Substantial accuracy was all that was needed, and if Mr. Ryan's plan were adopted, some one else would insist on Mr. Sprague's Select Tables. All estimates were fallacious which did not take into account the duration of the policy, and the difference between sum assured and reserve value, which far more important question than the rates of claim and death Mr. Ryan had hardly touched on. They certainly wanted a new term for "expected" deaths; but he could not follow the suggestion that the estimates may be made by another mortality table than that employed in the valuation. He did not see the practical good of them, save for ear-marking profit. He would ask Mr. Ryan whether he was aware of any company actually using these claim rates in a valuation: that would be an interesting fact. It was to be remembered that it was the Americans who first worked out and published loss tables, and thereby opened up a most interesting subject of enquiry.

Mr. G. Todd agreed with Mr. Higham in the general view he took as to the question of the higher rate of mortality amongst policies of larger amount. He had looked into the American Experience Table, and it seemed that the actual claims were only about 4 per-cent in excess of those expected, using the rate of mortality as the function for calculating them. He had the results of a certain office in such a form that he could conveniently work out similar results, and they came out very much the same, his actual average claim being 3·588 in excess of the amount expected. There were various causes which might lead to this excess of mortality among lives assured for more than average amounts, and he thought it was not entirely due to selection on the part of these lives, inasmuch as the excess was spread pretty evenly over the whole duration of the Thirty Offices' experience. One cause might be the suspended mortality amongst the younger lives, due to medical selection, and the consequent excess amongst the older ones. The correct cause for this mortality was not very far to seek, if they considered that most offices, when they got a large case, were apt to deal rather leniently with it. In fact, the selection really emanated from their own side. He could not agree with Mr. Ryan's reasons for wishing to take out the experience of losses. It would certainly be interesting, and would afford material for consideration, as was shown in the American cases that had been utilized by Mr. Ryan in his paper. Supposing the rate of loss amongst policies and the rate of mortality among lives did not closely agree, they should examine into their method of selection to see that they were not taking bad lives amongst any particular class of policies. If they calculated all their expected losses from such a loss table as Mr. Ryan advocated they would miss that warning which he thought was valuable. Referring to the suggestion in the paper to employ the $H^{M(0-4)}$ and $H^{M(5)}$ Tables in the calculation of expected deaths, he did not suppose any actuary in taking out such results overlooked the bearing of such an element as an abnormally large new business. He might use the H^{M} Table, but he did so because it coincided pretty well with the experience of his office. He had been in the habit of taking out the claims for successive quinquennial periods of policy duration, and periodically checking them off with the basis table, and also taking out the percentages of business existing at each age, and if those did not run on a level with the H^M Table he should look about for a table that suited him better. The comparison of actual claims with the expected really taught them nothing; it was the "strain" which was the point to be investigated. Looking at the experience of the office to which he had before referred in that light, he found that in two or three years, although there might have been an excess of actual over expected claims, there was in fact an appreciable profit in each case when the incidence of the claims was given due effect to. He agreed with Mr. Ryan, that students found a considerable amount of difficulty in understanding the method adopted by the Institute in dealing with the first year of insurance, but he was not sure that it was altogether right to use policy years as the correct element in calculating the exposed to risk. In the daily course of their official duties they were dealing with calendar years, and the experience as tabulated by the Institute was really what they required. Year 0 embraced only about six months of risk, and therefore the Institute Table was well suited to supply the facts required for that period. He was not sure that if they were now to take the old plan of regarding the commencement of the risk as being spread evenly over the twelve months, and taking, therefore, the half-year as being the amount of risk run, they would not bring out results somewhat wide of the truth. He had found in one case that the business had only been in force on the average ·24 of a year at the close of year 0. From the same experience he had found that the lives assured had, at the date of commencement of the risk, completed on the average ·81 of their current year of existence, and if they considered that the lives assured attained their "age next birthday" upon the completion of the business year, they would seldom be more than a small fraction out. Mr. Ryan approved the Scotch form of card which took account of months. He (Mr. Todd) could not see why they should use months. He was in the habit of using two-figure decimals, which could be written down without any difficulty.

After a few remarks from Mr. ANSELL,

Mr. B. NEWBATT said that, notwithstanding the view expressed by Mr. Todd, he was not prepared to accept the dictum of Mr. Ryan that the large policies were less valuable pecuniarily than the small ones. *A priori* they would all confess that that was contrary to what they would expect. It was only natural that those who were better housed, fed, clothed, those who could take better care of themselves, those who, when their health broke down, could go to sanatoriums, could be away from their work, and could be sustained in every manner which art and science could bring to bear, should live longer than those not so fortunately situated. Some little time ago they had an invasion from one of their colonies, which had since been withdrawn, and he believed that the fact that they were not now meeting the competition of a powerful colonial company was, that in the colony in question it was believed that the British aristocracy were, from an assurance point of view, not good lives. He had himself seriously combatted the doctrine, though, in the absence of authoritative facts, this was a point on which no one could attempt to dogmatize. He believed that, with the exception of the statistics which had been brought from America, there were none that showed the effect of mortality experience accurately with regard to policies of large amount. From the rough experience that many of them had had they would, he believed, be prepared to say the big policies were not inferior in quality to the small ones. The way in which he himself had considered this question was this: observing over a long series of years, he had found that his average claim had been considerably below his average amount at risk. That was an indication of some value. He did not put it forward as a proof; but, in the absence of anything better, it justified him in saying that he believed that big policies, carefully selected, were the best risks.

Mr. A. H. **BAILEY** said he had been struck with the criticisms of Mr. Ryan on the mortality statistics collected by the Institute. He might be permitted to recall the experience of the President and himself (Mr. Bailey) in connection with those statistics, when, in their capacity of honorary secretaries, it devolved upon them to go round to the offices and endeavour to induce them to provide the materials on which that mortality experience was based. The older offices, whose materials were the most valuable, were often unwilling to take the trouble, and they met with some disappointment in one or two quarters where they had hoped otherwise. Of those offices which contributed, the only information derivable from the earlier records was almost without exception that given on the Institute card. They could not have given the materials on the Scotch card for the reason that in the earlier registers the date of birth was not recorded, but the age at entry only; nor the date of death, nor the date of lapse, but only the years of these occurrences. It followed that there was no choice but to adopt the system of calendar years, and the assumption had to be made, when they had only the age next birthday given, that each man completed his current age on the 31st December following. He took some trouble in one company to find out how far that differed from the truth, and he believed that six or seven weeks was all the error, which he did not think material. There were certain advantages in this method as against taking the policy years, namely, that the deaths, surrenders and lapses, were tolerably uniformly distributed throughout the calendar year. Surrenders were almost invariably before the renewal premiums fell due, and lapses of course were always so; and thus this method was hardly open to the observations made in the paper. He had always thought it was agreed that lives assured for the larger amounts were better than lives assured for the smaller. Several reasons had been mentioned, but there was this confirmation: the mortality of individual assurance societies had been published: and was it not the case with three or four—one in particular—in which the policies were considerably above the average amount, that the mortality had been singularly favourable? Mr. Ryan stated that when it was found that a company had realized a certain amount of profit in an inter-valuation period, it was customary to trace out the various sources from which this surplus had arisen. He (Mr. Bailey) was very glad to hear it was customary. It was a most useful proceeding, but he had not thought it was customary at all.

Mr. M. N. Adler was glad that Mr. Bailey had said a word in explanation of the manner in which the Institute mortality experience had been taken out. It was important that members should be reminded of the practical difficulties under which the committee laboured at the time in obtaining the ages. He had lately looked into the different methods adopted by assurance companies in determining the ages for valuation purposes, and he found that in taking office ages, which was, in point of fact, the course followed in the construction of the Institute tables, the true age was understated by about forty days. This discrepancy arose from the fact that many persons about to assure deferred doing so till near their birthdays. On the other hand, they were not justified in assuming in all cases that assurances are uniformly spread over the year. A larger number of assurances were, as they all knew, on the average, effected in the latter portion of the year. This was an item of some importance, which could not be disregarded in taking out the experience of a group of assurance offices.

Mr. Geo. King agreed with Mr. Higham in his remarks about the very limited usefulness of the mortality experience of individual offices: it was practically useful only for the purpose of ascertaining the sources of surplus. For various reasons the experience of any one company taken out from year to year was otherwise valueless. In the first place, when the business got beyond a certain age it was entirely out of their control. It was not much use to know exactly how the mortality went unless they meant to change their rates of premium; and they could not do that from the individual experience of one office. On the other hand, the new business of a company was comparatively so small, that it was liable to great fluctuation, so that they could not base much on the experience of one year in a single company. If they were to take out the experience at all it should be done properly. The H^M Table understated the mortality at the younger ages, and overstated it at the older ages. The result was, that although the mortality of the office might exactly coincide with the real H^M mortality if it were analyzed, the H^M Table would almost always show that the rate of mortality had been favourable. It was a curious fact that the Carlisle Table would always show a larger amount of expected claims than the H^M. In his opinion, tables analagous to Mr. Sprague's select tables should be used, and that could be done with very little extra trouble. A further point was, to what were they to apply the calculations? Were they only to use them for the whole of life cases, or were they to bring in the special policies? Special policies were becoming more common than they used to be, particularly endowment assurances; and it was unmistakable that some of those special classes had a lower rate of mortality than the whole life policies, and some a higher. It was curious that the higher the rate of premium payable on the policy, the better was the rate of mortality. He had spoken to various actuaries of experience, and they confirmed his opinion that a limited premium policy had a decidedly better mortality than a whole life, and that the shorter the term for which the premium was payable the better. He was disposed to agree with the view of other speakers that the large policies were of a better class than the smaller ones. But they should distinguish between large policies. The large policies due to the normal assuring of the well-to-do classes would show a good rate of mortality, but the large policies resulting from loans would show a heavier rate. His experience was that these loan cases were mostly rated up, and in order to make a true comparison they ought to rate up those cases in taking out the mortality experience. It was absurd to take a man of thirty, charge him as forty, and then test the rate of mortality by his true age, thirty. If they took the rated-up policies according to the ages for which they charged premiums, they would, he thought, find the same gain that they did with other large policies. That was a point upon which he should like an opinion—the proper way of dealing with rated-up cases in taking out mortality experience. With regard to the second part of Mr. Ryan's paper, for premium purposes he believed Galloway's method was the best. For valuation purposes the Institute Table perhaps fitted in more with their habits; but, at the same time, it made very little difference in the construction of a table for valuation purposes, whether they used Galloway's or the Institute method.

Mr. H. J. Rothery said, that the way in which rated-up policies should be dealt with in estimating the expected claims depended on the method adopted in the valuation. If, in valuing the policies the ages had been rated up, then the proper function to use would be the q_x for the rated-up age; but if, as was common in many offices, the policies were valued at the true ages, the extra premiums being treated as sufficient to cover the extra mortality from year to year, the proper way would be to ascertain the amount of the expected claims at the true ages, adding thereto the total income received from extra premiums during the year. Having examined the reports of eighty offices for the year 1886, he found the following results: In forty-five cases no statement was made as to whether the mortality was excessive or the reverse; in nine cases it was stated that the number of deaths was less than provided for by the mortality table; in ten cases the amount of claims was less; in two cases the claims were exceptionally heavy, having fallen on policies of more than average amount; in seven cases the claims were also stated to have been exceptionally heavy, but no details were given; in six cases the number of deaths was less than expected, but the amount of the claims was greater than expected; and in one case only it was said that the number of deaths was greater and the amount of the claims less than expected. Thus, out of a total of thirty-five cases in which information was given in such a form that he could make use of it, in no less than fifteen cases the claims were heavier than expected. If it were true that when the claims were greater than expected that fact would generally be suppressed, it might be assumed that in a considerable number of offices the claims had, in the year 1885 (to which the reports generally referred), exceeded the expectancy. An important feature of the statement was that regarding the offices where the number of deaths were less than expected, but the amount of claims greater than expected, which seemed to show that the mortality among larger policies had in those offices been greater than among smaller policies. The suggestion made by Mr. Ryan as to tracing the surplus was a very useful one: he implied that this was usually done, but the results were rarely, if ever, published. In many friendly society investigations an analysis of the growth of surplus had been made, and results of great interest discovered. As to the correction for the lapsed policies, they would have to consider the exact conditions of the office regarding the fixing of the date of lapse. It was sometimes usual to take the *due date* of the unpaid premium; but if the policies were entered as lapsed at the date the receipt was returned by the agent, say at the expiry of the days of grace, then in the last year of exposure the risk would endure one month only, and the correction would consequently be $\frac{11}{12}$ths of a year for all annual cases.

Mr. A. F. **Burridge** disagreed with Mr. Todd's statement regarding the acceptance of large policies, and rather followed Mr. King, who noticed that large cases were, in his experience, usually subjected to a severe rating-up. Considerable criticism had been passed on Mr. Ryan's remark about the higher rate of mortality attaching to large policies. Mr. Ryan stated that it had been found by the American companies that among the larger policies this heavier rate of mortality prevailed, but he did not seem to imply that a similar state of things had prevailed in this country. Presumably the existence of that condition in America could be traced to a somewhat different mode of conducting business there. It might be that in America persons speculated on their own lives, but he did not think it was so in this country. The principal thing to be determined before they could accept the interesting method Mr. Ryan laid down for obtaining the expectation of claims was, whether really there was a different rate of mortality or rate of loss amongst small and large policies. He thought that, on the whole, pretty much the same rate prevailed; and unless Mr. Ryan could show that the one class exhibited a higher rate of mortality than the other, he did not think the new method was applicable at the present time. He imagined that each office would have its rate of loss depending to some extent upon the average amount of its policies. He did not gather whether each company should, under Mr. Ryan's suggestion, calculate its own rates of loss and act upon that, or whether the experience of several companies should be combined, as in the case of the Institute Table, and the result used as a test. If the latter, the fluctuations in the one company would no doubt be covered by those of others, and they would get a fair combination. Many actuaries would corroborate Mr. Rothery, that the actual claims were not always less than those expected. It was, however, the character of the claims and not their amount which made the difference between a good and a bad year.

Mr. H. W. **Manly** said that there had been such a diversity of opinion expressed on one of the chief points in the paper that it would be satisfactory to settle it by ascertaining, on the next occasion, whether there was any difference between the loss rate amongst large and small policies. He did not think there was for reasons opposed to Mr. Todd's, his experience being that when a very large proposal came before them they took more than usual care to inquire into the nature of the risk. With respect to the second part of the paper, he remained of the opinion that the proper way in which to make a table of mortality was by taking calendar and not policy years. His reason was that the advantage was in favour of the method which conformed most closely to the conditions under which they wanted to apply their tables. The first use of a mortality table was the calculation of premiums. For that object, if they wished extreme accuracy, it would be advisable to use policy years instead of calendar years; but the difference would be very small. And when they had got extreme accuracy with reference to their mortality, they calculated their premiums on the assumption that the moment they received a premium they were going to invest it at some fixed rate of interest, a thing which they never did; so that, for the purpose even of calculating their premium, the extreme accuracy which was to be obtained for that purpose by adopting policy years instead of calendar years in constructing a mortality table was entirely upset by the variable rate of interest they realized, and the unknown quantity of loading added for expenses, &c. The principal use of a mortality table, after they had calculated their premiums, was for valuation purposes; and what was wanted was a table that would conform to the actual conditions. They made their valuations on, say, the 31st December, and he held that to calculate their table of mortality as from that date was the better plan; it could be applied more easily, and would bring out truer results than if they adopted policy years, which were more properly applicable to the calculation of premiums. It was a happy thought that the two papers were put together, for the first part illustrated the second. They calculated the expected mortality at the end of a calendar year, not at the end of a policy year: there, again, they wanted their table framed to comply with the same conditions as to time, as the facts which they were going to deal with. The advantages, he thought, were overwhelmingly in favour of following the observations through the calendar year. Some reference had been made to the year "0" of the Institute tables, which seemed to puzzle a great many people. That year "0" represented the period during which the new policies had been under observation, from the time of their inception to the 31st December, the end of the calendar year; and the mortality in that year would represent the mortality amongst the new entrants during the year. If, instead of throwing the new assurances into the general body of observations, and reckoning that all their new policies had been on risk for half a year on an average, and then applying the average H^M Table to them (by which they would get an excessive expected mortality), they were to take the new policies by themselves and apply the rate of mortality of the year "0" to them, they would get the most accurate estimate of what their mortality ought to have been.

Mr. G. H. **Ryan**, in reply, said that he had no idea of passing any uncomplimentary criticism upon the work of the committee who took under their charge the preparation of the Institute Mortality Experience; for it was only with the Institute method as an abstract system that he disagreed. He (Mr. Ryan) saw certain defects in the system, which he thought might be pointed out and discussed; but he hoped he had not pushed his contentions to any unfair length. The chief criticism of the evening had centered round his comparison of the rates of mortality and the rates of loss. Several speakers had stated that it was quite unnecessary in their opinion that the comparison of the expected with the actual death claims should be made. He agreed with them, but to a partial extent only. In the case of companies of established position it might possibly be unnecessary that any such investigation into the claim experience should be made, or, assuming it to be made, that it should be based upon other than an average table like the H^M. But there were some companies whose new business was excessively heavy, and who, by computing their expected claims according to the H^M Table, represented their mortality as being unduly favourable. In discussing a question of this nature, he thought it desirable to take as broad a view of it as possible, and to remember that there were companies and companies. In the case of certain old companies the expectation of claims deduced on the basis of the H^M Table was almost absolutely accurate; and that resulted so because the distribution of risks either accorded entirely with that of the H^M Table, or did not differ from it to any appreciable extent. Mr. Higham had asked whether the loss rates were employed in any actual valuations. Upon the loss rates exclusively the valuable monetary results contained in the volume giving the Thirty American Offices' Experience had been constructed; and therefore the loss rates formed the elements which would, in the future, enter into the valuation of policies. A very important practical question had been raised having regard to the difference prevailing in the mortality among large and small policies respectively. On this point, Mr. Burridge had satisfactorily stated his (Mr. Ryan's) case. The only statistics available were those prepared by the Connecticut Mutual Company, and by Mr. Meech from the observation of the Thirty American Offices. Both those sources contributed data that pointed strongly in the direction of there having existed a larger rate of loss than of mortality; or, in other words, they established the fact that in America the larger policies carried a heavier rate of mortality than the smaller. He believed that in this country the opposite was the case, and his remarks were merely intended to be a paraphrase of the explanation given by the American authors concerning this feature of their results. It was to be hoped that their claim experience in Great Britain upon the larger policies would prove to have been more fortunate; but at present no statistics could be brought forward in support of that generally accepted idea. Mr. King had also stated the opinion that comparisons of the actual and expected deaths were quite useless except to ear-mark profits. There, again, it appeared that one most important element was overlooked, namely, the character of the company. Special cases and conditions could easily be named in which a true and faithful comparison of the actual with the expected mortality was an element of the greatest importance. Mr. King's question as to how rated-up policies should be treated in calculating the expected claims was met by Mr. Rothery's reply, that they ought to be treated strictly according to the mode in which they were dealt with in the valuation. Mr. Manly had ably expounded the view that calendar years and not policy years formed the better basis in arranging insurance statistics of mortality. He was reluctant to disagree on such a point with Mr. Manly, but he (Mr. Ryan) had never heard it suggested that the Institute tables were less applicable to the case of a company valuing as at the 30th of June than to that of a company valuing as at the 31st of December, and if Mr. Manly admitted that, the foundation of his argument would disappear.

A correspondent to the *Insurance Record* ("N. B. G.") subsequently pointed out that tables based on the claims paid, instead of the number of persons who died, had been published in this country (*see* Table G in the Report on the Mortality Experience of the Scottish Life Assurance Offices, 1869) and in Germany (Experience of the Gotha Life Assurance Company, 1880). He proceeded:

"In the experience of the Scotch companies, the rate of loss does not compare so unfavourably with the death rate as in the experience of the American offices, as will be seen from the following table of the ratio which the rate of loss bears to the rate of death: (1) according to the experience of the American offices, and (2) according to the experience of the Scottish offices—both being arranged in quinquennial groups of ages:

Ages	American Offices	Scottish Offices
20–24	·908	1·284
25–29	·988	·979
30–34	1·039	·883
35–39	1·090	1·055
40–44	1·032	·956
45–49	1·036	·926
50–54	1·023	1·078
55–59	1·055	·969
60–64	1·026	·939
65–69	1·086	·956
70–74	·993	1·021
75–79	·905	1·095
80–84	·911	1·225
85–89	1·089	1·107
90–94	1·000	·866

"The figures of the Gotha Office for the first fifty years of its existence have been arranged in every form which seemed likely to be useful; and among other methods of stating the facts, the lives have been separated into three classes, namely, those assured for £300 or less, those assured for sums ranging from £300 to £600, and those assured for sums above £600. The rate of mortality in the first class is considerably greater than that in either of the others, and is thus totally different from the American experience. The following are the ratios which the rate of death in each class bears to the death rate for all the classes combined:

Ages	£300 or Under	£300 to £600	Above £600
21–25	1·31	·92	...
26–30	1·38	·85	·76
31–35	1·14	·94	·95
36–40	1·11	·98	·89
41–45	1·08	1·02	·85
46–50	1·04	·99	·96
51–55	1·09	·96	·96
56–60	1·07	·95	1·01
61–65	1·02	·98	·99
66–70	·98	·99	1·06
71–75	1·01	·95	1·08
76–80	·99	1·02	·98
81–85	·90	1·02	1·11
86–90	1·28	1·12	·76

"A comparison of the different statistics shows that in the American experience the rate of loss is heavier among large than among small policies; the Scottish experience gives no decided indication in either direction; while in the German experience the small policies have been subject to the heaviest mortality rate. It is quite possible that the length of time during which the lives have been under observation may be one of the causes of this divergence. The figures of the American offices, which extend, on the average, over four-and-a-half years only, show a rate of loss higher than the death rate; those of the German office, in which the average duration is twelve-and-a-half years, yield a contrary result; while the experience of the Scottish offices, with an average of eight-and-a-half years of life, lies between the two others."

To this Mr. Ryan replied (*Insurance Record*, 25 Feb.), that although the belief that the idea of computing rates of loss from the amounts assured and claims had its origin in America appeared to be freely entertained by members of the Institute, there could now be no doubt that it was first conceived and put into practice by Mr. James Meikle. Admitting that the figures (given on p. 274) seemed to prove that, on the whole, the rates of loss did not compare so unfavourably with the rates of mortality in the case of the Scotch as in that of the American companies, he showed by the following ratios that the aggregate results for all ages combined pointed in the opposite direction:

Ratio of Rate of Loss to Rate of Death (all Ages).

Thirty American Offices	Males	1·066
	Females	1·065
Connecticut Mutual	Males	1·029
	Females	1·036
Ten Scotch Offices	Males and Females (Healthy lives).	1·105

On the Numerical Calculation of the Values of Complex Benefits, by means of Formulas of Approximate Summation. By George King, F.I.A., F.F.A., *Actuary to the Atlas Assurance Company.*

[Read before the Institute, 28 February 1887.]

I DO not pretend that much which is original will be found in this paper. Previous writers, notably the late Sir J. W. Lubbock, and Messrs. Woolhouse, Sprague, and G. F. Hardy, have, so far as actuarial science is concerned, nearly exhausted the subject of Formulas of Approximate Summation; and from time to time examples have been given of the practical use of the formulas, which have indicated the great facilities they afford to computers. But something in the way of numerical illustration remains to be done in order to bring the formulas into more general use, and to supply that want is my present object.

Lubbock's formula (*see J.I.A.*, v, 277; xi, 301; and xviii, 305), was the first of the kind to be applied to questions in Life Contingencies, but there are many objections to it. That it may be employed with effect, a considerable number of terms of the function the value of which is sought must be calculated; and if the function be a complex one, that course involves much labour. The series consisting of these calculated terms must then be differenced; and it will be found, that generally the differences run with great irregularity, and that after first diminishing they often rapidly increase again in proceeding to higher orders; so that it is difficult to say at what point to stop if the best available results are to be secured. The differences must be multiplied by inconvenient coefficients which do not lend themselves readily to purposes of calculation; and as the algebraical sign of the successive differences is uncertain, varying sometimes with values of the same function at different ages, the work cannot be arranged in a uniform manner; and unless great care be exercised, mistakes may easily creep in. Lastly, as Lubbock's formula is applicable to periodical and not to continuous functions, in using it a uniform distribution of deaths must often be presupposed; and that assumption is inadmissible in the case, for instance, of contingent assurances involving more than two lives. The mere statement of these defects in Lubbock's formula is probably sufficient to explain why it has been so little used.

Woolhouse's formula (*see J.I.A.*, xi, 61 and 301; and xv, 95) in some respects is an improvement on Lubbock's, because in using it there is no need to difference the values of the function;

but an equal number of terms must be calculated as with Lubbock's. Also, the differential coefficients of the initial term are required, and with some functions these cannot be readily obtained. Probably to find the values of annuities on three or more joint lives, Woolhouse's formula is frequently employed by many actuaries, but for more complex functions it is not available.

Mr. G. F. Hardy (*J.I.A.*, xxiv, 95) published a paper, the ultimate result of which will be to revolutionize our methods of dealing with complicated problems. These have hitherto been solved by resorting to a variety of expedients more or less reasonable, but the precise effects of which can never be foretold; or else the problems have been looked upon as insoluble, and mere guesses have been substituted for calculation. But Mr. Hardy, by an original and brilliant analysis, has furnished us with a number of formulas, simple, accurate, and brief, and which are applicable to functions of almost every kind. Nothing better seems to be required, and I feel sure that when these formulas come to be known and appreciated, they will be universally adopted.

There is one easy and obvious amplification of Mr. Hardy's analysis which I will here introduce, as by it we derive a formula as short as any of his, equally convenient in manipulation, and which by considerable practical experience I have found to produce results more trustworthy than those of his briefest formulas, results more than sufficiently accurate for ordinary purposes, although some of Mr. Hardy's longer formulas are still more to be depended on.

Mr. Hardy's formula A, (*J.I.A.*, xxiv, 107) is

$$\int_0^{6h} u_t dt = h\{\cdot 28(u_0 + u_{6h}) + 1\cdot 62(u_h + u_{5h}) + 2\cdot 2u_{3h}\} \quad . \quad . \quad (1)$$

where the value of h must be taken to suit the limits of summation.

By summing the series in sections, a very accurate formula may be derived from No. 1 above. We have

$$\int_0^{\infty} u_t dt = \int_0^{6h} u_t dt + \int_{6h}^{12h} u_t dt + \int_{12h}^{18h} u_t dt + \&c.$$

That is, after collecting together the terms,

$$\begin{aligned}\int_0^{\infty} u_t dt = h\{\cdot 28u_0 &+ \cdot 56(u_{6h} + u_{12h} + u_{18h} + \&c.) \\ &+ 1\cdot 62(u_h + u_{5h} + u_{7h} + u_{11h} + \&c.) \\ &+ 2\cdot 2(u_{3h} + u_{9h} + u_{15h} + \&c.)\} \quad . \quad . \quad . \quad (2)\end{aligned}$$

In using formula 2 any convenient value may be assigned to h, and by taking h comparatively small, greater accuracy is attained, though at the expense of including more terms. If h be so taken that u_{7h} falls just within or just beyond the mortality table, (ω being the limiting age), all higher values of u vanish, and we have

$$\int_0^\infty u_t dt = h\{\cdot 28u_0 + \cdot 56u_{6h} + 1\cdot 62(u_h + u_{5h} + u_{7h}) + 2\cdot 2u_{3h}\} \quad . \quad . \quad . \quad (3)$$

or

$$\int_0^\infty u_t dt = h\{\cdot 28u_0 + 1\cdot 62u_h + 2\cdot 2u_{3h} + 1\cdot 62u_{5h} + \cdot 56u_{6h} + 1\cdot 62u_{7h}\} . \ (4)$$

where the terms are arranged in consecutive order so as to be more easily calculated by the columnar method, presently to be described.

In formula 4, u_0 is generally either unity or zero, and need not be calculated if it be unity, its coefficient ·28 only being required; while u_{7h} either absolutely vanishes, or is insignificant and may be neglected. It thus happens that in the majority of cases only four values of the function need be computed, and the work is reduced to a minimum. It is formula 4 which will be employed throughout the remainder of this paper, but I do not mean thereby to depreciate any of Mr. Hardy's. On the contrary, as already remarked, I consider some of them to be still more trustworthy, and hardly less convenient.

In using the formula the logarithms of the coefficients are required, and they are here supplied for reference.

Term	Coefficient	Logarithm
u_0	·28	$\bar{1}\cdot 44716$
u_h	1·62	0·20952
u_{3h}	2·2	0·34242
u_{5h}	1·62	0·20952
u_{6h}	·56	$\bar{1}\cdot 74819$
u_{7h}	1·62	0·20952

The formula is a continuous one, and produces as results annuities payable momently, and assurances payable at the instant of death. In connection with these benefits the force of mortality comes constantly into request, but when the "Institute of Actuaries Life Tables" were published in 1872, that function was omitted; and the twin function, the force of discount, found no place among the constants on page 238. Even in those comparatively recent days, the continuous method seems to have been very little thought of.

Seeing that by formula 4 the value of the benefit is made to depend on at most six terms, it is necessary to secure the greatest accuracy attainable in the functions used in the calculations; and I therefore here give a table of the H^M force of mortality, computed by the formula, $\mu_x = \frac{7(d_{x-1}+d_x)-(d_{x-2}+d_{x+1})}{12l_x}$ (*see J.I.A.*, xxi, 64). In constructing this table the logarithm was first formed, and therefore that is the fundamental value. From the logarithm, the force of mortality itself to five decimal places was derived. It is hoped that this little original table will be useful in many ways. Mr. James Chisholm in his book of "Values of Policies" has also tabulated the force of mortality, prepared from the formula, $\mu_x = \frac{l_{x-1}-l_{x+1}}{2l_x}$; but that was hardly sufficient for present purposes, more especially as his table stops at age 79.

Values of $\log \mu_x$, *and* μ_x, *by* H^M *Mortality Table.*

Formula : $\mu_x = \frac{7(d_{x-1}+d_x)-(d_{x-2}+d_{x+1})}{12l_x}$.

x	$\log \mu_x$	μ_x	x	$\log \mu_x$	μ_x	x	$\log \mu_x$	μ_x
10	$\bar{3}$·75254	·00566	40	$\bar{2}$·01098	·01026	70	$\bar{2}$·78869	·06147
1	·64933	·00446	1	·01907	·01045	1	·82709	·06716
2	·55856	·00362	2	·02733	·01065	2	·86920	·07400
3	·48842	·00308	3	·04042	·01098	3	·91406	·08205
4	·44710	·00280	4	·05639	·01139	4	·95949	·09109
15	·44341	·00278	45	·07622	·01192	75	·99844	·09964
6	·47996	·00302	6	·10158	·01264	6	$\bar{1}$·03293	·10788
7	·54732	·00353	7	·12749	·01341	7	·06848	·11708
8	·63547	·00432	8	·15136	·01417	8	·10216	·12652
9	·72504	·00531	9	·17440	·01494	9	·13620	·13684
20	·78548	·00610	50	·19627	·01571	80	·17361	·14915
1	·81871	·00659	1	·21571	·01643	1	·21450	·16387
2	·83555	·00685	2	·23628	·01723	2	·25500	·17989
3	·83557	·00685	3	·26052	·01822	3	·29377	·19668
4	·82728	·00672	4	·28629	·01933	4	·33044	·21401
25	·82228	·00664	55	·31324	·02057	85	·35993	·22905
6	·82353	·00666	6	·34169	·02196	6	·38361	·24189
7	·83251	·00680	7	·37064	·02348	7	·40722	·25540
8	·84876	·00706	8	·39949	·02509	8	·42894	·26850
9	·86496	·00733	9	·42973	·02690	9	·44852	·28088
30	·88152	·00761	60	·46221	·02899	90	·48703	·30692
1	·89540	·00786	1	·49561	·03130	1	·54327	·34936
2	·90551	·00805	2	·52973	·03386	2	·60304	·40090
3	·91520	·00828	3	·56467	·03670	3	·67700	·47534
4	·92543	·00842	4	·59895	·03972	4	·78370	·60772
35	·93784	·00867	65	·63150	·04281	95	·91762	·82722
6	·95299	·00897	6	·66292	·04602	6	0·09636	1·24842
7	·96992	·00933	7	·69382	·04941	7	·43492	2·72222
8	·98557	·00967	8	·72325	·05288			
9	·99949	·00999	9	·75390	·05674			

In all the published examples which I have seen of the application of formulas of approximate summation, each term calculated has been worked out independently. (See for instance *J.I.A.*, xxiv, 109). Arranged in this way, the operations are tedious and confusing, and there is no easy and efficient check. The work may be much more advantageously set out in columns, because the elementary values required can thereby be expeditiously and accurately inserted, and, by comparing additions, useful verification may be obtained at each stage. At first the computer may find it irksome to add together the numbers when they are placed side by side instead of under each other in the more usual way, but after a little practice that difficulty is overcome. To make the work more easy, cross-ruled paper should be used, and the figures should be carefully made, and each placed in its proper position.

It will be convenient now to take a few numerical examples, and we shall commence with contingent assurances involving three lives.

Example 1.—Let it be required to find the annual premium for an assurance payable on the failure of the joint lives of (x) and (y) in the lifetime of (z); taking $x=30$, $y=45$, and $z=60$; and assuming interest at 4 per-cent.

The integral for the single premium, $\bar{A}^{\,1}_{\overline{xy}:z}$, is $-\int v^t.{}_tp_z.d({}_tp_{xy})$, or $\frac{1}{l_xl_yl_z}\int v^t.l_{x+t}.l_{y+t}.l_{z+t}(\mu_{x+t}+\mu_{y+t})dt$; and the divisor for the annual premium is $1+a_{xyz}$, or $\bar{a}_{xyz}+\frac{1}{2}+\frac{\mu_{xyz}+\delta}{12}$; and the integral for $\bar{a}_{xyz}$ is $\frac{1}{l_xl_yl_z}\int v^t.l_{x+t}.l_{y+t}.l_{z+t}.dt$. It thus appears that the whole of the work performed in finding $\bar{a}_{xyz}$, is available for $\bar{A}^{\,1}_{\overline{xy}:z}$, which contains only the additional factor $(\mu_{x+t}+\mu_{y+t})$. It will be convenient therefore, first to find $\bar{a}_{xyz}$, and then, adding another column, to find the single premium. As the risk will be finally determined on the death of (z) aged 60, and as 97 is the limiting age of the table, we have $7h=97-60=37$: whence $h=5$ nearly.

The following are the calculations in full:

(1) t	(2) log v	(3) log l_{x+t}	(4) log l_{y+t}	(5) log l_{z+t}	(6) log coeff.	(7) (2)+&c.+(6)
0	0·00000	4·95359	4·89164	4·76987	$\bar{1}$·44716	14·06226
5	$\bar{1}$·91483	4·93592	4·86169	4·69282	0·20952	14·61478
15	$\bar{1}$·74450	4·89164	4·76987	4·40978	0·34242	14·15821
25	$\bar{1}$·57417	4·82291	4·58120	3·73416	0·20952	12·92196
30	$\bar{1}$·48900	4·76987	4·40978	3·16435	$\bar{1}$·74819	11·58119
35	$\bar{1}$·40383	4·69282	4·14395	2·13033	0·20952	10·58045
	$\bar{2}$·12633	29·06675	27·65813	22·90131	0·16633	77·91885

t	(8) (7)+Colog l_{xyz}	(9) log $(\mu_{x+t}+\mu_{y+t})$	(10) (8)+(9)	(11) Antilog (8)	(12) Antilog (10)
0	$\bar{1}$·44716	$\bar{2}$·29070	$\bar{3}$·73786	·2800	·00547
5	$\bar{1}$·99968	$\bar{2}$·38703	$\bar{2}$·38671	·9993	·02436
15	$\bar{1}$·54311	$\bar{2}$·61183	$\bar{2}$·15494	·3492	·01429
25	$\bar{2}$·30686	$\bar{2}$·91403	$\bar{3}$·22089	·0203	·00166
30	$\bar{4}$·96609	$\bar{1}$·10934	$\bar{4}$·07543	·0009	·00012
35	$\bar{5}$·96535	$\bar{1}$·28321	$\bar{5}$·24856	·0001	·00002
	$\overline{10}$·22825	$\bar{8}$·59614	$\overline{18}$·82439	1·6498	·04592
				5	5
				$\bar{a}_{xyz}=8\cdot2490$	$\bar{A}^{\,1}_{\overset{\frown}{xy}:z}=\cdot22960$

From the two values just found we have $P\bar{A}^{\,1}_{\overset{\frown}{xy}:z}=\frac{\cdot2296}{8\cdot7563}=\underline{\cdot02622}$.

It will be observed that the last term, u_{7h}, is in this case insignificant both in the annuity and in the assurance, and might have been omitted without sensible loss of accuracy.

Had the question been to find the annual premium for an assurance payable on the death of the first of the three lives, (x), (y), and (z), the work would have been precisely the same as above, except that for $\log(\mu_{x+t}+\mu_{y+t})$ in column 9 we should have substituted $\log(\mu_{x+t}+\mu_{y+t}+\mu_{z+t})$: and had the question been to find the annual premium for an assurance on (x) against the joint lives (y) and (z), we should have proceeded as before, merely using $\log\mu_{x+t}$ in column 9.

Example 2.—Let it be required to find the annual premium for an assurance payable if (x) die after (y), and before (z). The premium will be payable as long as (x) survives with (z), and the work of finding $\bar{A}^{2}_{\underset{1}{x}yz}$ may be so arranged as to give also $\bar{a}_{xz}$, from which to derive $1+a_{xz}$, the divisor. We have

$$\bar{A}^{2}_{\underset{1}{x}yz}=\frac{1}{l_xl_z}\int v^t.\,l_{x+t}.\,l_{z+t}.\,(1-{}_tp_y)\mu_{x+t}.\,dt,$$

from which it is seen that the first three factors under the sign of integration give the annuity, and by adding the other two factors we pass to the contingent assurance. Taking the same ages and rate of interest as before, we must again make $h=5$, as the risk will be finally determined on the death of (z). By a preliminary process the values of $\log(1-{}_tp_y)$ must be found.

(1) t	(2) $\log v^t$	(3) $\log l_{x+t}$	(4) $\log l_{z+t}$	(5) log Coeff.	(6) (2)+&c.+(5)
0	0·00000	4·95359	4·76987	$\bar{1}$·44716	9·17062
5	$\bar{1}$·91483	4·93592	4·69282	0·20952	9·75309
15	$\bar{1}$·74450	4·89164	4·40978	0·34242	9·38834
25	$\bar{1}$·57417	4·82291	3·73416	0·20952	8·34076
30	$\bar{1}$·48900	4·76987	3·16435	$\bar{1}$·74819	7·17141
35	$\bar{1}$·40383	4·69282	2·13033	0·20952	6·43650
	$\bar{2}$·12633	29·06675	22·90131	0·16633	50·26072

t	(7) (6)+Colog l_{xz}	(8) $\log(1-{}_tp_y)$	(9) $\log\mu_{x+t}$	(10) (7)+(8)+(9)	(11) Antilog (7)	(12) Antilog (10)
0	$\bar{1}$·44716	$-\infty$	$\bar{3}$·88152	$-\infty$	0·28000	·000000
5	0·02963	$\bar{2}$·82374	$\bar{3}$·93784	$\bar{4}$·79121	1·07061	·000618
15	$\bar{1}$·66488	$\bar{1}$·38830	$\bar{2}$·07622	$\bar{3}$·12940	0·46225	·001347
25	$\bar{2}$·61730	$\bar{1}$·70818	$\bar{2}$·31324	$\bar{4}$·63872	0·04143	·000435
30	$\bar{3}$·44795	$\bar{1}$·82626	2·46221	$\bar{5}$·72642	0·00281	·000053
35	$\bar{4}$·71304	$\bar{1}$·91446	$\bar{2}$·63150	$\bar{5}$·25900	0·00052	·000018
	$\bar{9}$·91996	$\bar{3}$·66094	$\overline{11}$·30253	$\overline{19}$·55475	1·85762	·002471
		$-\infty$		$-\infty$	5	5
					9·28810	·012355
					$=\bar{a}_{xz}$	$=\bar{A}^{2}_{\underset{1}{x}yz}$

The last term is again insignificant, and might have been omitted. The first term, too, might in this case have been dispensed with, if before multiplying by 5 at the end of the operation, ·28 had been added to the annuity column. Had this course been followed, the values of both annuity and assurance would have been found by means of only four terms.

From the two values just found we at once have $a_{xz}=8·7944$, and $\mathrm{PA}^{2}_{\underset{1}{x}yz}=·001262$. It is perhaps worthy of note that the above value of a_{xz} differs by only ·004 in defect from that given in the Institute volume.

Example 3.—To find the annual premium for $\bar{A}_{\overline{xy}:\underset{2}{z}}$, an assurance payable on the death of the survivor of (x) and (y), if (z) be then alive. The most obvious integral for the assurance is

$$\frac{1}{l_xl_yl_z}.\int v^t.l_{z+t}\{\mu_{x+t}.l_{x+t}(l_y-l_{y+t})+\mu_{y+t}.l_{y+t}(l_x-l_{x+t})\}dt$$

and the divisor for the annual premium is $1+a_{\overline{xy}:z}$; but by a simple transformation a much shorter formula may be found. The assurance in question is evidently equal to an assurance payable at the death of (z) if he die third of the three lives, together with the value of the interest on 1 for such time as (z) may live after the survivor of (x) and (y): that is, $\bar{A}_{\overline{xy}:\underset{3}{z}}=\bar{A}_{xy\overset{3}{z}}+\delta\bar{a}_{\overline{xy}|z}$: because the assurances $\bar{A}_{\overline{xy}:\underset{3}{z}}$ and $\bar{A}_{xy\overset{3}{z}}$ depend on precisely the same probability of survivorship, the difference in their values being due only to the fact that they are payable at different times.

But $$\bar{a}_{\overline{xy}|z}=\frac{1}{l_xl_yl_z}\int v^t.(l_x-l_{x+t})(l_y-l_{y+t})l_{z+t}.dt$$

and $$\bar{A}_{xy\overset{3}{z}}=\frac{1}{l_xl_yl_z}\int v^t.(l_x-l_{x+t})(l_y-l_{y+t})l_{z+t}.\mu_{z+t}.dt.$$

These last two integrals are identical, except that the second one has μ_{z+t} as an additional factor; and we can find $A_{xy\overset{3}{z}}$ and $\bar{a}_{\overline{xy}|z}$ by one operation. Also $1+a_{\overline{xy}:z}$, the divisor for the annual premium, is equal to $1+a_z-a_{\overline{xy}|z}$, or to $1+a_z-\bar{a}_{\overline{xy}|z}$, because $\bar{a}_{\overline{xy}|z}=a_{\overline{xy}|z}$; and we therefore find the divisor at the same time that we find the assurance.

Taking the same ages and rate of interest as before, the following are the calculations:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
t	$\log v^t$	$\log(l_x-l_{x+t})$	$\log(l_y-l_{y+t})$	$\log l_{z+t}$	log Coeff.	(2)+ . . . +(6)
5	$\bar{1}$·91483	3·55437	3·71542	4·69282	0·20952	12·08696
15	$\bar{1}$·74450	4·07722	4·27996	4·40978	0·34242	12·85388
25	$\bar{1}$·57417	4·36832	4·59983	3·73416	0·20952	12·48600
30	$\bar{1}$·48900	4·49135	4·71790	3·16435	$\bar{1}$·74819	11·61079
35	$\bar{1}$·40383	4·60819	4·80611	2·13033	0·20952	11·15798
	$\bar{2}$·12633	21·09945	22·11922	18·13144	0·71917	60·19561

	(8)	(9)	(10)	(11)	(12
t	(7)+Colog l_{xyz}	$\log\mu_{z+t}$	(8)+(9)	Antilog (8)	Antilog (10)
5	$\bar{3}$·47186	$\bar{2}$·63150	$\bar{4}$·10336	·00296	·000127
15	$\bar{2}$·23878	$\bar{2}$·99844	$\bar{3}$·23722	·01733	·001727
25	$\bar{3}$·87090	$\bar{1}$·35993	$\bar{3}$·23083	·00743	·001701
30	$\bar{4}$·99569	$\bar{1}$·48703	$\bar{4}$·48272	·00099	·000304
35	$\bar{4}$·54288	$\bar{1}$·91762	$\bar{4}$·46050	·00035	·000289
	$\overline{13}$·12011	$\bar{4}$·39452	$\overline{17}$·51463	·02906	·004148
	$\delta\bar{a}_{\overline{xy}\vert z}=$	·00570		5	5
	$\bar{A}_{xy\overset{3}{z}}=$	·02074		·14530	·020740
	$\bar{A}_{\overline{xy}:\underset{3}{z}}=$	·02644		$=\bar{a}_{\overline{xy}\vert z}$	$=\bar{A}_{xy\overset{3}{z}}$

As $a_z = 9{\cdot}459$, the divisor for the annual premium is $10{\cdot}314$, and $\text{P}\bar{\text{A}}_{\overline{xy}:\underset{3}{z}} = {\cdot}00256$.

The foregoing examples are sufficient to illustrate the application of the formulas to the calculation of ordinary contingent assurances.

In the last example the value of a reversionary annuity was found incidentally. It is worthy of note that for this class of benefits the integral may be written in either of two forms. To take the simplest illustration, for an annuity to (x) after the death of (y) we may write, either

$$\bar{a}_{y|x} = \frac{1}{l_x l_y} \int v^t . l_{x+t} (l_y - l_{y+t}) dt$$

or

$$\bar{a}_{y|x} = \frac{1}{l_x l_y} \int v^t . l_{x+t} . l_{y+t} . \mu_{y+t} . \bar{a}_{x+t} . dt .$$

and similar alternatives are available for annuities on three or more lives. The formula to be selected will depend on the particular case in hand. The second integral becomes convenient if in the question two rates of mortality or two rates of interest are involved. For instance:

Example 4.—A married couple, husband aged 40 and wife aged 35, reside in the West Indies, but it is understood that if the husband die his widow will return to England. What is the value at 4 per-cent interest of a reversionary annuity to the wife after the death of the husband? Let it be assumed for purposes of illustration that in the West Indies mortality follows the Northampton Table, and in England the Carlisle; then in applying the formula, the values of l_x, l_y, l_{x+t}, l_{y+t}, and μ_{y+t}, must be taken from the former, and the value of $\bar{a}_{x+t}$ from the latter. Also we must take $h=8$, because 97 is the limiting age of the Northampton Table, and in this case $7h = 97 - 40$. Thus:

(1) t	(2) $\log v^t$	(3) $\log l_{x+t}$	(4) $\log l_{y+t}$	(5) $\log \mu_{y+t}$	(6) $\log (\frac{1}{2} + a_{x+t})$	(7) log Coeff.
0	0·00000	3·60314	3·56050	$\bar{2}$·31745	1·21856	$\bar{1}$·44716
8	$\bar{1}$·86373	3·53199	3·47914	$\bar{2}$·41295	1·17624	0·20952
24	$\bar{1}$·59120	3·32634	3·23350	$\bar{2}$·67230	1·01966	0·34242
40	$\bar{1}$·31867	2·92012	2·67117	$\bar{1}$·13501	0·75884	0·20952
48	$\bar{1}$·18240	2·46090	1·91908	$\bar{1}$·47009	0·60574	$\bar{1}$·74819
	$\bar{3}$·95600	15·84249	14·86339	$\bar{6}$·00780	4·77904	$\bar{1}$·95681

t	(8) (2)+ . . . +(7)	(9) (8)+Colog l_{xy}	(10) Antilog (9)
0	6·14681	$\bar{2}$·98317	·09620
8	6·67357	$\bar{1}$·50993	·32354
24	6·18542	$\bar{1}$·02178	·10514
40	5·01333	$\bar{3}$·84969	·00707
48	3·38640	$\bar{4}$·22276	·00017
	27·40553	$\bar{9}$·58733	·53212
			8
			4·25696 = Value required.

Compound survivorship annuities and assurances have always been a source of trouble and weariness to actuaries, but fortunately they are specially suited to formulas of approximate summation; and, in fact, their values are, as a rule, more easily found than those of benefits which include only simple survivorships. There are three compound survivorship benefits which involve not more than three lives, namely, $\bar{a}_{y\overset{1}{z}|x}$, an annuity to (x), to commence on the death of (z) if (y) be then alive; $\bar{a}_{\overset{2}{y}z|x}$, an annuity to (x) to commence on the death of (y) if (z) have died previously; and $\bar{A}_{\overset{3}{x}y\underset{1}{z}}$, an assurance payable on the death of (x), provided he die third of the three lives, (z) having died first. Mr. Makeham, in a masterly paper (*J.I.A.*, x, 241), showed that the values of the second and third of these benefits can be made to depend upon that of the first; and the relations he established are convenient, because it is easier to calculate $\bar{a}_{y\overset{1}{z}|x}$ by ordinary processes, than either $\bar{a}_{\overset{2}{y}z|x}$ or $\bar{A}_{\overset{3}{x}y\underset{1}{z}}$. An approximation to the value of $\bar{a}_{y\overset{1}{z}|x}$ is given in the equation $\bar{a}_{y\overset{1}{z}|x} = \bar{a}_{yz|x} \times Q_{y\overset{1}{z}}$, which is rigidly accurate on the assumption of Gompertz's law of mortality; and it is not difficult to get approximate values for $\bar{a}_{yz|x}$ and $Q_{y\overset{1}{z}}$. The final result is, however, far from satisfactory; and the arithmetical labour is greater than when formulas of summation are used. By employing the formulas of summation, each of the three benefits in question can be computed separately, the integrals being as follows:

$$\bar{a}_{y\overset{1}{z}|x} = \frac{1}{l_xl_yl_z}\int v^t . l_{x+t} . l_{y+t} . l_{z+t} . \mu_{z+t} . \bar{a}_{x+t} . dt$$

$$\bar{a}_{\overset{2}{y}z|x} = \frac{1}{l_xl_yl_z}\int v^t . l_{x+t} . l_{y+t}(l_z - l_{z+t})\mu_{y+t} . \bar{a}_{x+t} . dt$$

and

$$\bar{A}^{3}_{\underset{1}{xyz}} = \frac{1}{l_xl_yl_z} . \int v^t . l_{x+t} . l_{y+t}(l_z - l_{z+t})\mu_{y+t} . \bar{A}_{x+t} . dt .$$

Example 5.—As an example let it be required to find the value of $\bar{a}_{\overset{2}{y}z|x}$,—an annuity to (x), to commence on the death of (y), if (z) have died previously. Let the same ages as before be taken, namely, $x=30$, $y=45$, and $z=60$, and let interest be assumed at 4 per-cent. Here we must take $h=8$, because the risk cannot remain undetermined after the death of (y).

(1) t	(2) $\log v_t$	(3) $\log l_{x+t}$	(4) $\log l_{y+t}$	(5) $\log(l_z - l_{z+t})$	(6) $\log \mu_{y+t}$	(7) log Coeff.
8	$\bar{1}$·86373	4·92398	4·83972	4·20815	$\bar{2}$·26052	0·20952
24	$\bar{1}$·59120	4·83156	4·60684	4·71682	$\bar{2}$·75390	0·34242
40	$\bar{1}$·31867	4·58120	3·73416	4·76987	$\bar{2}$·35993	0·20952
48	$\bar{1}$·18240	4·26307	2·67117	4·76987	$\bar{2}$·67700	$\bar{1}$·74819
	$\bar{3}$·95600	18·59981	15·85189	18·46471	$\bar{6}$·05135	0·50965

t	(8) (2)+ . . . +(7)	(9) (8)+Colog l_{xyz}	(10) $\log(\frac{1}{2} + a_{x+t})$	(11) (9)+(10)	(12) Antilog (11)
8	12·30562	$\bar{3}$·69052	1·20614	$\bar{2}$·89666	·07882
24	12·84274	$\bar{2}$·22764	1·07375	$\bar{1}$·30139	·20017
40	10·97335	$\bar{4}$·35825	0·83206	$\bar{3}$·19031	·00155
48	9·31170	$\bar{6}$·69660	0·66076	$\bar{5}$·35736	·00002
	45·43341	$\overline{14}$·97301	3·77271	$\overline{10}$·74572	·28056
					8
					2·24448 $= \bar{a}_{\overset{2}{y}z\|x}$

In a second paper (*J.I.A.*, xii, 61), on "Solutions of General Problems in Survivorships", Mr. Makeham developed the theory of Compound Survivorships, and showed that the formulas for all annuities and assurances involving not more than four lives can be reduced to expressions including, besides simple benefits, only compound survivorship annuities of the form $a_{y\overset{1}{z}|wx}$ and $a_{xy\overset{1}{z}|w}$. When, however, an attempt is made to translate Mr. Makeham's expressions into numbers, the actuary is dismayed at the magnitude of the task, and I doubt if there are many members of our craft who would have the courage to persevere to the end. But to formulas of approximate summation even these complex cases present but trifling obstacles, and the values of benefits where four lives are concerned can be as easily found as when the number of lives is limited to three. The most complex annuity including four lives is $\bar{a}^{3}_{\underset{21}{xyz}|w}$, an annuity to (w) to commence on the death of (x), provided (x) die after (y) and (y) die after (z). It is evident that on the death of (y), if (z) have died first, and (x) be still alive, (w) will become entitled to a reversionary annuity on his life to follow the life of (x). That is

$$\bar{a}^{3}_{\underset{21}{xyz}|w}=\frac{1}{l_{wxyz}}\int v^{t}\,.\,l_{w+t}\,.\,l_{x+t}\,.\,l_{y+t}\,(l_{z}-l_{z+t})\,\mu_{y+t}\,(a_{w+t}-\bar{a}_{w+t:x+t})\,dt\,.$$

Example 6.—Let it be required to find the value of the above annuity at 4 per-cent interest when $w=30$, $x=40$, $y=50$, and $z=60$. Here we must take $h=7$, because it will be seen from the integral that the risk cannot remain undetermined after the death of (y), and therefore $7h=98-y=48$.

(1) t	(2) $\log v^{t}$	(3) $\log l_{w+t}$	(4) $\log l_{x+t}$	(5) $\log l_{y+t}$	(6) $\log(l_{z}-l_{z+t})$	(7) $\log \mu_{y+t}$
7	$\bar{1}$·88077	4·92811	4·88066	4·80381	4·14320	$\bar{2}$·37064
21	$\bar{1}$·64230	4·85471	4·75678	4·55331	4·67164	$\bar{2}$·82709
35	$\bar{1}$·40383	4·69282	4·40978	3·73416	4·76887	$\bar{1}$·35993
42	$\bar{1}$·28460	4·52271	4·00139	2·85914	4·76986	$\bar{1}$·60304
	$\bar{2}$·21150	18·99835	18·04861	15·95042	18·35357	4·16070

t	(8) (2)+ . . . +(7)	(9) (8)+Colog l_{wxyz}	(10) $\log(a_{w+t}-a_{w+t:x+t})$	(11) log Coeff.	(12) (9)+(10)+(11)	(13) Antilog (12)
7	17·00719	$\bar{3}$·50673	0·62572	0·20952	$\bar{2}$·34197	·02198
21	17·30583	$\bar{3}$·80537	0·66446	0·34242	$\bar{2}$·81225	·06490
35	16·36939	$\bar{4}$·86893	0·61202	0·20952	$\bar{3}$·69047	·00490
42	15·04074	$\bar{5}$·54028	0·52247	$\bar{1}$·74819	$\bar{5}$·81094	·00006
	65·72315	$\overline{\overline{13}}$·72131	2·42467	0·50965	$\overline{\overline{10}}$·65563	·09184
						7
					$\bar{a}^{3}_{\underset{21}{xyz}|w}=$	·64288

If more than four lives be involved, a process of double summation must sometimes be resorted to; because, instead of the reversionary annuity which appears in the integral of the last example, and the value of which can be obtained by a mere reference to the Institute Volume, we may have a compound survivorship annuity. Thus, the most complex case involving five lives is $\bar{a}^{4}_{\underset{321}{wxyz}|u}$,—an annuity to (u), to commence on the death of (w), if (x) die before (w), and (y) before (x), and (z) before (y). To find the value of this, we must consider the benefit on which (u) will enter at the death of (y). That benefit will clearly be an annuity on his life, to commence on the death of (w), if (x) die first. Assuming that (y) will die at the moment t, then the value at that moment of the benefit on which (u) will enter will be $a_{\overset{2}{w}+t:x+t|u+t}$, and therefore the required integral is

$$\overset{4}{\bar{a}}_{w\underset{3}{x}\underset{2}{y}\underset{1}{z}|u}=\frac{1}{l_{uwxyz}}\int v^t.l_{u+t:w+t:x+t:y+t}(l_z-l_{z+t})\mu_{y+t}.\bar{a}_{\overset{2}{w}+t:x+t|u+t}.dt.$$

To evaluate this integral,* we must first of all find, by a separate summation, the value of $\bar{a}_{\overset{2}{w}+t:x+t|u+t}$ at each of the epochs $t=h$, $t=3h$, $t=5h$, and $t=6h$, where h is so taken that u_{7h} falls just beyond the limit of the mortality table. Having then all the required values of the compound survivorship annuity, we may proceed exactly as in example 6.

In the majority of cases involving five, or even more, lives, the double summation will, however, not be necessary.

A problem often occurs in the consulting actuary's practice which is not easy of solution; namely, to find the value of a reversion with benefit of survivorship. For instance, a deceased father has by his will given a life interest in his property to his widow (u), and the property at her death is to be divided equally among such of his three children, (x), (y), and (z), as may be then alive. What is the value of the share of (x)? It can be shown that the value of the share may be represented by the integral

$$\int v^t.{}_tp_u.{}_tp_x\{1-\tfrac{1}{2}({}_tp_y+{}_tp_z)+\tfrac{1}{3}{}_tp_{yz}\}\mu_{u+t}.dt\,;$$

and to this integral the formulas of summation are very easily applicable.

If there be four children (w), (x), (y), and (z), the value of the share of (x) will be

$$\begin{aligned}\int v^t.{}_tp_u.{}_tp_x\{1-\tfrac{1}{2}({}_tp_w+{}_tp_y+{}_tp_z)\\+\tfrac{1}{3}({}_tp_{wy}+{}_tp_{wz}+{}_tp_{yz})-\tfrac{1}{4}{}_tp_{wyz}\}\mu_{u+t}.dt.\end{aligned}$$

The arithmetical work even in the case of the last integral is not very great; and the result is much better than could be obtained in any other way. The general expression for the value of such a reversion with benefit of survivorship is,

$$\int v^t.{}_tp_u.{}_tp_x\{1-\tfrac{1}{2}\mathrm{Z}+\tfrac{1}{3}\mathrm{Z}^2-\tfrac{1}{4}\mathrm{Z}^3+\&\mathrm{c.}\}\mu_{u+t}.dt,$$

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* For a much shorter and more elegant method see the remarks made by Mr. G. F. Hardy in the discussion which followed the reading of this paper.

where Z^r represents the sum of the probabilities that r joint lives will survive t years, for every combination of r lives that can be made out of m lives; the number of the existing children of the deceased testator being $m+1$.

To make the last expression still more general we may notice, that the letter u contained in it may represent a status instead of a life. If, for instance, it were to represent a status of two *joint* lives, (a) and (b), we should write ${}_tp_{ab}$ for ${}_tp_u$; and $(\mu_{a+t}+\mu_{b+t})$ for μ_{u+t}. If, however, u represents the *survivor* of (a) and (b) we must divide the integral into two parts, and write,

$$\int v^t.(1-{}_tp_a).{}_tp_b.{}_tp_x\{1-\tfrac{1}{2}Z+\tfrac{1}{3}Z^2-\&c.\}.\mu_{b+t}.dt$$
$$+\int v^t.(1-{}_tp_b).{}_tp_a.{}_tp_x\{1-\tfrac{1}{2}Z+\tfrac{1}{3}Z^2-\&c.\}.\mu_{a+t}.dt.$$

Unless the number of lives involved be great, it will, I think, be found, that the value of the reversion may be calculated by means of these general expressions, though apparently complicated, without insuperable difficulty, and that the result is much more satisfactory than can be obtained in any other way. It would, however, be interesting to learn how actuaries of experience have been accustomed to deal with such cases.

DISCUSSION.

The **PRESIDENT** (Mr. A. Day) said, that it seemed to him that the present paper was really a supplement to what Mr. G. F. Hardy had previously written, and might be regarded as practically a chapter in anticipation of the *Text-Book* upon which Mr. King was engaged. A letter had been written by Mr. Sprague to Mr. King upon the subject of his paper, which he would ask Mr. King to read as a part of the proceedings. It would be very interesting to hear something of the methods of approximation which had been in use among actuaries of experience, because these were not new questions but such as had come before them in times past, before they had the same facilities which had been afforded by the increasing knowlege of the science which the Institute had diffused.

Mr. **KING** then read the letter he had received from Mr. Sprague, in which he stated that the practical conclusion he was at present inclined to draw from the paper was, that the formulas therein explained were greatly superior to Lubbock's, which he had hitherto used, but that several of the objections to Lubbock's formulas were theoretical rather than practical, and were not entitled to much weight. In Scotland they often had occasion in disentail calculations to get approximate values of contingent annuities and assurances, depending on the order of survivorship among three or four, and sometimes even more, lives. He once had a calculation of that kind in which seven lives were involved. For such purpose he had always used Lubbock's formula, and found it worked very satisfactorily. When the differences began to increase they must not be used in the calculation. In practice he always arranged the work in columns, and generally found that when they had calculated a benefit depending, say, on three lives, and then required to get the value of a benefit depending on those lives and a fourth, comparatively little further calculation was needed. He thought Mr. King was not correct in saying that Lubbock's formula involved the supposition of a uniform distribution of deaths. Taking the case of only two lives, it certainly involved the supposition that the chance of x dying before y in the tth year was

$$\frac{(l_{x+t-1}-l_{x+t})\,(l_{y+t-1}+l_{y+t})}{2l_xl_y}$$

but although this was true on the supposition of a uniform distribution of deaths, it did not necessarily involve that supposition, but was true for other laws also. He inquired if the formula:

$$a_{\overset{1}{x}y|s}=a_{xy|s}\times Q_{\overset{1}{x}y}$$

—which was stated to be "rigidly accurate on the assumption of Gompertz's Law of Mortality"—was a new one, and, if so, suggested that a demonstration should be given of it. With regard to the benefit of survivorship in a contingent reversion, he thought the general plan had been to put no value on it, but to consider it as a sort of make-weight thrown in. Reversions were subject to so many unforeseen contingencies, which reduced the profit on their purchase, that some compensation of that kind was very desirable. He was inclined to think that most of those who bought reversions systematically gave too much for them.

Mr. WM. SUTTON said that, in discussing this paper, he proposed first to deal with the paper by Mr. Hardy, of which no abstract of the ensuing discussion was printed in the *Journal*.* Mr. Hardy's paper, like Mr. King's, began by criticising the familiar formulas which bore the names of Sir John Lubbock and Mr. Woolhouse, and he could not but think that, to some extent, those criticisms were hardly applicable. Sir John Lubbock's formula was never intended to do anything more than represent the approximate value of one summation of a series in terms of another summation involving a fewer number of the terms themselves. Mr. Woolhouse certainly did not appear to him to have contemplated that the formula he had given would do much more than that of Sir John Lubbock, and it was possible that, able mathematician as he was, he did not in those days anticipate that the continuous method would so soon become popular. It was only just to Mr. Woolhouse to mention that matter, for he, if anyone, was entitled to be considered the creator of the continuous method. The titles of the papers, both by Mr. Hardy and Mr. King, did not seem to be strictly correct, because what had been done in them was to give methods for the approximate evaluation of integrals, which was not quite the same thing as that denoted by the words "approximate summation." Both Mr. Hardy and Mr. King had adopted, in the strictest sense, Mr. Woolhouse's continuous method; in other words, they had represented the value of the benefit, however complex it might be, in the form of an integral, and, having obtained that integral, proceeded to establish methods for easily determining its value. If the integrals thus obtained were capable of exact evaluation, they would not want such papers as those; but the integrals, in the form in which they came before them in actuarial matters, were not generally capable of exact evaluation, and therefore some method of approximate evaluation became necessary. In this matter the members of the actuarial profession were not alone. There was another profession, that of a surveyor, in which the method of approximate evaluation of integrals was in constant use. No doubt some of them in their school-days had been taught the elements of surveying, and had been told that when they had a curvilinear area, of which they wanted to determine the extent, they were to draw their base line, and then erect off-sets at certain distances along the base line and measure them, and then, from certain formulas involving these off-sets (or ordinates), and the distances between them, they deduced more or less approximately the area which they were asked to survey. Some formulas for this purpose would be found in a capital little work written by the late Mr. Merrifield on arithmetic and mensuration, some of which, it would be seen, were identical with some of the formulas that Mr. Hardy had given them in his excellent paper. Mr. Hardy and Mr. King had set themselves to solve for actuaries a similar problem to that which the surveyors had constantly before them. They had a curvilinear area to measure, but they did not know exactly, or, at all events, could not express mathematically the laws in which the curvilinear outline was contained, and they had recourse to a similar method of approximation to that used by surveyors. This question had had great light thrown upon it by the great mathematicians. For instance, to Laplace they were indebted for a complete formula for what was known as "the method of quadratures." Other mathematicians had given formulas of one kind and another, and Mr. Woolhouse, in one of his valuable papers on summation, had illustrated in a very able way, although he had not put it so directly in an actuarial manner as, perhaps, they would have liked, this question of finding areas of curves; and, moreover, he actually gave one of the best-known formulas which bore the name of an old Cambridge man, Mr. Weddle. Since that time, other great mathematical minds have been engaged on the subject. Mr. Hardy, in his paper, had, in a very able manner, given the Institute the benefit of their investigations. He had used apparently one of the latest methods, known as the Gauss method, but there was no reference in his paper to the name of Christoffel, who, according to Mr. Boole, had investigated a similar problem to that which Mr. Hardy had dealt with. Mr. Hardy considered that in life assurance matters the initial and final values of those ordinates might, generally speaking, be taken as 1 and 0, and in the exceptional cases where they were not 1 and 0, their values could be easily determined. Christoffel had apparently investigated this problem under much the same conditions. He would be glad if Mr. Hardy could state whether he had studied Christoffel's investigations, as if so they would be very pleased to have the benefit of his views upon them. Referring to Mr. King's paper, he should be glad to know what were the exact reasons which led him to depart from the use of Mr. Hardy's formula (A) (*J.I.A.*, xxiv, 107). Mr. Hardy specially constructed that formula for life assurance benefits, the problem being: "given the initial and final values to obtain three other values from which the integral could be evaluated." Mr. King stated that he found that some of Mr. Hardy's formulas did not give such trustworthy results as the one he had submitted to them. No doubt he could tell them whether that remark applied to formula (A), in addition to other valuable short formulas that Mr. Hardy had given them. He would also like to know in what manner Mr. King arrived at his standard of accuracy. He presumed he had tested Mr. Hardy's formula in the case of some of those complex benefits, and found it more or less wanting. It would, therefore, be very useful to know how he had arrived at the conclusion that his own formula gave more trustworthy results. Mr. King's paper might be looked upon to a great extent as an addition by way of numerical illustrations to Mr. Hardy's paper, and they would all agree with him that they were greatly indebted to Mr. King for such a valuable paper.

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* [No general discussion followed the reading of Mr. Hardy's paper here referred to.—ED. *J.I.A.*]

Mr. T. G. **ACKLAND** said that, to his mind, the value of the paper lay in the fact that it dealt with the practical application of methods of approximate summation or evaluation of integrals. In the extremely important—he might say classical—paper of Mr. Woolhouse (*J.I.A.*, xv, 95), the continuous method was applied to the calculation of assurances and annuities, and the values of complex benefits were much simplified, being expressed in terms involving ordinary annuities and assurances. Mr. G. F. Hardy's paper (*J.I.A.*, xxiv, 95) dealt with the subject of integrals in a masterly manner, and some exceedingly simple formulas for their evaluation were obtained. All who read that paper must agree that it was a contribution of unusual value and excellence, and one which admirably illustrated Mr. Woolhouse's earlier paper. Mr. King had gone a step farther, and applied the continuous method of Mr. Woolhouse and the formulas of Mr. Hardy to the practical computation of benefits, the values of which had hitherto been only capable of expression in very intricate forms. Coming to the details of the paper, he would ask Mr. King by what process he arrived at the conclusion that his formula was more accurate than Mr. Hardy's. It would appear, from a general comparison of the formulas, that Mr. King's was likely to give a result numerically greater than Mr. Hardy's. Mr. King's method appeared to be a remarkably simple one, and the formulas and methods employed were very clear, and even simpler than the ordinary expressions with which they were more familiar. The continuous method of dealing with *moments* instead of years, and thus avoiding all assumptions as to uniform or other distributions of deaths, enabled them to write down, almost literally "with a flowing pen" the general expressions for the most complex benefit at a given instant. As an illustration of this, he might refer to the three compound survivorships dealt with by Mr. King in his paper. Some of those present would have a vivid recollection of studying those problems in the pages of Baily's treatise on life annuities and assurances (Filipowski's edition, problem xix, xx, and xlii). Three different cases were stated for each problem, the period of life had to be split up into two portions, and in the result an elaborate formula was arrived at, which was numerically quite indeterminable without some assumption of value. By the continuous method, as applied by Mr. King in his paper, these problems were reduced to the simplest forms, and were capable of being worked out in a very short time. Another advantage of the methods here set forth was, that the divisor for the annual premium for a given benefit could usually be expressed in a form similar to that employed for the single premium, and both could thus be found practically in one operation. He was surprised to find that Mr. King dismissed Mr. Woolhouse's formula for summation in a very few words, as he had always considered that formula to be of great value in the computation of complex benefits. It was, indeed, so applied by Mr. Hardy in his paper (*J.I.A.*, xxiv, 95), where it gave results almost identical with those given by Mr. Hardy's formulas. Referring to the table giving the force of mortality, he asked Mr. King whether the function so tabulated was based upon Woolhouse's graduation of the H^M table. Mr. Hardy had calculated the force of mortality as the average or central death-rate for a term including five years before and five years after the age. It would be interesting to know how the results from this formula would compare with those given in the paper before them. The arrangement in columns of the work of ascertaining the expression of the benefit appeared to him to be a good plan. Mr. Sprague had informed them that it was the custom in Scotland to arrange the work in columns, but most of those present would probably have followed the plan of working each term separately. It would be noticed that Mr. King gave alternative formulas for the value of a reversionary annuity, one of which represented the value for each year of life, and the other the value of the benefit on which the survivor entered on the death of the failing life in the form of a varying assurance. He might mention that this second form had already been given in print, and also that a problem, almost identical with that given by Mr. King in illustration of this point, was set in the intermediate examination paper for 1882. It was not quite self-evident that the continuous reversionary annuity was equal to the ordinary reversionary annuity as stated in the paper. It worked out so in the particular case given, but as the relation was not generally true as regards reversionary annuities, it would be as well to insert the proof in a note. He might add that he had applied the methods and formula of Mr. King to the computation of a rather troublesome benefit, namely, an assurance on (x) against (y) and for (t) years longer, and had found the formula to be extremely simple, and the result practically identical with that arrived at by the usual lengthy and complicated expression.

Mr. G. F. **Hardy** said he had to thank Mr. King for the manner in which he had referred to his previous paper. Mr. King had added greatly to the interest of the subject by giving them a number of practical illustrations of the use of the formula employed, and had no doubt improved the latter as regards accuracy by duplication, a step not contemplated in the original paper. Mr. Sutton's objection to the title of that paper was no doubt technically correct. His object in using the words "approximate summation" was to convey, perhaps, a more popular idea of its purpose and contents. He had not seen Christoffel's work at the time his paper was written, but it seemed to him a simple and obvious modification of the Gauss problem to assume the first and last ordinates known (as was invariably the case in life contingency calculations), and thus obtain a set of formulas specially adapted to these cases. With regard to the illustrations given in Mr. King's paper, the work might have been shortened if he had not been desirous of obtaining the annuities as well as the single premiums in one operation, as in many cases the annuities were already known. The object of the formulas being to save time, the arrangement of the work was of consequence. In example No. 2, for instance, the expression under the sign of integration might be put in the form

$$D_{z+t}(1-{}_tp_y)d_{x+t-\frac{1}{2}}.dt$$

or, with the Carlisle Table, in the convenient form

$$d_{y+t-\frac{1}{2}}\cdot M^{\;1}_{x+t\cdot z+t}\cdot dt\,,$$

the resulting value being multiplied by $(1+i)^{\frac{1}{2}}$ to allow for payment at the instant of death. As Mr. Ackland had pointed out, the force of mortality at age $(x+t)$ had in his previous paper been taken as equal to

$$\frac{l_{x+t-5}-l_{x+t+5}}{10l_{x+t}}$$

where the Carlisle Table was employed, and this was rendered necessary by the irregularity of the d_x column of that table. By using the above expression, this difficulty was overcome, the working formula became very simple, and no appreciable error was introduced.

There was one formula in his paper not of the class that Mr. King had specially referred to, which he considered in some respects superior to those taken as the basis of the present paper, especially in cases of double integration. These cases occurred very seldom, but occasionally where they had many lives to deal with, and the order of dying was important, they might want to effect a double integration, and the formula referred to was then very useful. It gave as the value of the integral of u_x from 0 to ∞, the series

$$2t[u_t+u_{3t}+u_{5t}+\ldots+\tfrac{1}{24}(u_{-t}-u_t)]\,;$$

or, making $t=5$,

$$10[u_5+u_{15}+u_{25}+\ldots+\tfrac{1}{24}(u_{-5}-u_5)]\,.$$

This had the advantage of equidistant values of u_x and of equal co-efficients, and lent itself readily to the process of double integration.

A numerical illustration would best show how the labour involved in a double integration may be minimised by the use of this formula.

Mr. King gave as the most complicated benefit that can arise involving five lives, the annuity $a^{4}_{\underset{321}{\overline{wxyz}}|u}$, giving as the value for this benefit, the integral

$$\frac{1}{l_{uwxyz}}\int_0^\infty v^t l_{u+t:w+t:x+t:y+t}(l_z-l_{z+t})\mu_{y+t}\bar{a}^{2}_{w+t:x+t|u+t}dt,$$

where the presence of the factor $\bar{a}^{2}_{w+t:x+t|u+t}$ necessitates a double integration. When the integral corresponding to this annuity is inserted, the entire expression becomes somewhat unwieldy; but it may be much simplified as follows. For $v^t l_{u+t}$ write D_{u+t} (substituting D_u for l_u in the divisor preceding the integral sign), for $l_{y+t}\mu_{y+t}$ write $\bar{d}_{y+t}$ ($\bar{d}_x = l_x\mu_x$, and $\log\bar{d}_x = \log l_x + \log\mu_x$, may be obtained from Mr. King's table of μ_x) finally for $a^{2}_{w+t:x+t|u+t}$ substitute the value

$$\frac{1}{\mathrm{D}_{u+t}l_{w+t:x+t}}\int_t^\infty \mathrm{D}_{u+t}l_{w+t}\bar{d}_{x+t}a_{w+t|u+t}dt$$

(since at the death of (x) leaving (w) and (u) surviving, the value of u's interest will be that of a reversionary annuity to commence at w's death).

The entire integral now assumes the form

$$\frac{1}{\mathrm{D}_u l_{wxyz}}\int_0^\infty \mathrm{D}_{u+t}l_{w+t:x+t}\bar{d}_{y+t}(l_z-l_{z+t})\cdot\frac{1}{\mathrm{D}_{u+t}l_{w+t:x+t}}\int_t^\infty \mathrm{D}_{u+t}l_{w+t}\bar{d}_{x+t}a_{w+t|u+t}dt.dt$$

$$=\frac{1}{\mathrm{D}_u l_{wxyz}}\int_0^\infty \bar{d}_{y+t}(l_z-l_{z+t})\int_t^\infty \mathrm{D}_{u+t}l_{w+t}\bar{d}_{x+t}a_{w+t|u+t}dt.dt.$$

A comparatively simple form. As a numerical example, take the ages of u, w, x, y, z, as 30, 40, 50, 60, and 70. The work will then stand as under. The expression under the second integral must first be computed for values of $t=-10$, 0, 10, &c., which, by the application of the formula above named, will give the values of this integral for $t=-5$, 5, 15, &c.; and these again being multiplied by the factors $\bar{d}_{y+t}(l_z-l_{z+t})$, will give the data for evaluating the entire integral. The differences of the numbers in column (7) are inserted in brackets; $\frac{1}{24}$th of these being subtracted from the numbers in column (8) and the result multiplied by 10 to obtain column (10). In all places where the characteristic of a log exceeds 10, the unit figure only is given. The remainder of the work explains itself.

(1) t	$\log D_{30+t}$	(3) $\log l_{40+t}$	(4) $\log \bar{d}_{50+t}$	(5) $\log a_{40+t\|30+t}$	(6) (2)+ … +(5)	(7) Antilog(6) (and Δ(7))	(8) Σ(7)
−10	4·6426	4·9536	2·9263	·5429	3·0654	1163·0	
						(−147·0)	2792·3
0	4·4426	4·9153	3·0580	·5909	3·0068	1016·0	
						(−91·7)	1776·3
10	4·2340	4·8617	3·2321	·6380	2·9658	924·3	
						(−273·1)	852·0
20	4·0100	4·7699	3·3699	·6639	2·8137	651·2	
						(−455·9)	200·8
30	3·7479	4·5812	3·3176	·6439	2·2906	195·3	
						(−189·8)	5·5
40	3·3889	4·1440	2·6514	·5536	0·7379	5·5	

(9) t	(10) $10\{(8)-\frac{1}{24}\Delta(7)\}=\int(7)$	(11) log(10)	(12) $\log \bar{d}_{60+t}$	(13) $\log(l_{70}-l_{70+t})$	(14) (11)+…+(13)	(15) Antilog (14)
−5	27984	4·4469	3·1361	$\underline{4}$·0482	$\underline{1}$·6312	−42·78
5	17801	4·2504	3·3243	4·0946	1·6693	46·70
15	8634	3·9362	3·4082	4·5146	1·8590	72·28
25	2198	3·3420	3·0941	4·5797	1·0158	10·37
35	134	2·1271	2·0480	4·5812	8·7563	·06

$\log D_{30} = 4·4426$
" $l_{40} = 4·9153$
" $l_{50} = 4·8617$
" $l_{60} = 4·7699$
" $l_{70} = 4·5812$

$3·5707 = \log$ Divisor

$$(u_5+u_{15}+\ldots) = 129·41$$

$$\tfrac{1}{24}(u_{-5}-u_5) = -\frac{89·48}{24} = -3·73$$

$$125·68$$
$$10$$
$$1256·8$$

$$\log = 3·0993$$
$$\log \text{Divisor} = 3·5707$$

$$\log \bar{a}^{4}_{\underset{3\ \ 2\ \ 1}{\overline{40.50.60.70}}|30} \quad \bar{1}·5286$$

$$\therefore\ \bar{a}^{4}_{\underset{3\ \ 2\ \ 1}{\overline{40.50.60.70}}|30} = ·3378$$

A calculation using three-figure logarithms throughout gave ·337 for the required value, and, I think, three-figure logarithms would be sufficient for most practical purposes.

If, in the annuity $\bar{a}^{4}_{\underset{3\ \ 2\ \ 1}{\overline{40.50.60.70}}|30}$, the life 70 is omitted, we have $\bar{a}^{3}_{\underset{2\ \ 1}{\overline{40.50.60}}|30}$, the value of which Mr. King has computed.

By omitting column (13) in the above calculation, and dropping the factor l_{70} in the divisor, we obtain an independent value for this last annuity.

Columns (14) and (15) would then become

t	(14)	(15)
—5	7·5830	3828 . .
5	7·5747	3756 . .
15	7·3444	2210 . .
25	6·4361	273 . .
35	4·1751	1 . .
$(u_5+u_{15}+\ldots)$		6240 . .
$\frac{1}{24}(u_{-5}-u_5)$		3 . .
		6243 . .

$$\log(+1) = 8.7954$$
$$\log \text{divisor} = 8{\cdot}9895$$

$$\log \bar{a}^3_{\underset{2\quad 1}{40.50.60}|30} = 1{\cdot}8059 \qquad \therefore\ \bar{a}^3_{\underset{2\quad 1}{\overline{40.50.60}}\,30} = {\cdot}6396$$

Mr. King's value for the same annuity is ·6429, a difference of only ·0033, or about $\frac{1}{2}$ per cent. Having regard to the totally different processes by which these results were obtained, and the complicated nature of the calculation as given above, so close an agreement must be considered very satisfactory. It would, perhaps, have been of interest had reference been made to some of the approximations used in practice, such as the substitution of an equivalent single life for two joint lives or for the survivor of two, it would be useful to know the extent of the errors introduced by those approximations. They could predict *à priori* from general reasoning the direction of the errors but not their extent. Recourse was frequently had to approximations in practice to save time, and no doubt in most cases the resulting errors were small, but it was desirable that they should know to what extent they might trust these somewhat rough though, doubtless, very convenient expedients. Neither Mr. King nor himself, in their references to Sir John W. Lubbock's and Mr. Woolhouse's formulas, had intended in the least to detract from the extreme value of their work, but it seemed to him needful, in bringing forward papers upon a subject already dealt with by previous investigators of such eminence, to offer some reasons for such a course.

Mr. D. A. Bumsted agreed with Mr. Sprague with regard to valuing benefits of survivorship, that the best plan was to put no value on them, but to consider them as a sort of make-weight. It was next to impossible to calculate their value accurately, and when calculated the value was so small as not to be worth noticing. The formula given by Mr. King in his last problem did not express the value of a reversion with benefit of survivorship. It rather expressed the single premium for an assurance against the contingency described, and had only a remote connection with the market value of the reversion. In dealing with these benefits of survivorship the first question to be determined was, what would be receivable at the death of the tenant-for-life. When they had determined the sum they were likely to receive at the death of the life tenant, the calculation was reduced to finding the value of a contingent reversion at his death provided the reversioner survived. There was another disturbing element in the calculation of values of these survivorship benefits, and that was the action of the Court of Chancery. A case had occurred where a person sold his reversion to half of a fund and also the benefit of survivorship in the other half which he would receive in the event of his brother dying in the lifetime of the tenant-for-life and the vendor surviving. The title was duly approved by counsel and twenty years ran on. At the end of this time the reversion fell in, but when the parties applied for the whole sum the Court of Chancery first decided that they were entitled to it, but on appeal it was decided that they were not. The end of it was that they had lost their benefit of survivorship although they had paid a consideration for it. He mentioned this to show that those securities were liable to a good many risks which needed very careful consideration, in addition to the mathematical estimate of the sum to be received. At the same time Mr. King's formulas would enable them more accurately to judge the nature of the risks they had to calculate.

Mr. G. H. RYAN, as one who had employed certain of Mr. Hardy's formulas with great advantage, said the meeting would acknowledge that Mr. King's paper possessed a thoroughly practical as well as theoretical interest for actuaries. Mr. Bumsted had fastened on the solution given by Mr. King of the value of a contingent reversion with benefit of survivorship attached; and he (the speaker) wished to add a few remarks upon the same point, though in a different line of criticism. Before so doing, he thought that, as Mr. King's paper was of the nature of an expository essay, fuller instructions might have been given on several matters for the guidance and assistance of students. The expressions appearing under the sign of integration appeared to need some explanation. They were not at all difficult to understand to members who were familiar with their subject; but students might be deterred from attempting to master the paper by finding no explicit directions that would enable them to recognize the expressions. Reverting to the problem of the value of a contingent reversion, with benefit of survivorship, Mr. King had, in his (Mr. Ryan's) view, laid himself open to the charge made by Mr. Bumsted—that he had dealt with a practical problem in a purely scientific spirit. He agreed with Mr. Bumsted, that the solution given in the paper did not satisfy the practical requirements of the case. A better way to regard it would be to separate, in the first place, the reversionary interest into the two elements of which it was composed. First, they had a simple contingent reversion to a share in the fund, the value of which could be ascertained in accordance with the methods already established. To protect this value insurance would naturally be set up. There then remained a small additional interest in the shape of the benefit of survivorship, to be separately calculated and allowed for in the estimated value of the whole. He found that the benefit of survivorship could be very readily expressed by a workable formula, and believed that it was much more convenient to approach the problem in this manner than in that recommended by Mr. King. They had heard it stated that the benefit of survivorship was of small pecuniary value, and might well be disregarded in valuing such contingent reversions. But circumstances could be imagined under which the benefit of survivorship would have an important influence upon the market price. It would be better in many cases to fix some value to that benefit, and it had, he believed, often been the case that no value had been so assigned, not so much because it was deemed worthless as because of the difficulty and trouble of ascertaining its value. He thought four-figure logarithms would be sufficient for such calculations.

Mr. A. W. **Sunderland** said Mr. King's paper might be regarded as a practical illustration of a particular use of Gauss's theorem for approximate integrations. In the formula Mr. King had applied his approximations in more than one way. In the first place, Gauss's formula was approximate, Mr. Hardy's formula was an approximation from Gauss's, and, in addition to this, in carrying out these approximations they were compelled to use approximate values of the force of mortality. For these reasons much interest would attach to Mr. King's reply to Mr. Sutton's inquiry as to the standard of accuracy that he had employed in judging the accuracy of these approximations. There were two methods by which they might form very accurate results for purposes of comparison. One was to take Mr. Hardy's formula, put $h=1$, and work out the summation to the end of the table by repeated applications of it; and a second way was by what he might call mechanical integration—plotting down on paper ruled in squares the ordinates of the curve $y=v_x$, then drawing through the ends of the ordinates, by the graphic method, a curve, and then forming the integration by merely counting the number of squares. With regard to the problem of benefit of survivorship, the last two speakers had attached to Mr. King's words a meaning different to that which he himself attached to them, namely, the actuarial value, or the values that would be attached to them by an actuary if he were asked to divide up the fund among the several beneficiaries. They were accustomed to deal with problems of that kind in a rough and ready way, and one plan, when the ages of the reversioners were not very far apart, was to find the value of each contingent reversion without benefit of survivorship, then to find the value of the absolute reversion to the whole fund, and divide the difference between the absolute reversion and the sum of the contingent reversions equally between the reversioners.

The **President** (Mr. A. Day) having proposed the usual vote of thanks,

Mr. **George King**, in reply, said he was aware that in Scotland a great deal more attention was paid to these complex benefits than in London. The Scotch had to deal with a much more stringent law of entail, and were obliged, therefore, to carefully estimate these very complicated functions. He thought Mr. Sprague had introduced into Scotland more accurate methods of treating the questions, because Lubbock's and other formulas were not much used previously. Mr. Sprague had said he (Mr. King) was not correct in saying that Lubbock's formula involved the supposition of a uniform distribution of deaths. He did not say so. What he said was that it sometimes did. With regard to the application of Gompertz's formula, it was not original, it was virtually to be found in Mr. Woolhouse's paper upon "Graduation of the H^M Mortality Table" (*J.I.A.*, xv, 402). He there gave the probability of survivorships, the expressions being based upon Makeham's formula, and if the constant a was put equal to unity, they at once arrived at the expression in question. With regard to the last problem of the paper, on which several speakers had commented, Mr. Sprague had said the general plan was to put no value on the benefit of survivorship, but to consider it as a make-weight thrown in; and he went on to say they were apt to give too much for reversions, and it was almost a necessity to get a small advantage through this benefit of survivorship. He (Mr. King) did not quite agree with the process, although the result might be desirable. He did not think they should so use their formulas as to give too large a value and then drop part of the benefit and take that for nothing. That was not really fair. They did not know what effect it was having, and his own view would be that it would be better to value the benefit more accurately on a basis that would give a smaller value. If the life, the value of whose interest he had to find, was pretty nearly of the same age with the other lives, and if those lives were not much separated and there were a number of them, he found they got a very good result by simply valuing the share of x as an absolute reversion for his own portion; but if x was the youngest or oldest life that process did not do; the benefit of survivorship to the youngest might be considerable, or it might be very small if he was the oldest. With regard to the last formula it would be noticed that it could be divided. The first part gave a contingent reversion payable to x after the death of u for the whole amount of the fund, and the portion in brackets, involving ${}_tp_y$, &c., was negative, and gave the deduction to be made on account of the probability of others of the beneficiaries surviving to take their shares. If they divided it in that way, and valued the first portion so as to obtain its market value, and then made a suitable deduction for the second portion, the required market value would be obtained. Notwithstanding the criticisms on that formula, it could be practically applied to market values to give results much more accurate than that got by other processes. He agreed with Mr. Hardy in his remarks regarding Sir John W. Lubbock and Mr. Woolhouse. There was not the slightest intention of detracting from their work. They were pioneers in this matter, and it was only natural that, as experience was gained, their processes should be improved upon. He had no means of testing the formulas for very complicated benefits. Mr. Sunderland had mentioned one or two which might be very good ways of testing. He had, however, worked out the values of a great many simple benefits—annuities and assurances—by Mr. Hardy's formula (A), by a number of Mr. Hardy's other formulas, and by his own, and the values of those benefits he was able to get absolutely so as to supply a standard. He tested formula (4) with a mortality table constructed on Mr. Makeham's hypothesis, and he still thought his formula (4) gave more accurate results than formula (A). He did not think his formula gave a larger result always than Mr. Hardy's. Mr. Hardy's summation was from 0 to $6h$, whereas his was from 0 to $7h$, so that although in appearance the same terms were used in the two, and in his some of the terms had larger co-efficients, yet really the terms were not the same, and it did not follow that formula (4) in the paper gave a larger value than formula (A) of Mr. Hardy's. He was pleased to hear Mr. Ackland's opinion as to the paper being a comparatively easy one. A very elementary knowledge of the differential and integral calculus was all that was necessary in studying life contingencies, hardly more than a mere knowledge of the symbols. Reference was made to using the force of mortality derived from l_{x-5} and l_{x+5}, and Mr. Hardy said for the Carlisle Table much better results would thus be obtained. His impression was that, using the formula he had done, practically equally good results would be derived. The formula of double summation shown by Mr. Hardy was extremely valuable. Mr. Hardy had also suggested that it would be very useful to investigate the effects of the ordinary approximations used in practice as compared with these formulas. It would be extremely interesting and useful, but somewhat laborious, and he might suggest that some of the junior members might spend their time much worse than in taking up that suggestion and working it out. The same remark applied to what Mr. Sunderland had said about testing the accuracy of these formulas. He was sure they gave accurate results, but he could not say how accurate. It would be very useful if they could learn the exact facts of the case.

OBITUARY.

The late Mr. Peter Gray, F.R.A.S.

THE demise of Mr. Peter Gray, one of our most esteemed honorary members, and one of the most prolific writers on the science of life contingencies, demands more than a passing notice in the pages of this *Journal*, to which so many of his valuable contributions were addressed.

Mr. Gray was a native of Aberdeen, and educated there in the Institution then known as "Gordon's Hospital" (since incorporated as "Gordon's College"), where he enjoyed the privilege then accorded under its constitution, as one of the most meritorious pupils, of being sent for two years to the University classes. Here he developed and improved the taste for mathematics, for which he was afterwards so distinguished, and (with the sole desire to assist the studies of a close personal friend) afterwards took a special interest in the study of life contingencies, securing a position rarely attained by an amateur not in the ranks of the actuarial profession. His numerous and valuable contributions to our columns are too well-known to require comment at this time, while his disinterested labours undertaken in organizing and preparing for publication the tables deduced from the Mortality Experience issued by the Institute, have well merited the warmest acknowledgments. At the date of his earlier works, nearly coincident with that of the establishment of this *Journal,* the amount of literature bearing upon the subject of Life Assurance was so limited, that Mr. Gray may be considered a pioneer in the cause; and his *Tables and Formulæ* is yet valued as a text-book for the training of the young student. His style of writing was luminous and exhaustive, and the generous assistance which he so readily accorded to any members requiring it, secured for him a lasting remembrance in their esteem and affection. He was a man of varied accomplishments, and as a Fellow of the Royal Astronomical and Royal Microscopical Societies, was distinguished by his knowledge of optics and of applied mechanics.

Mr. Gray was in his eightieth year, and leaves an only sister to regret his loss.

In addition to the introduction to the *Institute of Actuaries' Life Tables,* already spoken of, which may be regarded as the crowning work of Mr. Gray's long and useful life in this branch of study, the following contributions to this *Journal* are among his most important productions:

On the Doctrine of Successive Lives, ii, 1 and 271.
On Mr. Gompertz's Method for the Adjustment of Tables of Mortality, vii, 121.
On the Construction and Use of Commutation Tables, x, 84, 169, and 220.
On Mr. Makeham's Modification of Mr. Gompertz's Theory of the Law of Mortality, xi, 236.
On the Construction of Tables by the Method of Differences, xiii, 61, 149, and 293; xiv, 307.
On the Arithmometer of M. Thomas (de Colmar), xvii, 249; xviii, 20 and 123.

It should also be noted that Mr. Gray specially constructed for the *Institute Text-Book (Part I),* an extensive table of values of $\log_{10}(1+i)$, appending thereto an interesting note on the calculations.

History of Life Assurance in the United Kingdom. By CORNELIUS WALFORD, F.I.A.

(*Continued from* p. 132.)

5.—LIFE ASSURANCE—1825 TO 1843.

The state of the law in respect to Joint-Stock Companies, and associations for business purposes generally, remained in the same unsatisfactory condition which I have described in the previous section down to this time. Those great Insurance Associations, with millions of capital subscribed, in many cases, by the merchant princes of the land, for purposes admittedly the most beneficent, were simple partnerships, almost without legal recognition, except for purposes tending to their detriment or destruction. They could be attacked or pulled down by legal process readily enough; but they could only protect themselves against fraud, or recover their just debts by the most cumbersome of processes. What they had a right to expect, as institutions designed to aid largely in the accumulation of national wealth, was protection in carrying out their laudable designs; but of this they had not a vestige. Not only was every holder of stock in a proprietary company primarily and personally liable to his last shilling for the engagements of the partnership, but every policy-holder in a mutual society, being a member of, and therefore a partner in such society, was equally liable for all its engagements.

The only means by which they could obtain any legal recognition of their business necessities, was by going to Parliament for a special Act to enable them to sue and be sued. Under such an Act, the chairman or the principal officer might bring an action for money due to the association, and might be sued by claimants and creditors in the name of the entire partnership. About 1814 the Board of Trade and Plantations became so indulgent as to advise Parliament to insert in these special Acts a power under which the persons sued for the common purposes of the association could be recouped out of its funds—concerning which there had previously been some doubt. I believe the first Insurance Company which obtained the privilege of this new power of recoupment was the *Atlas;* but there was a penalty attached. All companies availing themselves of this power to do equal-handed justice amongst its members, were required to enrol a memorial of the members constituting the partnership, under oath, in the High Court of Chancery; and all transfers of shares had to be from time to time likewise so enrolled, until which the members originally enrolled remained liable. But compliance with all those requirements did not confer any corporate rights—the privileges so offered constituted a sort of apology for the withholding of justice. These remarks only apply to associations founded in Great Britain. In Ireland, as we have seen, a better understanding of legislative duties and obligations prevailed.

Every Insurance Association thus had to go individually to Parliament to obtain the powers requisite for the ordinary conduct of its business. The special Acts so obtained constitute quite a feature in the legislation affecting Assurance Offices. The necessity for additional powers, as they arose, involved a fresh application to Parliament. There have been passed within the last century and a half, more than a hundred of such Acts, involving enormous cost, and hampering the offices in a most unnecessary manner. The only remedy was to seek Incorporation; and this was a lottery. To make bad worse, in 1824 the Standing Orders of the House of Lords in relation to Private Bills were amended, and it was required that any company other than those for public purposes, as canals, railways, &c., requiring to be incorporated, must satisfy a Select Committee that three-fourths of its intended capital was paid up and deposited in the Bank of England. This was a most effectual barrier to the Incorporation of Insurance Associations, which in their early days, simply require the security of a subscribed capital without the incumbrance of any considerable portion being paid up; and this particularly applies to Life Assurance Companies.

This was the last vagary of this restrictive legislative policy. Common sense was awakened. There was enacted in 1825 the 6 George IV, chapter 91, which authorized the repeal of certain clauses of the "Bubble Act" of 1720, and conferred additional powers on His Majesty with respect to the granting of Charters of Incorporation to trading and other companies, namely, that by such Charters the measure of liability of the members might be agreed and defined, subject to the control of the law officers of the Crown. It was a modified consent that persons associating in Joint-Stock enterprize might make common-sense provisions for their own protection. The restrictions as to dealing in shares were entirely removed.

This amended law did not do all that was required towards the harmonious working of Insurance Associations. It was not convenient to go to the law officers of the Crown with special petitions for Charters of Incorporation. A refusal, on mere technical grounds, might, by reason of its publicity, produce most mischievous results; besides the cost was very considerable. Hence, in 1834, there was enacted the 4 & 5 William IV, chapter 94, enabling his Majesty to invest trading and other companies with the powers necessary for the due conduct of their affairs, and for the security of the rights and interests of their creditors. This was to be accomplished by a species of patent, which conferred corporate rights of a limited degree only. All such grants were to be recorded in the office of the Clerk of the Patents, and duly advertised. A list of members or proprietors was also to be filed in the same office; but decrees, judgments, &c., given against the companies were to extend to the property of such companies, *and to the persons and effects of every member thereof.*

This Act, like its predecessors, gave no relief, or means of relief to companies constituted previously to its enactment, and only two or three new Assurance Associations availed themselves of its supposed advantages. In 1837 yet another measure was enacted—the 7 William IV, and 1 Victoria, chapter, 73. This repealed many of the provisions of the two previous Acts, and enacted others, the chief of which was that the individual liability of the members *might be limited to such an amount per share as should be fixed by the letters patent;* and no execution should be issued against any member for a greater sum than the residue of his unpaid liability on the stock he held. There was also power to change the name of the company.

The principle of limiting the liability of the shareholders once introduced was certain to become developed in practice. But I must now resume the chronological narrative.

Friendly Societies.—The next event of general assurance interest occurring in 1825 was the appointment of a Select Parliamentary Committee to consider and report upon the Laws regulating Friendly Societies. These had been placed under legal protection in 1793, but by reason of subsequent legislation their legal status had become nearly as unsatisfactory as was that of Life Insurance Associations. Most of the leading actuaries of the day were called before this Committee, and the inquiry turned very much in the direction of the best data to be employed in the construction of tables of contributions for these societies. The question of the relative value of mortality tables in regard to accuracy was for the first time in a collective sense unfolded. The fate of the *Northampton Table* from this time henceforth was sealed. The Committee, indeed, after taking evidence enough to destroy it, went out of its way to bolster the table up; but the case was a hopeless one. The Committee wisely recommended that all laws relating to these societies be consolidated into one Act. There was in this report one passage of much significance: "Your Committee apprehend that, although the Act of 1793 "appears to begin by rendering lawful the Institution of Friendly "Societies, there neither was at that time nor is now any law or "Statute which deprives the King's subjects of the right of "associating themselves for mutual support." But we have seen and shall see yet further the difficulties surrounding those who did so associate themselves even for the most laudable of purposes.

It was to this Committee that Mr. Finlaison presented his Friendly Society Mortality Table. Other Committees on Friendly Societies followed, and a flood of light was thrown upon their working, and incidentally upon questions of mortality affecting life offices. (*See* 1827.)

At this juncture Mr. Griffith Davies published a table of mortality which he had deduced from the mortality experience of the *Equitable* Society, and it naturally attracted much attention. Its chief merit lay, perhaps, in its perfect graduation. This was contained in his *Tables of Life Contingencies* published this year. The table was adopted by some of the American Life Offices.

The new Life Assurance Offices of 1825 were the *Aberdeen* [now *Scottish Provincial*] (proprietary); the *Ægis* (Fire and Life, proprietary); *Alliance* [Irish] (Fire, Life and Marine, proprietary); *Crown* Life (proprietary); *Standard* (proprietary); *University* (proprietary); together with a few others of minor note.

In the early prospectus of the *University* Life was the following statement: "In order to show the profits likely to result "from assuring the lives of members of the Universities, it may "be observed that in the University Club, which consists of 1,000 "members, many of whom are not such lives as would be deemed "insurable, only 38 have died in 3 years; and if the average be "taken at the age of 35, which indeed is too low, the deaths will "be found much fewer than the ordinary rate of mortality, as "shown by the best tables, would produce." To which statement Mr. Babbage adduced the following rejoinder:

"Out of 1,000 persons aged 35 there die in 3 years:

38 by the University Club Experience.
33·1 by De Parcieux's (French) Table.
36·1 by Swedish Tables (1795), males alone.
34·7 by the same, males and females.
30·2 by Swedish Table (1805).
31·3 by the Carlisle Tables.
33·2 by the *Equitable* Experience."

Hence he inferred (I think not quite correctly), "that the "mortality amongst the members of the Universities is rather "high."

In 1826 Mr. Charles Babbage published his *Comparative View of the various Institutions for the Assurance of Lives*—a more popular work than that of Mr. Francis Baily in 1810. The author, in his *Passages from the Life of a Philosopher*, gives the following account of the origin of this book. He had been selected to fill the post of actuary, and, indeed, to aid in founding the *Protector* Life Office, 1824.

> "The information and experience I had thus gained led me to think that the public were not sufficiently informed respecting the nature of assurance on lives, and that a small popular work on the subject would be useful. I prepared such a work, as intervals of leisure admitted. . . . This little volume was soon translated into German, and became the groundwork upon which the great Life Assurance Society of *Gotha* was founded."

This work contained another Mortality Table deduced from the *Equitable* Society Experience; also a Mortality Table of Centenarians. It was in it, too, that the fanciful distinction between "Assurance" as applying to Life Contracts, as against "Insurance" applied to other branches of the business, was attempted to be set up.

Amongst the new Life Offices of 1826 were the *Promoter* (proprietary); *Scottish Amicable* (mutual); *Sheffield* (Fire and Life, proprietary).

The *Edinburgh Review* (March 1827) said:—"The late rage "for Joint-Stock Establishments produced an abundant crop of "new Assurance Companies. Upon the whole, 20 were brought "into being; and we believe we are accurate when we state that "the whole number of Life Assurance Societies at present existing "in the United Kingdom is 44. Recently there were 49, but 5 "have already become extinct." In a following section of this history, I propose to trace in detail the growth of Insurance Offices.

Friendly Societies.—Another Select Parliamentary Committee on Friendly Societies sat in 1827; again most of the leading actuaries were examined; again the *Northampton* Table was condemned for practical use in computations affecting these societies, and the *Carlisle* Table was spoken of on the whole favourably. While this Committee was sitting, Mr. Charles Babbage addressed a Letter to its Chairman, urging upon him the importance of collecting the mortality experience of the then existing Life Offices; and therein he said: "The system of Life "Assurance, so widely extended in England, and so thoroughly "indicative of the prudence and foresight of the people, is not "yet, in my opinion, carried to those limits which it might "reach, if those who deal in that species of security were perfectly "satisfied with the Tables they employ, and if the public were "informed, in a plain and popular treatise, of the many ways yet "unnoticed in which it might be desirable to have recourse to it." (*See* 1843.)

There were no new Life Offices founded in the United Kingdom during the years 1827 and 1828. In 1829 the *Clergy Mutual* was established; and in 1830 the *National Life* was founded as a mutual society.

In 1829 the Laws relating to *Friendly Societies* were consolidated under 10 George IV, chapter 56. *There was herein no limitation to the sum which might be insured on any one Life by a Society enrolled under this Act.* This presented an unexpected means of legal protection to Mutual Life Offices founded thereafter. (*See* 1834.)

In this same year Mr. John Finlaison published his *Table of the Mortality of Government Annuitants*; from which it became clear—whatever the causes may be—that Annuitants, on an average, live longer than persons of the same ages whose lives are insured. This table afforded, for the first time, the means of correctly measuring annuitant lives.

It was in 1830 that Assurances were effected for some £18,000, in various London Life Offices, upon the life of the beautiful Helen Abercrombie, by her step-brother, Thomas Griffith Wainwright; which event was speedily followed by her death; and for which he escaped the capital penalty, only to be transported for life for other transgressions. The entire record of this man's life is given in the late Mr. Serjeant Talfourd's *Final Memorials of Charles Lamb*. A full account of frauds upon Insurance Offices has yet to be written.

In 1831 there was published in the *Quarterly Journal of Agriculture, and the Prize Essays and Transactions of the Highland Society of Scotland*, a paper entitled "Remarks on "the Principles and Defects of the present Associations for Life "Assurance, with a view of the Preliminary Arrangements for "instituting the Scottish Economic Life Assurance Society." The paper was written with much force, and its main argument was in favour of the adoption of a lower scale of premiums for Life Assurance. The author was Mr. W. Fraser.

The principal Life Office founded this year was the *Scottish Equitable*, on the mutual principle. In 1832 the *Friends' Provident*, also mutual. In 1833 there was founded the *Argus* (proprietary); and in 1834 the *Mutual* Life; the *United Kingdom* Life (proprietary); the *Universal* Life (proprietary).

Friendly Society Laws.—In 1834 a new law, still more favourable to Friendly Societies, was enacted (4 & 5 William IV, chapter 40). The purposes for which they might be founded were—" For the mutual relief and maintenance of all and every " the members thereof, their wives, children, relations or *nominees*, " in sickness, infancy, advanced age, widowhood, or any other " natural state or contingency whereof the occurrence is susceptible of calculation by way of average, or for any other purpose " which is not illegal." The promoters of Assurance Associations were not slow to discern the advantages here offered, especially in the permission to issue nominee policies, whereby a man might appoint his wife or other person depending upon him to receive the sum insured directly on his death, without the expense and trouble of Probate, Letters of Administration, &c.

This Act gave great impetus to the founding of Mutual Assurance Societies. There was no limit to the amount which might be insured on any one life. Several had already been founded under the Act of 1829. The following is, I believe, a complete list of those so founded under the authority of these Acts:

1829. Clergy Mutual Life.
1829. West Herts Friendly Assurance Society.
1831. Aberdeen Mutual.
„ Western Annuity Society.
1832. Friends' Provident Life.
„ United Assurance Society.
1833. Prospective Endowment Association.
1835. National Provident Life.
1837. Inverkeithing Life.
1838. Widows' Fund and General Annuity Society.
1839. Stamp and Tax Office Assurance Society.
1840. Isle of Man Assurance Society.
„ Provident Clerks' Assurance Society.
„ Temperance Provident Life.
1841. Wesleyan Provident Life.
1844. Kent Mutual Fire and Life.
„ Tunbridge Wells Provident Life.
1850. Catholic Gild.
„ Church of England Schoolmasters.

Some of these associations speedily acquired a considerable business. They enjoyed exemption from stamp duties, facilities of arbitration, and other practical advantages, including the right of investing their funds with the Commissioners of the National Debt on very favourable terms. There can be no doubt that but for the events which followed, these Friendly Society laws would have exercised a considerable influence on the course of Life Assurance.

In 1835 Mr. Charles Ansell published his well-known work on *Friendly Societies*, in the preface to which were the following observations: "That the admission of varied objects into [Friendly] " Societies is not necessary to their success, seems sufficiently "evinced from the experience of the Life Assurance Offices "established in London. Every one of such establishments, "without exception, which has become distinguished for the "magnitude or success of its concerns, is marked by the extreme "simplicity of its plan; while others which embrace a vast "number of objects and options have also, without exception, "remained comparatively unpatronised, or not resorted to by the "public. The reason is abundantly plain—men engaged in the "ordinary business of life have little disposition to enter into "the critical and laborious investigations which are requisite for "the understanding of complex schemes in which Life Con-"tingencies are involved; and they have as little anxiety to "connect themselves with Institutions of which they do not "clearly comprehend the principles." These sagacious remarks by one who took an important part in the founding of many Life Assurance Associations—especially with those formed under the Friendly Societies Acts—are as true to day as they were at the hour they were written. One essential leading idea should characterize every Assurance Association: this should at once explain and justify its existence.

The principal Life Offices of 1835 were: the *Family Endowment* (proprietary); *Metropolitan* (mutual); *Monarch* (Fire and Life, proprietary); *National Provident* (mutual); *Protector* (proprietary).

The year 1836 marks an important epoch in the history of Vital Statistics in England—for there was then enacted 6 & 7 William IV, chapter 86—*an Act for Registering Births, Deaths and Marriages in England,* a step which had been too long deferred; for the old Parochial Registration system, established by Cromwell, Lord Essex, in the reign of Henry VIII (1530) had become inoperative; and Bills of Mortality were fitful productions issued upon no uniform system, and entirely voluntary. The Act came into force in the following year. Under its authority was erected the machinery of the Registrar-General of Births, Deaths and Marriages; from whose office has appeared since 1839 those annual volumes—the Registrar-General's Reports—which furnish a systematic digest of the Vital Statistics of the Kingdom; frequently in comparison with those of other countries.

A practical system of registration of births and deaths has other values from a life assurance point of view than that of supplying the records of the nation's standard of health. The facilities of obtaining "proof of age", and of ascertaining the "cause of death", are points of much value and have tended materially to lessen the difficulties which formerly existed in connexion with these requisites.

The same year was characterized by one of the most impudent frauds ever associated with the formation of Insurance Associations —this was the *Independent West Middlesex* project. I say "project" because it never had any legal constitution as a company—it was a sham from beginning to end. Its history was shortly this: Two men, one of whom had been a journeyman shoemaker and a smuggler, the other a tallow-chandler and a bankrupt, advertised life annuities for sale on terms about 30 per-cent more favourable to the purchaser than were granted by the established Assurance Offices. They opened handsome offices in London, Edinburgh, Glasgow and Dublin. They placed upon the prospectus the familiar names of bankers, members of Parliament, and others—varying the initials only. The newspapers of the day, particularly those in the provinces, teemed with their advertisements. The public were not long in responding to such inviting terms; and some £250,000 (it was estimated) was speedily obtained. There was at this date no publication devoted to the interests of insurance whereby the scheme could be readily exposed; but finally Mr. Peter Mackenzie, editor of the *Scottish Reformers' Gazette,* took upon himself the task, and the bubble was burst.

The state of the law, which had overlooked the interests of Insurance Associations, as such, entirely, was in a great degree responsible for this swindle; and no doubt the circumstance had a considerable influence in shaping the legislation which followed. (*See* 1844.)

The principal Life Offices of the year were: *Legal and General* (proprietary); *Licensed Victuallers* [afterwards the *Monarch*] (proprietary); *Liverpool and London* (Fire and Life, proprietary); *Minerva* (proprietary).

A return of the stamps on Life policies issued this year (1836) was published, namely, *England and Wales* £16,946. 11s. 0*d.*; *Scotland* (not kept separate from other stamps); *Ireland,* £1,556. 19*s.* 6*d.* Estimating the stamps used in Scotland to be three times greater than those used in Ireland—it may be considered that the *new* life assurances effected this year reached about 11½ millions in sums assured.

In 1837 an unusual number of Life Assurance Associations were founded, namely, the *Britannia, Dissenters and General, National Loan Fund* [afterwards *International* Life], *National Mercantile* Life, *Royal Naval and Military* (all proprietary); and the *Scottish Provident* (mutual).

In 1838 Professor De Morgan published his well-known *Essay on Probabilities, and on their application to Life Contingencies and Insurance Offices*—a work that did more to satisfy the public mind of the safety on which the calculations of Life Offices are based than perhaps all the works which had previously appeared: "*There is nothing in the commercial world which* "*approaches even remotely the security of a well-established* "*Life Office.*" That sentence deserves to be printed in letters of gold. It declared that what Life Assurance professed to be, so it actually was, under the conditions named. The present writer well remembers the impression this sentence made upon him when he first read it. He has often quoted it to merchants, bankers, and men in high places, and always with marked effect. If De Morgan had never written another line, he had still left a perpetual testimony to Life Assurance. But he also said: "The "theory of insurance, with its kindred science of annuities, deserves "the attention of the academical bodies. Stripped of its technical "terms and its commercial associations, it may be presented in a "point of view which will give it a strong moral claim to notice. "*Though based upon self-interest, yet it is the most enlightened* "*and benevolent form which the projects of self-interest ever* "*took.*" And more in the same way. (*Vide* preface, p. 15.)

The new Life Offices of this year (1838) were the *Albert, City of Glasgow, Edinburgh and Glasgow* [now *Life Association of Scotland*], *Freemasons' and General* [afterwards *Albert*], and *Victoria* (all proprietary).

Extent of Life Policies in force.—In Chambers's *Tract on Life Assurance,* it was estimated that there were 80,000 Life policies in force in the United Kingdom in 1839. The writer made no attempt to estimate the sum insured thereby. If we take the average of the policies to be £500—for the age of small policies had not yet arrived—this gives but 40 millions sterling insured. The number of policies given was probably an under-estimate. The *Equitable* Society alone had 7,481 policies in force, insuring over 14 millions sterling—the largest sum ever upon its books. (*See* 1843.)

This year gave rise to a number of Life Offices, namely, to the *Alfred, Australian Colonial and General, British Empire, English and Scottish Law, London Edinburgh and Dublin, Westminster and General* (all proprietary); and to the *Standard of England* [Britannia], *London and Westminster,* and one or two smaller mutual offices.

Earliest Insurance Journal.—The want of a Journal devoted to the interests of Insurance Companies generally, which had been felt in 1836 under the circumstances there stated, was supplied in 1840 by the establishment of the *Post Magazine*—a title derived from the fact that one page of the sheet (post size) was left unprinted, and intended for the purposes of correspondence. This journal was founded by Mr. J. Hooper Hartnoll, who had been originally a mathematical master at the Royal Naval School (Greenwich), and was now editor and proprietor of the *Kentish Mercury*. He conducted his new journal with spirit and thorough independence; and in a few years later—at a period when much needed—it became a power.

The Penny Post—Another important event of this year was the establishment of the Penny Post. The stimulus which this gave to all enterprizes which required the aid of publicity can only be properly estimated by those who are familiar with the state of matters as they previously existed. No institutions have benefited more largely by postal reform than have Assurance Associations.

The new Life Offices of this year (1840) were numerous. They were the *Agricultural and General*, *Church of England*, *Commercial* (Scotch) and *Royal Farmers* (all proprietary); and the *Provident Clerks*, *Reliance*, and *Temperance Provident* (mutual).

The *Friendly* Society Acts were now (1840) amended (by 3 & 4 Victoria, chapter 73) in the following particulars. No policy above £200 should be exempt from stamp duty. Societies issuing policies above this amount were not to invest their funds in Savings Banks or with the Commissioners of the National Debt. But societies exempted from the preceding advantages were allowed to issue *Nominee* policies, and to make the necessary changes in their regulations for this purpose. In a word, the Life Offices, as such, were shut out from the benefits of the Acts which had not been intended for their protection. The proprietary offices had been active in their protests against the privileges so gained by their mutual contemporaries. (*See* 1844.)

There was published by the Society for the Diffusion of Useful Knowledge the well-known Treatise of Mr. David Jones, *On the Value of Annuities and Reversionary Payments*. These volumes, which contained a series of practical tables deduced from the *Northampton* and *Carlisle* Tables respectively, became of great service to all engaged in the business of Life Contingencies.

Joint-Stock Companies Law.—In 1841 a Select Committee of the House of Commons was appointed to consider the Laws relating to Joint-Stock Companies. It was well known that Insurance Offices would demand a considerable share of the attention of the Committee; and several prominent actuaries were called upon to give evidence. Circulars also were addressed to the Assurance Offices generally, with certain questions to be answered for the guidance of the Committee. A number of merchants, bankers, solicitors and barristers, were examined, and a thorough effort was made to discover practical remedies for evils which were known to exist. The Report of the Committee was not published until 1844, when I shall make reference to it.

About this date various popular periodicals, such as *Chambers's Journal, Chambers's Information for the People,* and others, commenced to regard Life Assurance as a subject which might be discussed with advantage.

The new Life Offices of the year were the *Achilles, Commercial and General, Medical Invalid and General, National of Scotland, New Equitable, South of England* (Fire and Life) and *Wesleyan Provident* (chiefly proprietary).

The new Life Offices of this year were the *Anchor* (Fire and Life), *London and County,* and *Western* (all proprietary).

The year 1843 presented two incidents of a memorable character. These were:

1. The publication, in the 5th Report of the Registrar-General of Births, Deaths and Marriages, of the *English Life Table,* which is now known as "No. 1" to distinguish it from later National Tables. The value of a table of mortality based upon the deaths of the entire kingdom for many practical purposes cannot be overrated. It was indeed no part of the Registrar-General's official duty to prepare such a table. We owe it entirely to the genius of the late Dr. William Farr, who saw with the eye of a philosopher the many useful purposes to which it might be applied.

2. The completion, by a Committee of Actuaries, of a Life Table, based upon the returns of Insured Life, as furnished by seventeen of the more important British Life Assurance Offices, and which derived a largely increased value in the fact that it presented the combined experience of these offices; and so, in truth, furnished a true view of the value of Insured Life. Previously there had only been available to the actuary facts of this character derived from individual offices. This table is known as the "Seventeen Offices' Experience Table", or as "Experience Table No. 1" to distinguish it from a later table of the same character.

The coincidence of having these two tables simultaneously available—the one showing the measure of national life, the other of lives which had been selected from the mass, mostly with all the skill and judgment that could be brought to bear upon the business—was a very remarkable one. While the general coincidence in the results deduced from data so widely different in many respects is hardly less remarkable. Here is an abstract of results of the respective tables, placed side by side for comparison:

Age.	English Table No. 1. Expectation of Life. Years.	Experience Table No. 1. Expectation of Life. Years.
10	47·44	48·36
20	40·34	41·49
30	33·68	34·43
40	27·14	27·28
50	20·55	20·18
60	14·09	13·77
70	8·78	8·54
80	5·07	4·78
90	2·74	2·11

The new Life Offices of this year (1843) were the *Experience, Mariners' and General* Life, and *Star* Life (all proprietary offices).

Life Assurances in Force.—It was estimated by Mr. Ansell, Mr. Griffith Davies, and Mr. Kirkpatrick, that there were in force in Great Britain at this time (1843) 100,000 Life policies, averaging £1,000 each—giving the aggregate of sums insured at 100 millions. The invested funds of the Life Offices were estimated at £50,000,000—a very fair proportion.

Medical Selection.—It is not easy to determine when the practice of subjecting persons proposing to insure to a medical examination, *i.e.*, a personal examination by a medical practitioner retained in the interest of the Assurance Office, was first adopted. Dr. Price, in his preface to Morgan's *Doctrine of Annuities*, 1779, said (referring to the Equitable Society), "It would not, "perhaps, be amiss to appoint a medical assistant, whose particular "business it should be to inquire into the state of health of the "persons who are offered to be assured." That Society had, in its original proposal form, asked for reference to the usual medical attendant. It did not itself appoint a regular medical examiner until 1858. The early Life Offices all required the applicant to appear personally before the Board; not a very bad ordeal for ordinary purposes, but in the case of latent diseases, hereditary or acquired, quite ineffective.

I suspect the practice of a personal examination arose with the proprietary offices early in the present century, and had, by the period at which we have now arrived, come into almost general practice.

Mortality Experience of the Provident Life and Trust Company of Philadelphia, 1866 *to* 1885, *prepared by* ASA S. WING, *Vice-President and Actuary of the Company.*

IT would be a very useful and interesting task to examine the numerous mortality experiences which have been publisht of late years; to compare them carefully with each other, so as to see how far they agree and in what respects they differ; and to note what contributions (if any) they have made to the general theory of life insurance. On the present occasion I will state the results of my examination of the recently publisht experience of the above-mentioned company. The experience extends over 20 years, and contains 124,526 policy-years, the numbers at risk ranging from 301·5 policies in the year 1866 to 13,927 in the year 1885. In the introduction it is said: "This experience covers a longer period " and a larger number of exposures to risk, than the experience " upon which the American Experience Table of Mortality was " constructed by Sheppard Homans in 1858. That these observa-" tions were sufficiently numerous, and that the American Table is " a fair exhibit of the general mortality of assured lives in this " country, is confirmed by the mortality experience of 30 American " companies, collected by L. W. Meech, and publisht in 1881." I am not familiar with the figures of the American Experience Mortality Table, but it appears from a table given on pages 78 and 79 of the present work, that at the age of 25 the H^M mortality is 82 per-cent of that by the American Table; that the tables gradually approach each other, and that at the age of 36 the rate of mortality is practically the same in both; that from this point up to the age of about 70, where the tables again coincide, the H^M mortality is higher, the difference being greatest at the age of 49, when the H^M mortality is 16 per-cent in excess of the American. These features of the two mortality curves are well represented in Chart J, page 81, which also shows the mortality according to the Carlisle Table, the 17 Offices' Experience, and the recent 30 American Offices' Table, and lastly, the mortality of the company itself. The general result is, that the curve representing the mortality of the company lies considerably below all the others. Not content with this comparison of the mortality with that according to the principal standard life tables, Mr. Wing gives also the following comparison of the mortality of his office, with that of six other American offices.

	Ratio of Actual to Probable Loss by the American Table	Ratio of Actual to Probable Loss by the Combined Experience or Old Actuaries' Table
Mutual Life of New York, 1843–1873:		
On lives	·799	...
Connecticut Mutual, 1846–1878:		
On policies	·856	...
On amount	·879	...
On lives	·853	...
On male lives	·846	·778
On female lives	·945	...
Mutual Benefit, 1845–1879:		
On lives	·848	...
On amount	·894	...
Michigan Mutual, 1867–1881:		
On policies and lives	·763	·709
John Hancock, 1863–1883:		
On lives	...	·732
On female lives	...	·823
On amount	...	·843
Penn Mutual, 1874–1885:		
On amount	...	·785
Provident Life and Trust Company, 1866–1885:		
On lives	·657	...
On male lives	·647	...
On female lives	·800	...
On policies	·666	·619
On amount	·687	·637

It would appear from these figures that the mortality has been very low, and it is claimed that no such favourable mortality experience has been publisht of any other life insurance company in America. The fact is, of course, to be explained to a very large extent by the recent selection of the lives, more than half the experience relating to the years of assurance 0–3. I think, however, it is not unlikely that similar, and even more favourable results, would be exhibited by the mortality of some of the much longer establisht British offices that transact a select business. For instance, in the recently publisht eleventh quinquennial report of the *Clergy Mutual Assurance Society,* it is stated that the actual claims for the 5 years beginning 1 June 1881 and ending 31 May 1886, have been only 64 per-cent of the expected claims according to the Institute H^M Table, the experience at ages under 45 being still more favourable. Thus, for the 10 ages 35–44, the actual claims were only £24,961, as against the expected claims £86,303, or only 29 per-cent of the latter. Having regard to the age of the *Clergy Mutual* Office, it is clear that the proportion of the recently selected lives must be very much less than in the *Provident Life and Trust*; and this is borne out by the fact that the new assurances effected in the 5 years, numbered 1,976, as against 7,952 existing at the beginning of the 5 years.

The work contains 18 tables, of which the first 9 give particulars of the experience as regards policies. Tables 10 and 11 give the years of life, and the probable and actual deaths of males and females respectively, in each calendar year. The general results are, for males, 104,588 years of life, 1,229·770 probable deaths by the American Table, 796 actual deaths, being 64·7 per-cent of the probable; and for females, 6,849 years of life, 83·770 probable deaths, and 67 actual deaths, being 80 per-cent of the probable. In this case, as in so many (if not all) others, the experience of the female lives is less favorable than that of the male lives. In Tables 12 and 13 are given, for males and females respectively, the years of life at each age, and the probable and actual deaths; and in Tables 14 and 15 the figures are summed in quinquennial groups of age. In the following tables the results are combined in still larger groups:

Ages	Average Number of Male Lives Exposed	Probable Number of Deaths by the American Experience Table	Actual Number of Deaths	Percentage of Actual to Probable
11–25	6,730·0	53·656	35	65·2
26–35	34,967·5	298·479	163	54·6
36–50	46,618·0	491·922	330	67·1
51–60	12,757·0	238·543	160	67·1
61–82	3,515·5	147·170	108	73·4
Total	104,588·0	1,229·770	796	64·7

Ages	Average Number of Female Lives Exposed	Probable Number of Deaths by the American Experience Table	Actual Number of Deaths	Percentage of Actual to Probable
11–30	1,238·0	10·102	11	108·9
31–40	2,460·5	22·365	19	84·9
41–50	1,985·5	22·696	19	83·7
51–82	1,165·0	28·607	18	62·9
Total	6,849·0	83·770	67	80·0

It is difficult to say how far the progression of the figures in these tables may be due to the effect of the recent selection of the lives; but they appear to indicate that the experience has been less favourable among the male lives under the age of 25 than between 25 and 35; and as regards the female lives, they confirm the opinion derived from other statistics, that above the age of 50 the lives are better than those of males, but at other ages worse, and particularly so under the age of 30.

Table 16 contains a mortality table founded upon the experience of the company for male and female lives, which has been adjusted by Mr. Woolhouse's method. This indicates a maximum rate of mortality, ·0117, at the age of 15, and a minimum of ·0046 at ages 27 and 28; and throughout its whole course it exhibits the slight irregularities which, as I have elsewhere explained, are inseparable from the use of Mr. Woolhouse's method. Considering the very recent selection of most of the lives, the mortality table is obviously entitled to very little weight, and it is unnecessary to say anything further with regard to it.

The first 9 tables in the book deal (as already stated) with the experience of the company as regards policies, and compare the probable number of policies expected to be terminated by death, and the probable amounts of claims, with the actual numbers. This comparison is made in different ways, first, for each of the 20 years of the company's existence, and again, for each age of the lives assured and for quinquennial groups of ages; but the results given are of little general interest. This is not the case, however, with the comparison given in Table 7, of the actual and probable number of policies terminated by death for each year of exposure; for this seems to me to be a valuable contribution to the subject of selection among insured lives. The ratio of the actual number terminated to the probable, proceeds with tolerable regularity up to the year 5 of exposure, and then begins to run irregularly; but by grouping them suitably we obtain a satisfactory indication of the law, as will be seen in the following table:

Year	Average Number of Policies Exposed	Probable Number to be terminated by Death by the American Experience Table	Actual Number terminated	Percentage of Actual to Probable
0	12,272·0	120·389	43	35·7
1	20,724·0	209·670	103	49·1
2	16,419·0	171·878	101	58·8
3	13,379·5	144·993	112	77·2
4	11,050·5	123·990	98	79·0
5	9,204·5	107·384	85	79·2
6, 7	14,031·0	172·959	138	79·8
8–10	14,363·5	195·314	140	71·7
11, 12	6,025·5	91·722	65	70·9
13–20	7,056·5	126·624	90	71·1
Total	124,526·0	1,464·923	975	66·6

The figures in this table show a progression of the same character as I described in my paper on the rate of mortality among assured lives, as influenced by the duration of the assurance (*J.I.A.*, xv, 328). In the year 0 of assurance, the policies terminated by death are only 35½ per-cent of the probable number; but the percentage gradually increases until it reaches a maximum, 80 per-cent, in the 6th and 7th insurance years; and from this point it diminishes, being 72 per-cent in the years 8, 9, 10, and 71 per-cent in the subsequent years. As I have shown in the above-mentioned paper, this increase to a maximum and subsequent diminution, can be explained satisfactorily if we admit that the rate of mortality is injuriously affected by the numerous withdrawals that take place during the early years of insurance, but in no other possible way. In other words, the fact of the rate of mortality following such a law, is a conclusive proof that the lives which surrender their policies, or allow them to lapse, are, on the average, better lives than those which remain on the books.

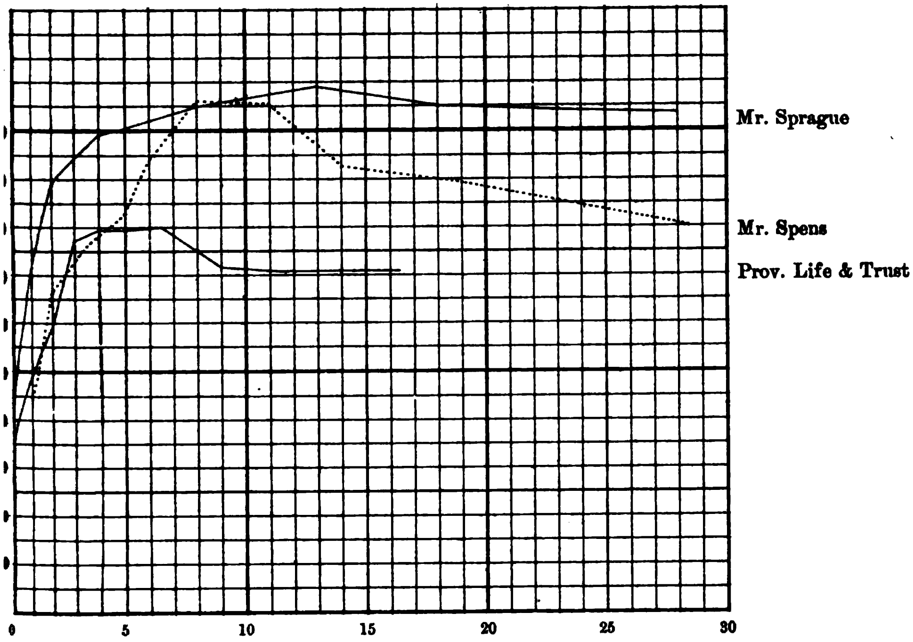
It may be interesting to my readers if I give here, for the purpose of comparison, a few figures from the paper in question.

Scottish Amicable (Mr. Spens)			20 British Offices (Mr. Sprague)		
Years of Insurance	Numbers at Risk	Percentage of Actual to Expected Deaths	Years of Insurance	Numbers at Risk	Percentage of Actual to Expected Deaths
1	8,482	44·5	0	63,644·5	44·6
2	7,079	66·3	1	116,565	71·5
3, 4	11,004	75·9	2	103,312·5	88·9
5	4,069·5	83·4	3–5	252,291·5	98·6
6	3,365	94·0	6–10	288,416	105·1
7–9	6,698·5	106·4	11–15	170,529	108·7
10–12	3,018	105·0	16–20	96,560	105·2
13–15	1,895	93·3	21–25	52,939·5	104·5
16–22	2,084	89·3	26–30	26,232·5	103·5
23–34	653	80·2	31–63	18,468	102·1
Total	48,348	...	Total	1,188,958·5	...

In order to show more clearly the law that the different figures follow, I have represented them graphically in the following diagram, in which the abscissa is the year of insurance, and the ordinate is the percentage of the actual deaths (or claims) to the expected. Here we see that the features of Mr. Spens's curve are very similar to those of the *Provident Life and Trust* Office; but in the former the maximum of the curve is much larger than in the latter, and the time in which it is attained is somewhat longer. These differences should admit of explanation by means of the different rates of lapse experienced in the two companies; but as no information is given by Mr. Wing as to the rate of lapse in his office, it is not possible to make any comparison on this point.

T. B. SPRAGUE.

The abscissa is the Year of Insurance; and the ordinate is the percentage of the Actual to the Expected Deaths.



THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1886.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	144,649,252	3,702,510	148,351,762
Adjustment for two Returns discontinued (£7,566, and a Deficiency Balance of £4,976) .	– 2,590	...	– 2,590
	144,646,662	3,702,510	148,349,172
Premiums	12,846,925	3,550,035	16,396,960
Consideration for Annuities . .	601,187	...	601,187
Interest and Dividends (less Tax)	6,047,412	139,392	6,186,804
Increase in value of Investments .	160,055	...	160,055
Fines, Fees, &c.	7,239	224	7,463
Capital Paid-up	10,040	16,213	26,253
Customs Timber Measuring, &c. .	2,795	...	2,795
Donations (Itinerant Methodists) .	3,946	...	3,946
Miscellaneous	58	796	854
	164,326,319	7,409,170	171,735,489

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	11,349,172	1,366,537	12,715,709
Cash Bonuses and Reduction of Premiums	950,747	...	950,747
Annuities	658,331	15	658,346
Surrenders	780,575	7,799	788,374
Commission	607,064	934,877	1,541,941
Expenses of Management . .	1,258,784	568,572	1,827,356
Bad Debts	5,366	318	5,684
Decrease in value of Investments .	33,765	483	34,248
"Losses, Bad Debts, and Defalcations" (Briton Med. and General)	43,061	...	43,061
Interest on Capital and Dividends and Bonuses to Shareholders .	695,950	6,129	702,079
Transfers to other Accounts . .	9,237	246	9,483
Miscellaneous	686	...	686
Balance* at the end of the Year .	147,933,581	4,524,194	152,457,775
	164,326,319	7,409,170	171,735,489

* This Balance includes the whole of the Life and Annuity Funds (£147,144,390), and, in addition, the Capital of those Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1886).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,407,309	133,240	11,540,549
Life and Annuity Funds	142,751,707	4,392,683	147,144,390
Fire Funds of Companies transacting Life business	8,565,461	...	8,565,461
Marine Funds of Companies transacting Life business	606,169	...	606,169
Reserve Funds	3,328,387	...	3,328,387
Other Funds	427,434	533	427,967
Profit and Loss Balances	2,112,760	...	2,112,760
Depreciation and Investment Balances	256,608	100,000	356,608
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,515,576	21,331	3,536,907
Outstanding Accounts	466,491	129	466,620
Temporary Loans	80,937	...	80,937
Sundries	11,887	20,000	31,887
	174,633,526	4,667,916	179,301,442

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	75,547,609	131,328	75,678,937
Loans on Policies	8,195,510	5,081	8,200,591
" Rates (and Rent-charges)	21,579,122	1,508,117	23,087,239
British Government Securities	5,018,436	598,925	5,617,361
Indian and Colonial Government Securities	11,309,846	...	11,309,846
Foreign Government Securities	4,067,940	...	4,067,940
Debentures	12,750,576	409,466	13,160,042
Shares and Stocks	11,387,202	...	11,387,202
Companies' own Shares	590,565	...	590,565
Land and House Property, and Ground Rents	9,777,524	1,604,366	11,381,890
Life Interests and Reversions	3,273,184	50,893	3,324,077
Loans on Personal Security	1,290,134	16,227	1,306,361
Agents' Balances and Outstanding Premiums	3,560,034	216,247	3,776,281
Outstanding Interest	1,598,747	47,595	1,646,342
Cash, Bills, Stamps, &c.	4,473,860	76,664	4,550,524
Customs Timber Measuring Balances, &c.	1,924	...	1,924
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Outstanding Accounts	4,591	...	4,591
Deficiencies, Preliminary Expenses, &c.	131,722	3,007	134,729
	174,633,526	4,667,916	179,301,442

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's* SUMMARY (1886), *when compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
	£	£
INCOME		
Premiums	+ 291,128	+ 260,854
Consideration for Annuities	− 43,087	...
Interest and Dividends (less Tax)	+ 129,354	+ 19,096
Net Increase in Value of Investments	− 31,836	− 711
OUTGO		
Claims	+ 873,636	+ 116,287
Annuities	+ 25,240	...
Surrenders	+ 7,670	+ 2,094
Commission	+ 27,072	+ 64,284
Expenses of Management	+ 63,650	+ 40,374
LIABILITIES		
Paid-up Capital (including sundry Shareholders' Balances)	− 2,496	+ 16,213
Life and Annuity Funds	+ 3,205,695	+ 805,472
ASSETS		
Mortgages (including Loans on Rates and Rent-charges)	+ 868,708	+ 264,194
Life Interests and Reversions	+ 58,774	+ 2,282
Loans on Policies	+ 196,614	− 62
British Government Securities	+ 169,989	+ 104,824
Indian and Colonial Government Securities	+ 609,679	− 4,510
Foreign Government Securities	+ 170,131	...
Debentures	+ 338,250	+ 193,080
Shares and Stocks	+ 989,004	...
Companies' own Shares	− 22,492	...
Land and House Property and Ground Rents	+ 393,061	+ 263,796
Loans on Personal Security	− 15,221	− 2,645

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 102, of which 93 have been classed as Ordinary, 8 as Industrial, and 1 appears in both Classes, this Company's Return showing its Ordinary and Industrial business separately.

During the year two names have been removed from the official List of Companies, namely, that of the Emperor, which went into liquidation, the Policies having since been largely exchanged; and the Masonic and General (Limited), which went into liquidation.

CORRESPONDENCE.

A FEW LAST WORDS ON GRADUATION.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Mr. Sprague has conclusively proved both his propositions, which, I think, may be shortly stated as follows, namely:

First—A formula calculated to third differences will not give fourth differences correctly.

Second—An arithmetical graduation is not suited to a geometrical series.

The formulas which you have done me the honour to publish were stated in distinct terms to be arithmetical graduations correct to third differences only. It has not been claimed for them that they are applicable to every order of differences and to every conceivable series.

The extent to which formulas of the kind "distort the facts", &c., in graduating a table of mortality, has been shown by Mr. Woolhouse (*Journal* xv, 396).

Anyone who is satisfied with the measure of accuracy attained when the formulas are used for the purpose for which they were furnished, will find in them a ready means of testing what a graphist has been doing—whether he has* "faithfully reproduced every well pronounced characteristic in the original"; or whether he has applied his method with the courage of a railway manager who, desiring to represent his line as more direct than another, draws a curve differing little from a straight line, and groups his facts so as to bring every important town in the neighbourhood upon that curve.

My belief that Mr. Sprague himself will always employ the graphic method with skill and fidelity has been expressed in a previous communication. It is, however, only fair to notice that, with my untouched results before him, he has been able to rectify and beautify his own to any extent necessary for claiming superiority; and that, supposing this attained in the latter case, it is possible that similar labour bestowed upon the former might produce something better still.

I am, Sir,
Your obedient servant,

February 1887. **J. A. HIGHAM.**

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* Mr. Peter Gray's introduction to the Institute Tables (p. ix).

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

A Proposal for the Amendment of the Law relating to the Assignment of Policies of Life Assurance. By CHARLES DANIEL HIGHAM, *Assistant Actuary of The Metropolitan Life Assurance Society.*

[Read before the Institute, 28 March 1887.]

"THE Institute has now fairly started on its new career of usefulness as an incorporated society." Usefulness, then, in the admirable words of the Council on the 5th June last, is to be the ruling principle of the Institute's new and corporate life, as few, who are acquainted with what has been done with but small opportunity, will deny it has been of its action in the past. But for its usefulness to be thorough, the Institute must be concerned with both the theoretical and practical sides of our profession in all its branches—recollecting as well its commercial basis as its mathematical speculations, considering its connection with medical and legal specialties, collecting data, comparing methods, proving uncertainties, destroying chimæras, teaching students and testing their acquirements, not to mention accumulating a library for reference and instruction—always advancing, suggesting, restraining, reforming. That the public will, on all occasions, rightly appreciate our work is not so certain, nor is it material; but there are some matters which may well come before us for discussion of which the value will appeal to the understanding of the least instructed, and one of them is now submitted for consideration. A reform is proposed which, when it has been put into shape by those better qualified for the purpose, will be conducive, as I believe, of much public good; and the many who are interested in life assurance policies, will recognize in one of the early actions of the Institute's renewed youth, an acceptance of the responsibility, and a justification of the dignity, conferred by Her Majesty's Charter.

Few, probably, will deny that some radical change is desirable in the modes by which policies of life assurance, and such only are referred to in this paper, are legally and equitably assigned. The law is complicated, perhaps from the relaxation of the chose-in-action doctrine having been brought about gradually, and the ordinary person cannot or will not understand it. He persists in believing that the possession of the policy is the principal thing necessary; or he registers a notice, under the statute as he imagines, that it has been assigned (without ever a deed), thinking his title is thereby secured, and he is bitterly disappointed when he finds that he cannot raise a loan, or that the consent of the assured, or possibly an official receiver in bankruptcy, must be obtained before a surrender can be carried through; moreover he tears up the documents when an advance has been repaid, under the impression that there is an end of it. Notices, too, of deeds are sometimes never served at all, either by accident or design, a way being thus opened for trust policies to be mortgaged in error or fraud; and "understandings" go on between friends who dread a lawyer's bill, and many are wronged. Meanwhile, through all, the companies decline, as firmly as politely, to give an opinion as to title until they are compelled to make the investigation for their own purposes, nor can they well do otherwise. It is proposed then, that the Institute shall, with the assistance of counsel, draft a Bill to abolish anything that remains of the chose-in-action theory, and to provide that, in future, a policy shall be assigned as far as possible as a chattel, much as a share in a joint-stock company is, the deeds being lodged at the company's office, and a register of policyholders and mortgages kept there, so that at any time a certificate can be issued showing who is entitled to the benefit of any assurance, and what loans have been raised upon it. To promote such a Bill in Parliament would probably be too serious an undertaking for the Institute's powers and resources; but if the Bill were drafted, and copies sent to the Board of Trade and other public departments, as well as to any private Members of either House who might be willing to give consideration to the subject, it may be hoped that if ever there should be a lull in burning legislation, and some much-needed social measures receive the attention for which they have long waited, a Bill such as is proposed would be thought of sufficiently wide-spreading interest to be supported and passed. Anyhow, the Institute would have done its best, a better way would have been pointed out, and those who declined to be led off the old paths would have but themselves to thank for any inconveniences they suffered.

The criticism which this paper will presently receive, will, it is hoped, draw forth some information, from those who are in possession of it, concerning the law in this matter abroad, as to which it is not easy to learn anything; but I have not been able to ascertain that the practice in Canada and the United States differs materially from those of the United Kingdom: I have been told, too, that the same remark would, to some extent, apply to the Continent also, save that there is ordinarily much less formality there (assignments, for instance, with a space left for the name of the beneficiary to be filled in at a future date not being unknown), while nomination by deed of the parties who are to profit by a policy is permitted under certain restrictions. In the Australasian Colonies, however, there have been various attempts to remedy the evils complained of; and though the Colonial Acts are not always very carefully drawn, it will be remembered to the credit of the new country, that several efforts have been made in it to simplify the procedure in question, before any action worth speaking of has been taken in old England. In Victoria, by The Life Assurance Companies Act, 1873 (37 Vict., No. 474—see *J.I.A.*, xx, 68 & 73), every assignment of a policy *must* be by an endorsement thereon in an absolute form given in a schedule to the Act, notice must be served on the company, and the transfer itself registered at the office for validity; mortgages and trusts are effected by way of defeasance or declaration of trust by a separate document with which the company is not concerned, nor is it bound to see to the application of the policy moneys. In Tasmania, The Life Assurance Companies Act, 1874 (38 Vict., No. 6) provides that assignments *may* be as above specified, the other provisions being similar: The Life Assurance Companies Amendment Act, 1885 (49 Vict., No. 21), however, besides arranging for the endorsement and registration, as if assigned, of titles acquired through death, bankruptcy, &c., and for the issue when necessary of certified copies of policies, lays down that a company need not take notice, save in case of fraud, of any interest created since the passing of the Act of 1874, unless it be shown by a memorandum duly endorsed and registered according to such principal Act. In South Australia, the Life Assurance Companies Act, 1882 (45 & 46 Vict., No. 277—see *J.I.A.*, xxvi, 38 & 50) makes the procedure the same as in Tasmania before 1885, but the form of memorandum of transfer provides also for the assignment, if desired, of a part instead of the whole of the sum assured. Finally,* in New Zealand, The Life Assurance Policies Act, 1884 (48 Vict., No. 31) is a bold and comprehensive measure, including law on somewhat similar lines to that which I venture to propose, and I greatly regret not to have had an opportunity of becoming acquainted with the enactment until after this paper had been fully prepared. This Act, *inter alia,* supplies forms which, or words to the effect of which, are compulsory in case of sale or mortgage, whether to the company itself or other persons, and it furnishes sets of covenants, besides providing for bankruptcy, &c.; it lays some stress on the possession of the policy, though provision is made for the issue of certified copies in case of loss; and it requires a memorial of all transactions to be endorsed on the policy at the office, and a record thereof kept there, absolute assignments being given up, and mortgages, or certified copies thereof, lodged: trusts must be raised by separate instruments, the company being unaffected thereby; and the company may disregard unregistered interests, and a registered policyholder is safeguarded, except in fraud. The Life Assurance Policies Act 1884 Amendment Act, 1885 (49 Vict., No. 20), moreover makes it obligatory in addition for any future assignment, other than a mortgage, by way of ordinary transfer to be itself endorsed on the policy.

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* Probably there are other Acts than those mentioned dealing with the matter, but it has not been my good fortune to light upon them. In our own country, too, Mr. Scratchley and others in 1863 petitioned the House of Commons that policies should, *inter alia,* be made assignable by simple endorsement like bills of exchange: and in 1864 he published the draft of a Bill which contained a scheme for nominating, by endorsement on the policy to be acknowledged by the company, certain relatives, when desired, to receive the sum assured; as also provisions for recording at the office, and noting on the policy, the particulars of assignments thereof, which were then to be legally and equitably efficacious. Copies of both Petition and Bill will be found in *Tracts, Vol.* 18, in our library.

If, with these examples, it be agreed that legislation of some kind in this matter is both desirable and to be accomplished, it may not be thought too presumptuous to detail certain provisions which it would seem a statute with such an object ought to contain. The proposed Act then, after its preamble, short title, and interpretations—among which a further definition of "policyholder" is perhaps advisable—will, no doubt, begin by decreeing that, in the case of all policies issued after a defined future date, the methods to be designated, and none other, shall be valid for all purposes of assignment. Moreover, after allowing a year's grace from such date for the service of notices which there has been failure to send in, companies will be empowered, if they think fit (which option will allow them the opportunity of freeing themselves from expenses in the matter), to register the names of the holders of old policies; and if a company decline to act on such power, a claimant will be permitted, at his own cost, to prove his title in a County Court (if the sum assured be small) or in Chancery, the Judge's order being decisive to require the registration of name as above: all such policies will then be on all-fours with those issued after the stated date, the company being completely indemnified in the matter, save for any *laches* of its own. Assignments will have to be, without exception, in certain specified forms, but, seeing that years may elapse before any action thereunder may become necessary (there being no question of dividends, as on a transfer of shares, to force earlier attention to the matter), it is very desirable that such forms, instead of being purchased of any law stationer, shall be supplied by the company itself, of the size and quality it may find most suitable for filing (in a guard-book, probably); and all future policies must specify the places at which such forms are to be obtained, and registered when completed. The company will be entitled to a fee on giving out a form of assignment (the requirement of a separate one for each policy will be convenient), and perhaps a charge not exceeding ten shillings will be welcomed by the assured as a less amount than he is at present liable to pay in law costs, and at the same time be a sufficient remuneration to the company for its labour and expenses. The fee, however, will also cover the subsequent registration of the deed, properly executed, within six months of the issue of the form (including the grant of the certificate of holding shortly to be described), but after that period a second fee will be payable therefor.

Three forms will be necessary, namely, for transfer, mortgage, and discharge, and the various wordings given in schedules to the Act will have to be strictly followed, but each form must also have a place prepared for its number, and for noting the date of its issue from the office, and the day (and hour) of its return for registration. The form of transfer will be to the effect that A. B., of &c., in consideration of &c., as beneficial owner (or as settlor,* if such be the case, or as mortgagee,* in the case of a transfer of mortgage), conveys and transfers to C. D., of &c., all his interest in Policy No. of The Life Assurance Company assuring £ on the life of ; and that the transferee duly accepts such interest. The mortgage will show that A. B., of &c., as mortgagor,* charges (or further charges) in favour of C. D., of &c., Policy No. of The Life Assurance Company assuring £ on the life of , for securing payment of the principal sum of pounds as the mortgage money advanced by the mortgagee. The discharge need not be more than that C. D., of &c., as mortgagee, discharges Policy No. of The Life Assurance Company assuring £ on the life of , from being security for pounds, being (or of) the principal sum secured to him thereon. An acknowledgment of receipt is not required in these forms, as it should be a separate document not given up to the company; but it is desirable that the transfer and mortgage should be executed by both parties, that the company may become acquainted with the individuality of a purchaser or lender, and in the former case, moreover, that he may be clearly brought under any possible liability which may have rested upon the assured. As in the Conveyancing and Law of Property Act, 1881, the Act will contain all general words and covenants for title according to the estates under consideration, including a declaration that if the assignment be to more persons than one, they will hold on a joint account unless they are specially described as tenants in common, their respective interests in this latter case being set forth in a separate deed with which the company will not be acquainted: it is possible that partial dealings with a policy will be sometimes conveniently effected by a conveyance to the assured and others as tenants in common. This recital in the Act of general words and covenants is preferable to a reference to any other statute for such clauses, in order that the whole subject may be dealt with in a single enactment, and that not necessarily a very long one.

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* It is desirable that the intention should be shown in one word, the Act making it equivalent to the "as settlor and as beneficial owner", "as mortgagor and as beneficial owner", or "as mortgagee and according to his estate" which are, I believe, now considered better. "Mortgagor" will also have to include the grantor of a further charge.

Among these required provisions, it must be laid down in the case of a sale that the transferor conveys absolutely to the transferee all his rights—of voting for instance if he have any such power, and herein a proviso will be needed that in the case of a policy standing in more names than one, the vote will appertain to the policyholder whose name comes first—and all his liabilities under the contract of assurance, so that the latter holds subject to the several conditions on which the former did at the time of the execution of the transfer, and will be entitled, for example, to receive all notices, and, of course, to sue and give receipts in his own name: it may be added, too, that a policy thus sold ought to cease to obtain return of income tax to the transferor.* Or as regards mortgages, it is necessary to provide that, if no other arrangement be made by separate agreement (in duplicate if so desired) between the parties, the date of repayment of the principal shall be six months from the date of advance, with interest at five per-cent. per-ann.; or, failing such repayment, at six months' notice on either side (but at once if the policy become a claim), the like interest being paid half-yearly on the anniversaries of the two dates specified. Covenants also for such repayment (with reconveyance to follow) and payments, and for due discharge of all premiums, and production of the renewal vouchers to the mortgagee within five days of each renewal date, as well as that the mortgagor (if the life assured) will not by any action of his invalidate the policy &c., are requisite; as, moreover, is a power for the mortgagee, at his option, to make any essential disbursements, and recover the amount thereof with interest. Particularly, too, must be included a power of sale (or surrender to the company) in whole or in part, its exercise, however, being withheld until there has been default for three months, after service of notice requiring it, in fully repaying the mortgage money; or unless some interest has been in arrear and unpaid for two months after becoming due; or there has been a breach of some other condition of the loan: and further such a vendor must be clothed with any necessary authority to give a title to a purchaser, a statutory declaration by the former (that an amount is in arrear, for example, and he authorized to sell) being made conclusive evidence to the latter of the truth of the facts contained in it (any person damnified having his remedy in damages against the declarant), the buyer being also freed from any responsibility as to the disposal of the purchase-money in the due order to be specified in the Act. Or if a settlement be in question, the provisions as to keeping a policy in force must be specified, and so forth, without going into further detail now, save to suggest that sometimes the settlor may reasonably be included as one of the policyholders.

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* To what extent (if any) the premiums on sold policies are permitted to procure return of tax, on production of an official certificate of renewal, cannot be known, but an additional reason is given for the requirement by the Department of the actual renewal voucher.

Other implied covenants (for instance, as to joint and several action when there are two or more holders of a policy) need not be mentioned here, but it will not be forgotten that all these provisions will be liable to any desired alteration, not inconsistent with the Act, by agreement between the parties, with which, however, the company will have nothing to do, for a statutory declaration as to facts will always be a sufficient testimony to it and others, if action have to be taken before the sum assured becomes payable. In particular, the rate of interest will very frequently be so varied, but it is better for it to be managed thus, rather than recited in the deed lodged with the company, as not only it does not really concern the office, but it cannot be always readily specified (in repayments by way of annuity for example), nor is it even necessarily pounds, shillings, and pence at all. There does not appear to be any need for special clauses in the Act to meet the case of a company lending on its own policies, though an affirmation is desirable that the registration of the name of the company as a mortgagee does not affect its character as assurer; all question will thus be avoided, and any doubt as to the advisability of the intervention of trustees.

The Act will provide that mortgages rank according to priority of registration, save that the transferee of a mortgage takes the place of his transferor; also, that if a mortgagee who has more than one charge is partially paid off, the latest of his loans shall be first got rid of: it will make clear, too, that a sum assured, or surrender-value, is to be paid to the possessor of the earliest charge without any further concurrence, and he, after satisfaction of his own advance or what remains of it, for in cases of repayment by instalments it will probably not be thought necessary to formally discharge at the company's office each several amount refunded, will distribute the balance in order among the successive mortgagees, and finally the policyholder himself.

When a deed therefore, in one of the forms specified, has been issued from the office and duly filled in and executed, it must needs be stamped; and it would be very convenient if an opportunity were to be taken—probably no fiscal provision could be included in a general Act—to make the stamp duty on a transfer of a policy, except in the case of a mortgage or discharge, dependent on the sum assured only: possibly half-a-crown per-cent. thereon will be considered sufficient if the proposed alteration of procedure take place, bearing in mind that there will then be no escape for any assignment, while further deeds will very often be necessary too. Having been stamped, the Act will require that the deed be given up to the company for registration, and at the office, the time of its receipt will be noted and the deed numbered, the company having been first satisfied as to identity, if necessary, and that the person who has executed the deed is either a duly-registered policyholder or mortgagee, or the legal representative of such: the officials will be entitled to call for any reasonable evidence on such points; reasonableness to be defined, if it be requisite, in Court as above, and the costs imposed on the party in the wrong, the Judge's order being conclusive. If all be in order—and possibly, to repress fraud, a private letter will have been sent to the last known address of the assignor mentioning what is proposed to be done—the company will be bound to receive and preserve the assignment, and at once register the particulars of it in a book to be kept for that purpose, the only incident of title noted being "as tenants in common" where such words appear in the deed. The following columns will suffice in such a book, the pages being also divided by horizontal lines into spaces of a size which it is probable will be large enough to contain the record of all dealings with any one assurance, and to such spaces reference from the policy register must be made: possibly in some cases a separate book for mortgages and their

Policy No., Life Assured, and Sum Assured	Transfers				Mortgages *(and Transfers of Mortgages)*					Discharges					
	No. of Deed	Date of Deed	Name of Policyholder to whom transferred	Date of Registration	No. of Deed	Date of Deed	Name of Mortgagee	Amount of Loan	Date of Registration	No. of Deed	Date of Deed	In or towards Discharge of		Amount Discharged	Date of Registration
												Deed No.	Name of Mortgagee		
								£						£	

discharges will be preferred. The book had better not be encumbered with addresses &c.; the date of registration will be that of the deed's receipt by the company, which will not be until any necessary further evidence has been submitted; and in the case of a transfer of mortgage, the name of the original mortgagee will be ruled through in a different coloured ink, and a reference made to the number and name of the deed noted below, against which latter, too, no amount of loan will be specified, but a reference given instead to the number of the deed above.

On transmission of title by force of law, namely by death, bankruptcy, lunacy, writ of execution, &c., the parties becoming entitled will have the power, though not be compelled, after submission of the proper evidence, to execute a deed from themselves in their representative character to themselves as individuals, or to any other persons; but until such a step is taken their capacity will not be recognized, and the person to be displaced will remain the policyholder. In many cases, however, it will be sufficient to let the registered title run on until the policy become a claim or be surrendered, the amount being paid, as at present, to those entitled at the time, for it will not be necessary to register the name of a new holder for the mere purpose of giving a receipt. No alteration, however, must be made in the assignment register without a proper deed, save only, after production of the usual proof, the removal of the name of a deceased holder from a joint account, or the necessary rectification on any change of name by marriage or otherwise: nor, of course, will any other persons than the executors or administrators be recognized when representation of a dead man is necessary, no legatee, for instance, having any *locus standi* with the company.

The registration of the deed, whatever its nature, having been thus accomplished, the company will issue within seven days a certificate of holding under the hand of the principal officer, detailing the title as it then exists. The wording thereof will be provided in a schedule to the Act, to the effect that A. B. is the holder of Policy No.   of The   Life Assurance Company assuring £   on the life of   ; and, when necessary, it will be added, subject to an existing charge of   pounds in favour of C. D. under registration date of   18   , of   pounds in favour of E. F. under registration date of   18   , &c. The date of the certificate will probably be the date of registration when the acknowledgment of a new deed is being thus completed. Any person interested in the policy will, however, have the right to obtain a certificate of holding at any future time, on requesting it in writing and paying a specified fee—perhaps not more than half-a-crown, or one shilling. The certificate should be limited to the names of the persons in question, tenants in common being so described, but no addresses or professions, which are liable to much variation, being given.

The Act will lay down in the clearest possible terms, that in the case of policies which come under it, its forms are the only ones for valid assignments, and that no deed is of any force until it has been duly registered. The certificate of holding will be good evidence of title, and the persons registered will, according to their powers, stand in the place of the assured for all purposes. No limitations (except as to tenants in common) or trusts will be permitted to be expressed either in the deeds or in the policy itself, they being always effected by way of defeasance or declaration of trust by a separate instrument; and the company will be freed from concern with express, implied, or constructive notice of any kind, as well as with the application of moneys it pays away. Nor will notice that any dealings have been irregular, or even fraudulent, affect the company, until an order of Court has set them aside; but power will remain to the Judges to enforce any necessary equities against parties who have received payments on account of policies registered in their names. The Act will, moreover, enable companies to dispense on all occasions with production of the policy itself, and protect them in doing so, and that without any requirement of a bond or letter of indemnity; it will also compel such a practice when the contract has become a claim, for, with this provision and the proposed regulations as to assignments, the possession of the policy will become of less and less consequence, save as evidence of the various conditions of the assurance. A form of deed-poll for declaring trusts in favour of a wife, or husband, and children, with all necessary provisions that cash (save for paying premiums) or assurance subsequently raised out of the policy shall follow the trust, must be given in a further schedule, that the facilities afforded by the Married Women's Property Acts may be maintained, though in a different form. If there be any abrogation of existing statutes, or abolition of current procedure as to assignments or notices, necessary, it will follow and conclude the Act, which, it may be hoped, will not contain a clause excepting any part of the United Kingdom.

It is much to be desired that any enactment of the nature proposed should be by a separate statute dealing with the one matter only, entirely distinct from any revision of the Life Assurance Companies Acts, in order that the complete law as to these assignments may be found in a comprehensive and handy form, for the use of the very much larger general public who are holders of policies, than the special class to whom the various requirements as to life offices are of interest. And, at the risk of being thought to have wandered from the subject under consideration, I would add that if any legislation be attempted, an admirable opportunity would present itself of, at any rate, empowering companies to make, of their own accord, advances against the surrender-values of policies, for the purpose, when necessary, of keeping them on foot, the amount of premiums so paid, with interest at five per-cent. per-ann., being a statutory first charge, as from the registered holder, in priority to all mortgages whatever. A memorandum of such a charge would, of course, be made in the assignment register, with a reference to the statute instead of to a deed.

It only remains to mention briefly some of the advantages and disadvantages of the innovation contemplated. And, first, the officials of a company, in respect of policies under such an Act, would always be able to give a plain answer to a plain question, and say definitely who is at any given time the holder of a given policy, and to what charges it is subject; which, it will be readily admitted, would be a more satisfactory reply than the guarded answers, and references to future consideration by solicitors, which are now necessary. They would also be freed from the need of constantly watching all communications which came to hand, to see that they did not contain implied or constructive notice of any claim on, or disclaimer of, a policy; and they would be spared, too, the having to answer with considerable caution enquiries, becoming yearly more and more frequent on the part of proposing assignees and others, as to previous notices received, for a certificate of holding could always be procured and would suffice. The public would not be in need of legal advice on many occasions of loans and sales: while the assurance offices, on the other hand, would be delivered from the trouble and expense, and responsibility withal, investigation of title as at present causes; not to mention the erroneous imputations, or, at any rate, uneasy suspicions, sometimes produced by the calling for old deeds, forgotten by, or not known to, the claimants. Lawsuits, friendly or otherwise, with the company for a party, would almost cease; nor would the suggested registration be much more onerous than the receiving and acknowledging notices of assignment according to the existing statute. Trustees could, without question by the office, for they would not be known in that capacity, effect surrenders in exchange for paid-up policies, or raise money, out of bonus or otherwise, to pay premiums or for advancement of children; and mortgagees could make the best of a bad borrower by a speedy sale of the security in their hands: if it be hinted that such powers are greater than it is in all cases wise to confer, it must be recollected that no more is asked for than is already customary when stocks and shares are settled or mortgaged; and also, that while, as to probably a few, the Courts are stern to require the fulfilment of trusts and performance of contracts brought to their notice, many are now hampered and kept in by legal fences from desirable objects, and that which was intended for the solace of future anxieties, becomes itself sometimes a present burden to be borne. Private deposits and secret assignments, as well as informal notices, would be put an end to; agreements as to discharged loans might be, after an interval, destroyed; and little harm would be done when a man mislays his policy, while he would never have a chain of title to either preserve or lose. Solicitors would probably welcome relief from having to serve notice on the assurer after completion of a transaction; but, as such notices are not invariably remembered, they may, perhaps, object to any clause, as proposed, empowering the registration of the title to previously existing policies, after a year's grace from the time at which the new law begins. If they feel that accustomed work will be taken out of their hands, they will not forget that many agreements and deeds of trust will remain to be drawn, and often they will have the care of transactions of this nature, filling in the forms, and carrying the matter through. One of the reasons, moreover, which makes for the proposed changes, is that so many persons in the past have been unwilling to consult them, though often at the cost of learning the truth of the well-known proverb; and, after all, they will be the first to recollect that to provide occupation for their profession is not one of the objects of life assurance.

It may be added, that if an Act on the lines proposed be ever passed, the constant effort on the part of the companies to render facilities to the public, stimulated by friendly rivalry and the competition which is never a disadvantage until it is carried on at unhealthy pressure, may be expected at no very distant date to lead to the offices themselves, when so desired, arranging transfers and the like—receiving the moneys as stakeholders, procuring execution of the deed, and delivering the cash to the one party and the certificate to the other.

Such is the scheme advocated, and if it be thought that the sketch given has been needlessly filled in, I would apologize for the detail, and plead the importance of any proposal for legislation being made plain to others beside ourselves—if ideas, and occasionally a few words, be recognized out of existing statutes, they at any rate may be assumed to be correct. For the rest, no claim is made that it is for more than a beginning, but if the wisdom of the Council should appoint a committee, to obtain learned assistance and endeavour so beneficial a change in the law, the good that would be effected could not be measured by the labour, and in the general approval of a public spirit which is the glory of any profession, the Institute would have its reward.

Finally, I would crave the indulgence of those of us who are also at the bar, for the manifold shortcomings of a paper on a legal subject, by a writer without legal acquirements.

DISCUSSION.

The **PRESIDENT** (Mr. A. Day) said that the possession of the policy seemed to him to be a very important element. A case had come under his notice in which a man who had insured his life borrowed some money from his mother and handed her his policy; he gave her no memorandum of deposit and no notice was given to the office. The mother afterwards desiring the return of some of her money applied to him, but he wrote expressing his inability to make the repayment, and saying that she need be under no apprehension as she held his policy. In this letter he did not refer to its being a policy in any particular office, nor did he give its number, or describe it in any way. When he died, no one would administer to his estate on account of prospective liabilities, but the possession of the policy and that letter were held to be sufficient, and the office paid her the claim. The habit of destroying an assignment or mortgage after the money had been repaid was very constant and caused serious inconvenience. Many of the difficulties which they met with arose from the obstinate way in which people refused to consult their legal advisers. Members of a mutual office seemed to have the notion that they had a right to go to the office for advice, and he was not sure that there was not some reason on their side. It would not be a very great trouble for the offices to keep certain forms of assignment, for instance, which they might issue to members; and provided they printed abundant instructions as to the use of those forms, and cautions about the preservation of the assignments and re-assignments, companies might very well assist members in that way. Some people had a clever way of getting opinions as to title, which was coming to the office to borrow money upon the policy, in which case, of course, the title was investigated, and what was good enough for a loan would equally answer for a death claim. One interesting point in Mr. Higham's paper was the reference to the colonial Acts of Parliament, whereby policies were transferred, principally by endorsement, and forms of transfer were given, generally confined to one form, namely, an absolute assignment. Mr. Higham seemed to follow that idea at first, but subsequently he spoke of having three separate forms, one of mortgage, one absolute transfer, and one of settlement. If the colonial plan was found to work advantageously it would, he thought, be adopted in due time in this country. He might say that the Council were not responsible in any way for the views that had been expressed. Mr. Higham would probably bring his proposition in due course before the Council, as the executive of the Institute, if he thought fit to proceed with it.

Mr. G. **HUMPHREYS** said they had to consider the paper from two standpoints—the official and the legal. Speaking solely as to the effect that such changes as those proposed would make upon the working of an office, he thought they would be the means of increasing the office labour without proportionately improving and facilitating matters. Mr. Higham proposed that the Institute, with counsel's assistance, should draft a bill for abolishing anything that remained of the chose-in-action theory. He, however, thought the Institute would be travelling out of its province in doing so, and that it would be more appropriate for the Law Society, for instance, with actuarial assistance, to undertake such a duty should it be thought desirable. The suggestion that policies should be regarded as shares in a joint-stock company would bring about these alternative results: either the register of the company was to be the real evidence of the title, according to Mr. Higham's paper, certificates being issued from time to time—a policy such as it was at present would not be needed—or there would have to be issued both policy and certificates. In the first case, considerable confusion and annoyance would arise. Holders of certificates would have imperfect means of knowing the date of the last certificate issued; and the second condition would only tend to complicate matters and increase the labours at the office. Mr. Higham laid down in his paper, as a necessary consequence of the transfer under the proposed Act, that all rights, including those of voting, should be transferred. Now, as it did not appear that the company under the Act would have any power to veto the transference of a policy, such a condition as that would be very inconvenient and impolitic, because any company, especially a mutual society, might have a partner forced upon them without any possibility of preventing it; whereas at the present time most insurance companies had power to prevent the transference of shares. That would be a strong objection to making a policy like a share in a joint-stock company. It was proposed to make the stamp duty on a transfer of policy, except in the case of mortgage or discharge, dependent on the sum assured only, but that would make a small loan on a large policy pay more than a large loan on a small policy. With regard to making loans on policies, the new plan would not do anything to facilitate the matter, either to the owner of the policy or the office. At present the owner brought the policy to the office, and, the title being clear, he could, on signing the ordinary loan document and paying the stamp, at once receive the money. The repayment of the loan was equally simple. If, however, a policy was to be treated as a share, it must in every case be properly transferred, whether with or without the equity of redemption clause, and at the time of repayment the signature of the company's trustees would be required, which would cause delay. The transfer would also take a very much longer time to carry out than at present. Mr. Higham had drawn up a new record book. This did not appear to be intended to supersede any other, and it looked like adding further troublesome labour upon the officials of insurance companies. The difficulty would be considerable in so arranging such clauses and making such an Act of Parliament that the officials of an insurance office would be enabled to act under it without some fear of getting into difficulties. He agreed that some important changes were desirable. And no doubt many of the difficulties which seemed to interfere with the scheme would be smoothed away if the subject were taken in hand and dealt with effectually.

Mr. J. Whitcher said that in all matters relating to life insurance, the Institute had a right to propose legislative improvements and to be foremost in such measures. In this matter, he thought it was quite within bounds for the Institute to ask the advice of the

Law rather than for the Law to ask the advice of the Institute, as had been suggested. With regard to the possession of a policy, he was not sure that in the instance cited by the President, the letter from the assured saying that his mother had got the policy was not after all of more importance than the holding of the policy itself. At any rate it was of equal importance, and that was all that Mr. Higham intended by saying that the actual holding of the policy was not by itself quite enough. It had happened within his (Mr. Whitcher's) experience, that such possession was not sufficient. A policy was held by assignees, who became holders owing to an absolutely illegal act on the part of the assured, who had no right either to have had it in his possession, or to have assigned it. In the absence of such legislation as Mr. Higham had suggested, that policy was transferred from one hand to another, and it was only by way of family arrangement, involving loss to some parties, that the evil of that transfer was remedied. Possession of the policy in that case was certainly not an effective proof of title. Another point that had been alluded to was the supply of forms. Offices that supplied forms only did it in a friendly way; but he thought that lawyers would not thank them for so doing, because such action would be held to be an interference with solicitors' rights and privileges. As to the existence of policies being unnecessary if certain suggestions made by Mr. Higham were carried out, the speaker thought that such existence at present was simply a form of registration of right, because there were assurance societies in which no policy was issued whatever. It was possible for a contract of assurance to be a chose-in-action as at present, even if it simply took the form of the entry of the assured's name in certain books, as (in other cases) that of an inscribed stockholder is entered. The bare existence of a policy was not absolutely necessary, that document being simply an evidence, as it were, of the registration of the assured. That was a side issue, and Mr. Higham would not attach very much importance to it. It might be awkward to confine registration of transfer of policies to the office which issued those policies, but it would not be beyond the spirit of Mr. Higham's scheme if, in the event of such an Act being passed, they should ask the Government to establish a public registration office—such as the Land Registry Office, only, perhaps, on amended lines.

Mr. H. C. L. SAUNDERS asked whether the Act of 1867 had proved an utter failure. He could not personally say that the Act of 1867 was a very great reform over the old law, but he believed from his own observation anterior to its passing that it was a reform, and that if there were two or three minor amendments to it, they would not need the heavy machinery which the foreshadowed Act would put upon them. Viewing the subject from its more practical side, he thought it was a great disadvantage to teach the assured that the policy was of no value at all. What they ought to do was to implant in their minds the idea that it was a certificate which they should treasure and take care of. If they wished to transfer it, let them do so by the ordinary process. It seemed to him they were going out of their way to teach the public that dealing with a life policy was a difficult and arduous thing; it ought rather to be shown that it was a very simple thing, and that if they would comply with the Act of 1867 they could effect all their purposes at a very small cost. Mr. Whitcher had mentioned the land registry, and he (Mr. Saunders) thought that an independent bureau was necessary, as it was not right that an assurance company should be its own judge of the law.

Mr. G. E. **HUMPHREYS** (a visitor) agreed that there was great obscurity in the present state of the law on this subject. There could be no question that to have the law on any subject in simple and symmetrical form was a great advantage, and he dissented from a remark made by Mr. G. Humphreys, in which he suggested that this was a matter for the incorporated, or some other, law society. If this particular reform was to be carried out, it was, he thought, peculiarly the province of the Institute (particularly under its new conditions) to come forward and help the public to what would be a great benefit. Mr. Higham proposed a radical change in the "law by which policies of life assurance are assigned", and the question arose, what was the nature of the change, and whether it was a really radical one. The actual changes proposed were, he gathered, first, to carry further the principles of the Policies of Life Assurance Act of 1867, and section 25, sub-section 6, of the Judicature Act of 1873. By the 1867 Act, a person who could give in equity an effectual discharge to the company was enabled to sue in his own name on certain conditions. Mr. Higham wished to change this part of the law for the benefit of the public, and to say that registration on the books of the company was to be evidence of title between the parties to a transfer of a policy. It was often supposed that the Act of 1867 made notice to the company the evidence of title between the parties as well as between the company and the parties. If notice was given to the company it was evidence as against the company, but not as against other people, and that was to be altered so as to make registration on the company's books the only evidence of title. Mr. Higham also wished to benefit the companies by relieving them of all responsibility with regard to informal notice. He further wished to simplify the transfer of policies by the use of certain statutory forms and simplified words and covenants, and to relieve companies from being in any way bound by informal or formal notice of trusts created upon policies. He was not able to follow Mr. Higham in his proposal to "abolish what remained of the chose-in-action theory", because a chose-in-action, or the right to recover the benefit of contract by legal process, was now, as regards most mercantile contracts, assignable at law as well as at equity. By the Act of 1867 policies of life insurance were on certain conditions brought among the exceptions to the old rule as to chose-in-action. The section in the Judicature Act provided that any legal chose-in-action might be absolutely assigned in writing though not by way of charge, but the Conveyancing Act of 1881, by permitting the receipt in writing of a mortgagee to be a sufficient discharge for any securities comprised in his mortgage, seemed pretty nearly to complete the emancipation of life policies. What remained in the chose-in-action theory that ought to be abolished? He did not see what there was. The policy of insurance must remain a chose-in-action. It was suggested that a policy was to be transferred like a share, but there was a most important difference between the policyholder and the shareholder. A shareholder was a member of a partnership firm, and the evidence of his title to be a member was properly the presence of his name in the list of members kept at the office of the company. A policyholder was one of two contracting parties. There was no connection and partnership between him as a policyholder and the company, but there was a contract with peculiar incidents which made a deed necessary as a binding evidence of what the conditions of the contract were. Mr. Higham recognized the practical necessity of having a deed, and that made a most important difference, because shares could in general be transferred without a deed. A deed was not necessary in the same way as with a policy. He did not say that this was fatal to the proposed system of registration, but it was an important point as bearing upon the manner of registration. It had been objected that the new process would make more work; but, bearing in mind the distinction between a policy and a share, registration should be on somewhat less simple a plan even than that which Mr. Higham suggested, because the policy ought to be produced in every transaction. The policy was the deed evidencing the conditions of contract, and so far from it being desirable to diminish the importance of the deed, it ought, if anything, to be increased. Mr. Higham suggested that the secretary of a company should decide as to the validity of title, but should be entitled, if the evidence offered were not sufficient, to call for such further reasonable evidence as he thought proper, reasonable evidence being defined by a reference to the Court. That would be throwing a very unreasonable burden upon life assurance companies. As to compulsory forms of transfer, there were very strong objections to any compulsory forms whatever, the intentions of the parties varying in each particular case. Even in the New Zealand Act, it was only said that the assignment should be "in the form or to the effect of" the words in the schedule. Mr. Higham suggested that any variation in the statutory covenants and provisions should be by separate deed, so that they would have one deed by its terms implying all statutory words, covenants, and powers lodged with the office, and another deed varying that deed in the possession of the parties; and this, to say the least, would give rise to considerable confusion and difficulty. He did not see why trusts should not be recognized by a company. When a trust was accompanied by an absolute assignment, the company could not do itself any harm by recognizing it. And it would be a benefit to an intending transferee to know that he was taking from a holder who held only as trustee. On the whole, he thought that Mr. Higham had made out a good case for simplifying the law on this subject.

Mr. A. H. Bailey said for several years past he had thought there was no part of the practice of life assurance more urgently needing amendment than that relating to the assignment of policies. Such assignments were on the increase. But he was surprised at the complicated reforms that Mr. Higham sought to introduce, and was rather inclined to agree with Mr. Saunders that an extension of the Act of 1867 would meet the case, the idea being, not that notices of assignment, but that the assignments themselves, should be compulsorily registered. As Mr. George Humphreys had said, he could not understand why policies should not be assigned in the same way as stocks and shares were assigned,—that is, that there should be a statutory form (and the form in the Act of 1867 would probably answer the purpose), that the transfer should be prepared by the office, given out to be executed, sent in for registration, and a certificate of registration issued. The policy so assigned might be subject to settlement, mortgage, or to any other kind of trust, but those were not matters which should concern an insurance company. Bank of England stock was subject to all kinds of trust, but all that was known to the Bank of England was that the names on their books were A, or A and B, or A, B and C. The same thing applied to shares in companies. Mr. George Humphreys said they were not assigned by deed. [Mr. G. E. HUMPHREYS—They need not be assigned in all cases.] As a matter of fact, in almost all cases they were so assigned. Railway stock was transferred by deed, included in marriage settlements, made the subject of mortgage and the like. Large sums of money were borrowed on stock. The stock was transferred to a banker or any other lender, and the smallest possible piece of paper would be given to say that on the repayment of the money the stock would be re-transferred. That was done constantly. Why could not some such machinery as that be resorted to with life assurance policies? He could not understand the difficulty, but if they were to introduce the various complexities that Mr. Higham proposed, they would be no better off than they were at present. He agreed that the Act of 1867 was a distinct improvement on what went before, but thought it might be extended.

Mr. MARMADUKE MATTHEWS (a visitor) said that the paper seemed to be written entirely from one standpoint—that of an actuary of a life assurance company. What he understood to be suggested was this: a simple statutory form obligatory upon everybody, to be registered with the company, and which should in itself and alone, so far as the company was concerned, form the title of the person claiming. Upon that basis, nothing more was needed than a few words to this effect, namely, "I, the holder of a particular policy, assign it to B", and then B would, by virtue of the assignment and the statute, become possessed of the policy and the whole interest under it. Deeds and declarations of trust were contemplated to regulate the equitable claims, and declarations carrying with them all the consequences of perjury were also suggested. Comparing the present state of things with that which they were invited to adopt in substitution, he failed to see where the advantage would be to the British public. Besides their interest as insurance companies in making the work of the office as simple as possible, they should give the public a contract which was both negotiable and valuable. From that point of view, he would not ask the assured to part with his policy or with any one of the documents connected with it, since persons who invested in policies of assurance, or any other property, very much prized the emblems of their property, and liked to have them in their own possession. To his mind the present mode of transfer seemed to be as simple as possible.

He knew of no form of deed, and no particular form of words, which was necessary in order to make a valid and effectual assignment of a policy. The great difficulty was in the law upon the question of notice, and if they could simplify and define that branch of the law they would get over a large part of the practical difficulties which affected the insuring companies. Mr. Higham's scheme put upon everybody the obligation of giving notice, but there was nothing new in that. They all knew that the notice to the insurance company was the essential thing. He was much interested in the illustration given by the President, but he might carry it even further by saying that without even the letter held by the assignee in that case, the mere passing of the policy would in itself have proved sufficient if, in fact, it was *bonâ fide* and genuinely done. The great difficulty in that particular case arose from the fact that no notice of the transfer was given at the time: it was not until the man was bankrupt and dead that the question of the ownership of the policy was brought into view. The whole difficulty in passing policies from hand to hand lay in this, that such informal transfers tended to assist fraud; but no amount of care and precision could afford an absolute protection against fraud. Cases of fraud were, however, very exceptional, and might reasonably be left to be dealt with upon their individual merits. In the interest of the insurance companies, as well as of the public, it seemed desirable to make transfers of policies as simple as possible, and not to hamper any holder of a policy by requiring the use of any particular form of words. His (the speaker's) suggestion would be, that any change in the law should be in the direction of simplifying and defining the effect of notice, and not in providing a statutory form of words for effecting an assignment. The Act of 1867 was a valuable step in that direction, but it did not go far enough. It was also of value as giving the assignee of a policy power to sue in his own name; but one effect of that, he was afraid, had been a disadvantage to the companies. Prior to the Act an assignee had to make out his title at his own cost, but sueing under that Act the companies were constantly exposed to risk of costs; a man was entitled to bring his action to trial, and if at the trial he proved his title, the company might be mulcted in the whole costs. If the law had been let alone they would have been spared that risk at all events.

Mr. M. N. ADLER expressed regret that Mr. Higham should not have alluded a little more fully to the law prevailing on the Continent with regard to the assignment of policies. Policies of assurance were not much known on the Continent at the time the Code Napoleon was framed, and therefore there was no express allusion to them in that Code. It was the general practice to regard policies very much like bills, and with a bare endorsement on the back by the assured they would pass to bearer and thus become negotiable.

Mr. AUGUSTUS HENDRIKS said, in dealing with assignments with regard to registration and notification, an absolute assignment ought to be registered on the books of the company and a title given in the same way as the assignment of shares or stock. The paper was altogether silent upon the Act of 1867, which gave one of the most simple forms, enabling anyone to assign a policy with the greatest ease. It was so simple a form that it was hardly ever adopted. It would be a very great hardship on the public to make any one form obligatory. A very simple form such as that might not only be noted by the company, but they might have a title given to them by way of certificate just the same as holding railway stock. It was held by nearly all companies that the registered name upon their books was for their purposes the absolute holder, leaving it to him to deal with any subsidiary forms of charges which might exist. If they went beyond that they would get into great danger, and to lay down any particular form to be applied to all cases of equitable liens or charges would be a practical impossibility. Most of the colonial Acts had gone some distance in the direction of simplifying the proceedings, but they would never get over the very great difficulty which existed. On the Continent they endeavoured to follow as much as possible the same procedure as with bills. He had lately had a case in which a policy was effected in this country by a Portuguese, who subsequently went to the United States and died there. The policy bore an endorsement of this sort, "I declare that this policy is the property of my wife." There was no actual assignment. He believed that according to English law that would not be sufficient, but according to Portuguese law it was ample—it gave the wife a clear assignment to the policy. She proceeded to endorse it by these simple words, "Transferred to Messrs. A. B., of New York." She meant it was transferred to them for collection, but according to Portuguese law that gave Messrs. A. B. an absolute assignment of the policy, and if, instead of collecting the money, they had put it into their own pocket there would have been no remedy. This proved that the process of assignment might be made too easy. On the Continent a very general practice existed of making policies payable, not actually to bearer "*au porteur*", but to the holder "*der inhaber*" of the policy. This did not mean that the man who merely held and presented the policy was to take the insurance money, but that he had to prove that he was the legal holder of the policy. There might be some means by which to simplify those matters in this country, and one would be not only to register but to give a certificate to the holder of the policy by absolute assignment. But to grant such certificates on mere liens on mortgages would be going further than there was any necessity for, and instead of helping the public they might place them in a much more difficult position than they were in at the present moment.

Mr. A. F. BURRIDGE said the chief difficulty was as between two characters of assignment, a charge with a proviso for redemption and an absolute mortgage. They were called upon to deal with both those classes of securities. He thought there were numerous difficulties in the way of applying to a mortgage such a mode of registration as had been suggested; but there was no difficulty in applying some such plan to an absolute assignment. He would recommend that, provided a holder became possessed absolutely of his policy by an assignment or a succession of deeds, or supposing it should revert to the original grantee, he might send in his deeds to the office and get his title admitted, and be supplied with a certificate of holding. That should be final proof of ownership, and the title should start afresh from that point. He could not see the difficulty mentioned by Mr. Higham with regard to the chose-in-action. Choses-in-action or the rights under a contract must be the principle of the policy, and all choses were now practically assignable, subject to one or two conditions, namely, that the assignment must be absolute, notice must be given, and the assignee take subject to equities. Mr. Higham's suggestions would almost do away with an equitable title altogether, and an equitable title would become an absolute title. In a great many instances a double form would be necessary—the deed supplied by the office, and the subsidiary deed of defeasance which the mortgagee would hold. This would lead to costs, and there would probably be the double stamp duty. There was a great objection to any fixed statutory form, and Mr. Higham himself would, probably, not think it prudent to reject a notice, if his proposed Act were passed, because it was not given in the statutory form. The paper suggested that the office should refuse to acknowledge any limitation of title to the policy, but that such limitation should appear in a separate deed. What would the position be supposing a husband effected a policy under the Married Women's Property Act? The husband must be registered as the holder, whereas, in reality, he would only be a nominal holder, and it would be subject to a trust of which the office distinctly refused to recognize the existence. Mr. Higham suggested that trustees could effect surrenders and commit other acts with regard to a policy, irrespective altogether of whether they were in accordance with their trust deed, which would be entirely outside the cognisance of the office. But this would probably tend to the increase of law costs.

Mr. F. **Bell** did not propose to discuss whether the fresh Act desired by Mr. Higham was called for or not, but would suggest that the existing Act (1867) could be made to work more smoothly by endorsing extracts from the Act, or a note explanatory of its provisions on all receipts issued by the office, in much the same way as it is now the practice of several companies to indicate their "principal place of business at which notice of assignment may be given", as mentioned in the 4th section of the Act. By this means, since probably the great majority of assignees see "the last renewal receipt", attention would be called to the necessity of having a proper assignment executed, and proper notice given. At present, offices were often unable to give "a plain answer to a plain question", when that question referred to the title to a policy as shown from the notices received; but he thought that this was caused in a great measure by want of clearness in the notices themselves. He suggested that a further schedule should be attached to the Act, giving the form of the notice to be given to the office. That schedule could be arranged under certain heads, each requiring a short and definite statement—for instance, name of assignor; name of assignee (to include the names of any other parties entering into the deed, if any); number of policy; date of policy; sum assured; date of assignment deed; nature of assignment, being "absolute", "by way of mortgage", "in trust", &c.; and also name and address to be registered for future renewal notices, &c. Then, in case of enquiry as to the notices of assignment which had been received, copies of these could easily be made and the desired information furnished.

Mr. C. E. **Brown** (a visitor) said he should like to make a few remarks on the subject of assignments on the Continent. His experience was, that the only safe way to effect an assignment of a policy in France was by Notarial Act or private deed in the form of a transfer or of a sale; but the transferee, in order to guard himself against third parties, should, in accordance with Article 1,690 of the Civil Code, give notice of the transfer to the assuring company. This mode of transfer necessitated two things—first, registration of the policy, and then registration of the transfer itself. The great objection to this mode of transfer was the expense it involved. The duty on registration of a policy amounted to 1 per-cent on the value of the premiums, which he supposed meant the surrender-value of the policy, and the duty charged on the registration of the act of cession amounted to a further 1 per-cent on the sum assured, and, in addition, expenses of notification were incurred. In France that form of transfer was very much objected to. They had a very common practice in France and other countries of effecting policies in favour of third parties or beneficiaries. The beneficiary might be either the wife of the assured, or wife and children, or children only, or a creditor of the assured. In the case of a policy issued for wife, or wife and children, it was generally a benevolent disposition, capable of being revoked by the assured during his lifetime, and did not amount to an assignment. However, in certain continental countries, if the beneficiary accepted the benefaction and gave written notice to the assuring office that he did so, the beneficiary was considered to have acquired a right to the assurance contract, subject to the conditions of the contract being duly observed. In the case where a policy was effected, say, as security, in favour of a banker, when the policy became a claim, if the banker could give evidence of the indebtedness of the assured to him, his discharge would be accepted, the concurrence of the legal representative of the deceased sometimes being required. In Belgium, by Article 42 of a law passed in June 1874, policies were assignable by endorsement in a very short form, by which the transferee acknowledged the acceptance of the transfer at the foot of the transfer itself. The companies themselves were also bound to acknowledge the existence of the transfer at the foot thereof, although, on the part of the company, that was usually done without accepting any responsibility as to the validity or legality of the transfer.

Mr. W. H. J. **Whittall** considered there was a very great difference between the stock of a company, or consols, and a life policy. Where there were trusts, great injustice would be done if the offices were bound to deal only with the person whose name was formally standing on the books. For instance, if the mortgagor of a policy found that it had been surrendered by a mortgagee to whom he had given no such power, it would be a poor answer to him to say he had a remedy against the mortgagee. However simple it might be for the offices to arrange their affairs in that manner, the public would never stand a system which involved injustice to them.

The **President** (Mr. A. Day) having conveyed the thanks of the meeting to the author, Mr. C. D. **Higham**, in reply, said it was gratifying that nearly every speaker had admitted that something ought to be done, and the point was what that something ought to be. He did not say that his scheme was the best, or anything like it. It was only a sketch, to be varied as might be necessary. The possession of a policy was at present of value when the assured was dead, but in the cases in which difficulties arose, to expedite the claim was always the very thing the man would not do, and much of the criticism had hardly kept in view the troubles of the holder of an existing policy, without funds to keep it up or power to surrender. Offices did to some extent give advice to their own clients; and a good many used to supply forms of assignment, but he fancied the practice was being dropped, as it was found out what difficulties and complications unlearned use of them caused. He did not think, with Mr. G. Humphreys, that the proposed scheme would on the whole entail more office work than the present system. Mr. G. Humphreys objected that there would be no veto on an undesirable assignee, but he (Mr. Higham) believed the majority of companies had to accept any transferee of shares whatever; at any rate, companies under the Act of 1862 had no such power of veto. [Mr. G. **Humphreys** said that almost all insurance companies had that power.] He thought only those that were under special deeds. The old corporations refused no one, and in such experience as he had had of a company with a special deed, although three directors used to sign each transfer in confirmation, he never recollected the veto being exercised. The stamp duty, he had suggested, could never be more than 2*s*. 6*d*. per cent. on the sum assured, unless the mortgage money were greater than it; and the new register proposed would be instead of, and not in addition to, the register of notices now kept. With regard to any difficulties as to the validity of deeds, in the simple forms he proposed, brought to the office for registration, he thought the officials of life offices were quite capable of being trusted in such matters. Mr. Whitcher's proposal to have a public registry office seemed to him hardly feasible; but some of the colonies appointed a valuable officer known as the *Public Trustee, who some day might make his way into this country. Mr. Matthews had described the simplicity of the form of deed at present necessary, but complained as to the difficulties in giving notice, whereas he (Mr. Higham) had always understood that any form giving the number of the policy and the date and purport of the deed would suffice for notice, though he bowed to the better opinion of an expert. Mr. Saunders protested against the scheme proposed, but commended the colonies for their progress in these matters, forgetting that many of them already had Acts of this nature. He did not presume to controvert Mr. G. E. Humphreys' opinion as to choses-in-action, but he would refer him to the introduction to the last edition of Mr. Bunyon's *Law of Life Assurance.* Had Mr. Humphreys not forgotten also how many of the offices were mutual, for a policyholder in a mutual office was in much the same position as a shareholder in a proprietary company? If production of the policy at the office were necessary on every change of ownership, there would be difficulty in the case of a second mortgage, because the second mortgagee could not always get possession of it. In the New Zealand Act of 1884 there was a covenant by which the first mortgagee might be required to produce the policy at the office whenever it was reasonably required. Mr. Bailey agreed with him, so far as the one absolute form was concerned; but the point was, was the one enough? Most of the colonial Acts had only one form, and that absolute, limitations being arranged by separate documents. If the Council should ever appoint a committee, they would decide what was the best thing to be done, though it seemed to him that forms for use in case of loans were almost necessary for the protection of the man of low estate. He did not anticipate any difficulty if the use of certain forms issued by the companies were made compulsory, for it would soon become a matter of course to get one from the office when wanted, just as now in the case of companies using special forms for the transfer of shares. In answer to Mr. Burridge, his idea was to let every man who lent on a policy, whether a large or small sum, have the powers now included in a well-drawn mortgage deed. If the proposed statutory covenant should be varied by consent, an agreement with a 6*d.* stamp would, he imagined, generally be considered sufficient evidence thereof. Trustees would have large powers, but if a trustee was a man to be trusted, though in some cases he might do wrong, in far more, probably, would he carry out the intention of the person who had created the trust, for the benefit of the *cestui-que trust.* If Acts akin to that he proposed worked well in the colonies, they might assume that the Council would be able to discover a scheme which would be acceptable, and of great advantage to the general body of the assured.

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* [Since Mr. Higham's paper was read, we have learned that a Bill (House of Commons, No. 121) has been prepared and brought before Parliament, the object of which is to provide for the appointment of a Public Trustee. The explanatory memorandum which prefaces the Bill is as follows:—

"The object of this Bill is to meet the difficulty which both public bodies and private individuals frequently experience in finding suitable trustees. The choice of an executor is frequently not less embarrassing. In some of the States of America, and some of the British Colonies, notably in New Zealand, a public functionary and a public office exist for the purpose.

"The Bill provides for the payment of fees on a regulated scale upon all receipts of income under a trust, or on realizing property under a will, as for other duties connected with the administration of a trust, or the execution of a will. The Public Trust Office will therefore be not only self-supporting, but in all probability a profitable source of public revenue.

"The Bill constitutes the Public Trustee as a corporation sole, with perpetual succession. The public would thus be able to appoint a trustee or executor who would never die, never leave the country, and never become incapacitated. The State would, under certain reservations, guarantee the fulfilment of all trusts placed in the Public Trust Office, and the execution of wills accepted by the Board of Advice."—Ed. *J.I.A.*]

A correspondence subsequently arose in the columns of the *Insurance Record,* on the subject of Mr. Higham's paper, from which we make the following abstracts.

Mr. Sprague wrote:

All persons who are practically interested in the business of life insurance will probably agree with Mr. Higham that the present law and practice as to the assignment of life policies admitted of great improvement; but I do not think that many will entirely approve of the alterations he suggests.

In order to place the law on a thoroughly satisfactory footing, it will be necessary to obtain the assistance of Parliament, but the life insurance offices have the matter to a certain extent in their own hands, and might do much to introduce reform without any legislation. Probably all will agree that it is desirable that life policies should be capable of being transferred from one holder to another in the same way as the shares or stock of a joint-stock company; and that, when a policy has been transferred into the name of a new holder, his title shall be recognized by the company without further reference to the original holder. Life offices might safely do all this if they were so disposed. When shares are transferred into a new name, the company retains permanently the transfer, and is thus always in a position to prove that a proper transfer has been executed, if, at any time, the contrary should be alleged by the former holder or his representatives. Similiarly, if a life office recognizes a new policyholder, it should retain the assignment in virtue of which he claims. In the case of shares, the old share certificate is usually delivered up, and a new one issued by the company; and, similarly, when a policy is to be transferred to a new holder, the policy should be produced, and a memorandum endorsed on it by the office, to the effect that it has been transferred into the new name. As the law now stands it is necessary for the assignee to give notice to the office of the date and purport of the assignment to him. He would very effectually do this by lodging the assignment with the office, and leaving it in their custody, but possibly it might be held that this was not literally in accordance with the provisions of the Policies of Assurance Act; and it is also desirable, as Mr. Higham points out, that the office should become acquainted with the individuality of the assignee. Both these objects would be gained if the assignee signed a notice to the following effect, which might be conveniently placed on the same sheet of paper as the assignment itself: "I, C. D., of , hereby give notice that A. B., of , has by the above deed dated , assigned and transferred to me Policy No. , granted by you on the life of , and I hereby request you to register me as the holder of the said policy; and for this purpose I deliver up to you the above assignment to be permanently retained by you."

The company, on receipt of such a notice and request, might safely and properly endorse the policy to the effect that "C. D. is the registered holder" of it; and thereafter recognize C. D. as the absolute owner for all purposes. Similarly, the policy might be afterwards assigned by C. D. to E. F., and by him to G. H., or back again to A. B., without any reference to the previous transactions.

Another way in which the companies might, as the law now stands, facilitate dealings with their policies, would be to express their willingness to investigate the title of any person who claims to be the absolute owner of a policy, on payment by him of a moderate fixed fee; and to give him, if his title be found satisfactory, a certificate of ownership that would render it unnecessary for him to prove his title to them again. In this case he must either deliver up his title deeds to the office, or give the usual undertaking to produce them if that should be necessary.

Mr. Higham says that, if his scheme were adopted, trustees would be able, without question by the office, to effect surrenders in exchange for paid-up policies; but this also can be done under the present law. If, for any reason, the premium is not duly paid by the trustees, the policy will become forfeited; but, according to the non-forfeiture regulations which have been adopted by some offices the policy will, after an interval of, say, two months, be revived by the directors as a paid-up policy, without any application by the trustees or any other person.

So far, then, it is a change in the practice of the offices, and not a change in the law of the land, that is wanted.

The principle involved in Mr. Higham's proposals is that the office shall be entitled to deal exclusively with one person (or set of persons), as regards each policy, and to disregard the interests of all other persons. When a policy has been transferred into the name of a new holder or holders—for instance, the trustees of a marriage settlement—they are to be allowed, regardless of the trusts under which they hold the policy, to deal with it as they think fit, without any question by the office; and when a policy has been mortgaged, the office is to be entitled to deal with the mortgagee, regardless of the bargain that may have been made between him and the mortgagor. Mr. Higham proposes that these regulations shall apply, not only to policies issued hereafter, but, after the lapse of a year, to all policies then subsisting. The changes that would be thus introduced appear to me to be far too sweeping, and to be calculated to relieve the offices of trouble and responsibility at the expense of the assured. In many cases, also, the course advocated by Mr. Higham would cause additional expense to the assured—for instance, if a loan of £500 is granted on security of two policies in different companies and other property, three separate mortgage deeds would be necessary instead of one. From another point of view, I regard the proposed changes as an undue interference with the freedom of contract. If a policy is included in a marriage settlement, and the parties interested think it desirable to give the office notice of the trusts on which the policy is held by the trustees, they should be allowed to do so; and if a mortgagor has refused to give his mortgagee an absolute power to surrender the policy at his discretion, it would not be just for the law to import this condition into the mortgage without his consent. In other words, persons should be allowed to make their own bargains with each other; and life offices should not object on the ground that this occasionally causes them a little extra trouble.

Mr. Higham says that what he proposes is nothing more than is already in vogue when stocks and shares are settled or mortgaged; but in making this remark he has lost sight of the fact that distringases may be put upon most or all of the stocks which usually form the subject of settlements. On the other hand, when stocks or shares are mortgaged, it is essential for the protection of the mortgagee that he should have the absolute power to sell them when he thinks fit; and probably no person would lend money on security of them without such power. This arises from the fact that the selling values of many stocks and shares are occasionally subject to sudden and sharp variations in price; but there is no similar reason why the mortgagee of a life policy should have the absolute power to sell it without a day's notice when he thinks fit.

Perhaps the principal object to be aimed at in any reform of the law is to get rid of the necessity which at present exists of carefully preserving all deeds that relate to past transactions, mortgages that have long since been paid off, and settlements of which all the trusts have expired, so that the policy belongs absolutely to the original assured. What is wanted, in order to gain this object, is the power of giving a notice to the office that shall operate in the same sort of way as a distringas; that is to say, a notice that shall entitle the giver of it to be immediately informed of any proposed dealing with the policy, and shall prevent any such dealing for a time sufficient to enable him to produce his title, or, if need be, to institute proceedings to defend his rights if he thinks they would be prejudiced by the proposed dealing. At present, if the assignee of a policy gives notice to the office that he is interested in the policy by virtue of a certain deed, the office must, for its own protection, call for production of that deed before it can allow any dealing with the policy. I would suggest an amendment of the Policies of Assurance Act, to the effect (1) that the assignee should not be required to give notice to the company of the date and purport of the deed under which he claims, but that it should be sufficient for him to give notice to the company that he is interested in the policy, and that they are not to allow any dealings with it until, say ten days after they have given him notice; and (2) that the date on which such notice shall be received by the office shall regulate the priority of all claims. Under such a state of the law, if the assured, A. B., borrowed money from C. D. on mortgage of his policy, it would only be necessary for C. D. to give notice to the office that he was interested in the policy; and upon his debt being paid off he could withdraw the notice, and the mortgage deed might be destroyed. Or, if C. D. thought that his interest was not sufficiently protected by this course, he might insist on having the policy transferred into his name by an assignment *ex facie* absolute, in which case he would give A. B. a declaration of trust setting forth the conditions on which he holds the policy. The terms of this declaration of trust would, of course, not be communicated to the office; but if A. B. wished to protect his interest he might give notice to the office that, notwithstanding the assignment to C. D., he (A. B.) was still interested in the policy. Then, on the debt being paid off, the policy would be re-transferred to A. B., and the declaration of trust destroyed. Or, lastly, the policy might be transferred by A. B. into the joint names of himself and C. D. as tenants in common, in which case neither of them could deal with the policy without the consent of the other or of his representatives.

The cases of second and third mortgages would be dealt with on precisely analogous principles. If A. B. executed a second mortgage in favour of E. F., and a third mortgage in favour of G. H., it would be necessary for E. F. and G. H. respectively to give notice to the office that they were interested in the policy; and the dates when the office received the several notices would regulate the priorities as between C. D., E. F., and G. H. It would, as at present, be open to the several assignees to give notice to the office of the deeds under which they claim; but A. B. might reasonably stipulate that they should not do so, the difference being that a notice by an assignee, stating only that he has an interest in the policy, can be withdrawn by him at any time without any further formality, but the office cannot allow the notice of a deed to be withdrawn without calling for the production of the deed in order to satisfy themselves that it does not contain some trust of which they must, for their own protection, take notice.

In cases where two or more policies are included in one mortgage, or policies are mortgaged along with other property, it may be difficult to say what is the proper consideration money to be inserted in the transfer of a policy; and for this and other reasons it would, perhaps, be most suitable that every transfer of a policy should bear the same stamp as the original policy.

If we examine the list of anticipated advantages given by Mr. Higham, I think we shall find that all those which are likely to benefit the assured are put within his reach by means of the very simple change in the law which I have suggested. It is not for his benefit that all transfers and mortgages of life policies should be in certain prescribed forms, and that all deeds in other forms should be wholly inoperative, nor that the offices should occasionally be saved the expense and the trouble of investigating the title to his policy, and of deciding to which of two rival claimants the policy moneys are to be paid; and it would be unfortunate for the life offices if they were to endeavour in any way to relieve themselves of trouble and responsibility at the expense of the assured. Their wiser course will be to consider what measures are likely to be advantageous to the assured, and to adopt those measures, heartily and consistently, without regard to their own ease and comfort.

Mr. C. D. Higham wrote:

It should be borne in mind that there must be other assignments than absolute ones; that exceptionally we have to deal with dishonest men; and that existing policies of large present value, yet with a heavy burden of future premium payments, must be kept in view. Not that the possibility of occasional wrong-doing ought to bar the adoption of great present improvements, for it may be that general facility is cheaply purchased at a small risk of fraud.

To sketch the present law would have been a bold undertaking, which seemed hardly necessary; nor had I intended to travel beyond actual legislation. I do not suggest the alteration of any existing contract save by consent, except that when a man shall have proved his title in a court of law at his own expense, a company may well be then required to register his name as the holder of the policy. But, for the future, I would have always one person (or set of persons) recognized in the books as the owner, the chain of title being in the office strong-room, instead of all over the country. To have to act on any deed that might be produced would throw too onerous a responsibility for a variety of wordings, not to mention limitations and covenants, on the company; while if it is to be required that all shall be absolute, it is but a short step to a compulsory form, which would soon become a matter of course. If the wording be not fixed, the objection to the secretary having to decide on an assignment is reasonable enough; but it needs little intelligence to see if a form is filled in and executed, and that the party assigning is entitled, as a policyholder, or by probate or other evidence of legal transmission, to do so. If a document be not in accordance with statutory requirement, Mr. Burridge will admit that it is not only prudent but necessary to reject it; and the clear terms for which Mr. Humphreys dares not hope are at any rate as probable in a public statute as in private deeds.

An absolute owner may wish to borrow, and the proposed Act would at once come to his aid. I would have no distinction between legal mortgages and equitable charges, but every capitalist who lent £10 on a policy should have by statute the ordinary powers of a well-drawn mortgage deed. If he wanted a little more interest, or the borrower wished his loan for a fixed term, they would, by a separate agreement, bar or vary any of the statutory covenants, the others remaining in force. Should the policy become a claim during the existence of the loan, the lender would take the sum assured, and account for it to the others interested; should there have been default in repayment, and he desire to dispose of his security, he would, in the absence of other proof of breach of covenant, make a declaration of the facts (of three months' arrear, for example, after notice given), as well as that, by the statute, or a private agreement, he was entitled to sell, and he would then be empowered to do so; but that he should so act at discretion I have not advocated. Such a scheme is less open to objection than for the office to recognize absolute assignments only, which might often work injustice, as on a poor man, for instance, in the clutches of some usurer. A policy might be surrendered wrongfully, and so it might now under a mortgage deed, but ordinary care is a great safeguard, and a company is not bound to redeem a contract, nor likely to do so if suspicions have been aroused.

With settlements the office would be little concerned, though often the settlor will have his own name included with the trustees' as one of the holders of the policy. Still, it is desirable that there should be a statutory form for appointing trustees and declaring simple trusts such as are now included in policies under the Married Women's Property Acts. Law stationers would, no doubt, keep such forms in print, but it would not be the company's affair. Perhaps the Act might declare the interest of the trustees in the life of the settlor an insurable one, the necessity for a transfer being thus avoided: if there were no trustees, the assured's executors would act at time of claim. The admission by the company of the status of trustees introduces at once all the difficulties of dealing with the policy during its currency; without it the nominal holder is the only holder as far as outsiders are concerned.

Mr. Sprague suggests a plan of securing limited control by a system of notices to run concurrently with existing practice. But to have alternative procedures will not conduce to simplicity, and it will often be hard to decide under which scheme an informal notice is to be classed, while there is the difficulty also of proving due communication when necessary with the person (perhaps abroad) designated to receive it. Non-forfeiture regulations can scarcely give the facilities in contemplation; but it would seem better to have a general law on the subject than a variety of practices. To put a policy into the names of both the mortgagor and the mortgagee will probably never be acceptable, either to the solicitors in a large case or a loan office in a small one, for it will be considered to unduly limit the rights of the lender in case of need; and for this reason of freedom I did not mention the use now made of a distringas on stocks. When Mr. Sprague proposes that the stamp on any assignment should be the same as on the policy, he intends, no doubt, to treat old policies as if they had been stamped under the present charges. One shilling per-cent on the sum assured seemed too moderate a conveyance stamp to be looked for, while a mortgage stamp dependent on the sum assured, bears hardly on the borrower of a small loan-value. If the amount to be secured on each of several policies cannot be determined, it must be estimated, and a separate consolidation deed drawn; and there is always, too, the last resource of an absolute transfer, with an independent declaration in defeasance.

It did not fall within the scope of my paper to suggest possible improvements in our present practice, though I do not think the proposed instructions to be printed on renewal vouchers would be attended to; nor that the *form* of notice is worth much trouble, for, however admirable the notice, the deed itself may be invalid. But to investigate and register title, and accept the custody of documents, would be of great service; and to refuse to receive a notice unless a deed were recited therein would protect many who now suffer from want of acquaintance with the law.

I was prepared to have it objected that the facility with which a company lends on its own policies would be impaired, and if the matter ever be taken up, it will be worth consideration whether special exception ought not to be made for such cases. Mr. Sprague protests against a supposed desire to relieve ourselves of trouble at the expense of the assured. This last intention I utterly disclaim, for it is the assured rather than the company (both would, however, be gainers) I have had in view, though, truly, to procure benefit for the former is the best possible policy the latter can adopt.

In connection with the subject of Mr. Higham's paper, it may be interesting to point out that the desirability of an easy mode of vesting the title to a policy of assurance in an assignee or transferee was recognized as far back as 1712. In that year the articles were issued "for the establishment of a company under the name of the *Most Advantageous Insurers*" (see *History of Life Assurance, J.I.A.*, xxv, 261–2), in which the following clause (numbered XVIII) appeared:

> "That one person may assure upon as many other persons lives (not before insured upon in this Company) as he pleases, until, the number of 2,000 are insured upon; and upon the selling, transferring or assigning any of the Company's policies (which the members may do as often as they please), the purchasers or persons to whom they are sold, transferred or assigned, shall come to the office within 15 days after such purchase, transfer or assignment, to receive new policies in their own names, but upon the same lives, and have the old ones cancelled, or otherwise to be excluded all benefit of this Company (unless the legal proprietor of such policy or policies shall, after the expiration of the time limited as aforesaid, make an affidavit, that such policy or policies could not be produced sooner, and in the said affidavit set forth the cause of such delay or neglect), which will certainly prevent all frauds and effectually secure the right of the persons to whom they are sold, transferred or assigned; for every which new policy and stamps they must pay the Registrar 7*s*. 6*d*.

Ed. *J.I.A.*

On Bonuses arising from Surplus Premiums and Surplus Interest.

By Arthur W. Sunderland, M.A., *Actuary to the National Life Assurance Society.*

[Read before the Institute, 25 April 1887.]

THE great importance of the subject of distribution of surplus, I think, no actuary would be inclined to dispute. With regard to the magnitude of the interests involved, I will content myself with referring to Mr. Hewat's paper, read before the Actuarial Society of Edinburgh on 8 January 1880, and of which an abstract is given in the *Journal* (xxii, 286–292). In this paper he stated that, from the Board of Trade Returns relating to seventy-seven offices, he found that £2,285,000 was, on the average, annually divided among the policyholders by way of bonuses. Of this sum he estimated that £890,000 arose from surplus premiums or loading, and £1,040,000 from surplus interest. The equitable distribution of such enormous sums of money among those entitled to participate in them is a problem which demands, from all actuaries who respect their profession, the most earnest and careful consideration; yet it seems to me to be one to which attention at all commensurate with its importance has not been paid. I believe I have seen it written, I cannot at the present time recall where, that any system of distribution is equitable which the policyholders understand, and to which they assent. This, however, does not help us. The policyholders cannot be expected to, and do not, understand the various modes of distribution, and practically have to acquiesce in those systems provided for them by actuaries. They rely on the offices to deal justly between them, and the burden of responsibility is again thrown on the actuary's shoulders.

At the same time that the problem is one of great importance, it is also, in my opinion, one of the most difficult in actuarial science. The claims of equity have to be reconciled with those of practicability under circumstances which are continually undergoing change. Our theories must be carried out in practice, and must harmonize with existing conditions. In the present paper I do not attempt to do more than trace out, under certain limited hypotheses, the bonuses which arise from surplus premiums and surplus interest. It will be assumed that all surplus is divided among the policyholders, and, unless otherwise stated, that interim bonuses are allotted so that those who survive any valuation period take no part of the surplus arising on the policies which have become claims within that period.

In the 24th volume of the *Journal* (pp. 173–184) is a paper by Mr. C. J. Harvey, "On a Formula for returning to the Assured their Contributions to the Surplus." This formula is approximate, and, with regard to it, I am uncertain whether Mr. Harvey contemplates the allotment of interim bonuses or not. I shall assume with him that the claims are paid at the end of the year in which death occurs, that interest is convertible yearly, that valuations are quinquennial, that the policies are effected at the beginning of valuation periods, and that the valuations are by the ordinary or pure-premium method.

Let letters with dashes denote theoretic or "valuation" quantities, *e.g.*, π'_x = the pure premium according to the mode of valuation employed, and undashed letters "experience" quantities, *e.g.*, i represents the rate of interest actually realised, and l_x is the symbol for number living according to actual experience. Consider l_{x+n} unit policies n years in force at the beginning of a valuation period on lives aged x at entry, to each of which a reversionary bonus ${}_nB_x$ is attached. Let X be the surplus premium, that is the amount of the loading which remains after providing for expenses of management. Then the sum which these policies contribute at the beginning of the $\overline{n+1}$th year to the funds of the office is

$$l_{x+n}(\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n}+X)$$

and multiplying this by $1+i$ we get the actual contribution at the end of the year. Now, on the assumption that the "valuation" and "experience" mortality are the same

$$l_{x+n}(\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n})(1+i')$$

is the contribution required to provide for the claims and the reserves to be made at the end of the year; in other words

$$l_{x+n}(\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n})(1+i')$$
$$=(l_{x+n}-l_{x+n+1})(1+{}_nB_x)+l_{x+n+1}({}_{n+1}V'_x+{}_nB_xA'_{x+n+1})$$

an equation which admits of easy verification. The difference between the actual contribution and that required for claims and reserves is, of course, the surplus; so that

$$l_{x+n}\{(\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n})(i-i')+X(1+i)\}$$

is the surplus arising on the l_{x+n} policies in the year. Dividing this by l_{x+n}, we have the surplus per policy =

$$(\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n})(i-i')+X(1+i)$$

We are making the hypothesis of interim bonuses, and shall therefore assume that each of the $l_{x+n}-l_{x+n+1}$ policies which lapse by death during the year receives its share of the above surplus, and that for each of the others this surplus is set apart as it were, and accumulated at rate of interest i. Proceeding in this way for each of the years of the quinquennium, we finally get as the total cash surplus on each of the policies existing at the end of the valuation period

$$
\begin{aligned}
&{}_{+1}(1+i)^5+X_{n+2}(1+i)^4+\ \dots\ +X_{n+5}(1+i)\\
&+\pi'_x(i-i')\frac{(1+i)^5-1}{i}\\
&+{}_nV'_x(i-i')(1+i^4)+{}_{n+1}V'_x(i-i')(1+i^3)+\ \dots\ +{}_{n+4}V'_x(i-i')\\
&+{}_nB_x[A'_{x+n}(i-i')(1+i)^4+A'_{x+n+1}(i-i')(1+i)^3+\ ..\ +A'_{x+n+4}(i-i')]\ .\ (1)
\end{aligned}
$$

Dividing this cash surplus by A'_{x+n+5} we get the reversionary bonus to be allotted to each policy at the end of the quinquennium, and by repeated applications of the formula we shall get the successive bonuses and the total bonus at the end of any period.

Let us turn our attention to the first line of expression (1). Suppose that on certain suppositions—*e.g.*, "valuation" H^M 3 per-cent, experience H^M 4 per-cent, age at entry 20—we calculate the table of bonuses giving assigned values to X_1, X_2, . . . which may, if necessary, all be different; and again suppose we calculate another table of bonuses in which everything is the same, except say X_m, which we may suppose increased by some specific amount, say 5*s*. on a £100 policy. This increase in the surplus premium at a particular epoch we may regard as a donation made to the policyholder to be applied in increasing his bonuses, and it is obvious that the increase in the bonuses will be proportional to the amount of the donation. An increase in X_m of 10*s*. will obviously produce increments in the bonuses twice as great as will arise from an increase in X_m of 5*s*. Of this principle, namely, that the bonuses arising from the surplus margin in any particular year are proportional to the amount of that surplus margin, and are independent of the amounts of the bonuses arising from any other sources, an algebraic demonstration could be given, but it appears unnecessary. Again, if we suppose $X_1 = X_2 = X_3 = \ldots = X$, the additional bonuses arising from an increment in X will be proportional to that increment. Now, referring to our formula (1), let us assume that $X_1 = X + Z$, $X_2 = X_3 = X_4 = \ldots = X$. This, Z being negative, will represent the case of uniform expenditure, together with initial expenses of setting a policy on foot. The expression may then be written

$$X(1+i)\frac{(1+i)^5-1}{i} + Z(1+i)^5$$
$$+P + {}_nB_x . Q,$$

where for brevity P is written for the second and third lines of expression (1), and Q for the coefficient of ${}_nB_x$ in the same expression.

From this last expression, subtracting the value which it takes when X is put $=0$, we have

$$X(1+i)\frac{(1+i)^5-1}{i} + Q . \Delta_n B_x$$

as the expression for the increase in the cash surplus corresponding to the increase X in X. Here $\Delta_n B_x$ is the increase in the reversionary bonus.

In the same way, for Z we have

$$Z(1+i)^5 + Q . \Delta_n B_x$$

as the expression for the increase in the cash surplus corresponding to the increase Z in Z.

In the annexed tables marked A, B, and C, I have set out the bonuses calculated from the three formulas for cash surplus:

$$P + {}_nB_xQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$X(1+i)\frac{(1+i)^5-1}{i} + {}_nB_xQ \quad . \quad . \quad . \quad . \quad (b)$$

$$Z(1+i)^5 + {}_nB_xQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)^*$$

The valuation throughout is taken to be H^M 3 per-cent, and the bonuses are calculated for the ages at entry 20, 30, 40, 50, and 60, at each of the five "experience" rates of interest, 3, 3½, 4, 4½, and 5 per-cent. Table A, calculated by application of formula (*a*), shows the bonuses from interest, all loading being supposed spent in expenses of management, and of course for 3 per-cent interest the bonuses of this table are all zero. To illustrate the process of calculation, suppose 4 per-cent the experience rate. Then formula (*a*) may be written

$$\pi'_x \times {\cdot}054163 + [\alpha . {}_nV'_x + \beta_{n+1}V'_x + \gamma_{n+2}V'_x + \ldots + \epsilon_{n+4}V'_x]$$
$$+ {}_nB_x[\alpha A'_{x+n} + \beta A'_{x+n+1} + \gamma A'_{x+n+2} + \ldots + \epsilon A'_{x+n+4}],$$

where

$$\alpha = {\cdot}011699$$
$$\beta = {\cdot}011249$$
$$\gamma = {\cdot}010816$$
$$\delta = {\cdot}0104$$
$$\epsilon = {\cdot}01$$

---

* It should be noted that ${}_nB_x$ is a general symbol for total reversionary bonus, and has different values in the three expressions. It is used in the case of each formula as the symbol for the total reversionary bonus resulting from the cash bonuses given by that formula. Thus in (*a*) ${}_nB_x$ means the total reversionary bonus found by application of this formula, so that ${}_{n+5}B_x = {}_nB_x + \frac{P + {}_nB_xQ}{A'_{x+n+5}}$.

Setting up upon an arithmometer the constant multipliers $\alpha, \beta \ldots \epsilon$, the portion of the first line within brackets, and the coefficients of ${}_nB_x$ are easily tabulated when the policy-values and single premiums are known. The tables of these two functions being formed, the successive cash surpluses for any age at entry are formed in order, each as it is formed being converted into the corresponding reversionary bonus by multiplying by the reciprocal of the appropriate single premium. Probably a simpler way of effecting the calculations would have been found by expressing the policy-values and single premiums in terms of annuities. In Table A the decimal points are so placed that the amount of the policy is £100.

The mode of forming Table B from formula (*b*) will be obvious from what has been already said. As a matter of fact, for the three "experience" rates of interest 3½, 4 and 4½, Table B was formed by the following more difficult process. The bonuses given by the cash surplus formula

$$X(1+i)\frac{(1+i)^5-1}{i}+P+{}_nB_x.Q$$

were calculated, and then the corresponding bonuses of Table A subtracted from these, the remainders being, of course, the bonuses of Table B. In this table X has the value ten shillings, so that it gives the bonuses arising from an annual surplus margin ten shillings.

In Table C, formed by application of formula (*c*), Z has been taken £1 for $n=0$, and is, of course, zero for any other value of n. It therefore gives the bonuses arising from an initial surplus margin £1.

A survey of these three tables will, I hope, enable us to form, in the case of offices which value at 3 per-cent interest, a general idea of the effects upon their bonus-giving power produced by the investment of their funds at higher rates, and also by their expenditure in the conduct of their business. On this subject I do not propose now to dwell. I proceed to point out and illustrate one of their uses.

One of the difficulties which has presented itself in the investigation of the subject of distribution of surplus is this—that no two offices being exactly alike, it is impossible to make hypotheses which shall fit them all. Each office has its own table of rates of premium, while the rate of interest realized varies considerably, as well as the amount and character of the expenses of conducting the business. Now, from the three tables A, B and C, we can without difficulty form a bonus table to fit the case of any ordinary office which experiences the H^M mortality, values by H^M 3 per-cent, and applies surpluses in making additions to the sums assured. As an illustration, take an office whose profit rates are:—

Age 20	. .	£1	19	0	per £100 assured.
„ 30	. .	2	9	0	„ „
„ 40	. .	3	5	0	„ „
„ 50	. .	4	12	0	„ „
„ 60	. .	7	0	0	„ „

and which spends, in conducting the business, on each £100 assured, £1 out of the first premium, together with 3*s.* and 5 per-cent out of every premium. In this case $Z=1$, the annual expenses for a £100 policy are ·2475, ·2725, ·3125, ·38, ·5, for the respective ages at entry 20, 30, 40, 50, 60, so that the values of X for the same ages are ·2753, ·2980, ·3484, ·4195 and ·5126.

To find for each of the tables A, B and C the total bonuses corresponding to the "experience" interest 4¼ per-cent, we interpolate in each case between the total bonuses for 3½, 4, 4½ and 5 per-cent by means of the formula

$$*u_{4\frac{1}{4}}=\tfrac{1}{2}(u_4+u_{4\frac{1}{2}})-\tfrac{1}{16}\{(u_{3\frac{1}{2}}+u_5)-(u_4+u_{4\frac{1}{2}})\}.$$

These being formed, we must, for age at entry 20, multiply the B table by ·5506, add the results to the bonuses of the A table, and subtract from these the bonuses of the C table. For ages at entry 30, 40, 50 and 60, the corresponding multipliers are ·5960, ·6968, ·8390 and 1·0252. The results are shown in the following table, the fifth column of which gives the total bonuses of the hypothetical office.

---

[*] See *Notes on Finite Differences*, by A. W. Sunderland, p. 32, equation (4).

Total Reversionary Bonus—Valuation H^M 3 *per-cent, Experience* H^M 4½ *per-cent.*

	TABLE A	TABLE B	TABLE C	Hypothetical Office	
Age at Entry 20					
25	·580	7·919	3·440	1·50	25
30	1·951	15·669	3·659	6·92	30
35	4·130	23·285	3·892	13·06	35
40	7·099	30·811	4·140	19·92	40
45	10·863	38·282	4·407	27·53	45
50	15·433	45·760	4·687	35·94	50
55	20·801	53·297	4·986	45·16	55
60	26·965	60·961	5·309	55·22	60
65	33·942	68·834	5·655	66·19	65
70	41·715	76·978	6·024	78·08	70
75	50·319	85·483	6·420	90·97	75
80	59·752	94·429	6·849	104·90	80
85	70·043	103·903	7·303	119·95	85
Age at Entry 30					
35	·665	6·605	2·865	1·74	35
40	2·231	13·059	3·055	6·96	40
45	4·698	19·391	3·249	13·01	45
50	8·061	25·649	3·457	19·89	50
55	12·292	31·887	3·679	27·62	55
60	17·387	38·169	3·915	36·22	60
65	23·337	44·561	4·170	45·73	65
70	30·110	51·117	4·444	56·13	70
75	37·729	57·924	4·736	67·52	75
80	46·173	65·049	5·046	79·90	80
85	55·447	72·574	5·385	93·32	85
Age at Entry 40					
45	·768	5·490	2·379	2·21	45
50	2·600	10·854	2·534	7·63	50
55	5·450	16·141	2·697	14·00	55
60	9·290	21·400	2·871	21·33	60
65	14·081	26·705	3·059	29·63	65
70	19·774	32·099	3·261	38·88	70
75	26·358	37·656	3·477	49·12	75
80	33·794	43·437	3·707	60·35	80
85	42·064	49·516	3·956	72·61	85
Age at Entry 50					
55	·925	4·585	1·990	2·78	55
60	3·075	9·099	2·119	8·59	60
65	6·377	13·602	2·258	15·53	65
70	10·744	18·142	2·402	23·56	70
75	16·134	22·778	2·560	32·69	75
80	22·459	27·580	2·732	42·87	80
85	29·666	32·604	2·914	54·11	85
Age at Entry 60					
65	1·149	3·910	1·700	3·46	65
70	3·688	7·819	1·815	9·89	70
75	7·528	11·777	1·934	17·67	75
80	12·500	15·847	2·064	26·68	80
85	18·483	20·089	2·196	36·88	85

It has been assumed in the foregoing that the policies participating in the surplus shown at any valuation take no share of the surplus arising from the policies which have become claims within the valuation period, this latter surplus being, in fact, supposed to be distributed in the form of interim bonuses on the policies which have terminated by death. It would, I think, be interesting to trace the values of these interim bonuses, taking the above or any other hypothetical office, but to this point I have not been able to give attention.

If any considerable initial expense is incurred in placing policies on the books of an office, these interim bonuses would, for the early years of a policy's existence, come out negative quantities, and the same might even be the case with the bonuses arising at the first quinquennial distribution. Such results will, of course, indicate that under the hypothesis as to mortality which has been made, the premiums are at first insufficient to provide for the expenses, reserves and claims, and the amount of the deficit is given by the cash value of the negative bonuses. As a matter of fact, the mortality experienced during the first five years will generally differ very considerably from that which has been assumed; and I now proceed to investigate a formula for calculating bonuses when the mortality experienced is not the same as that on which the valuations are based. I shall further assume that no interim bonus is granted to a policy unless five years' premiums have been paid upon it, this hypothesis according, I believe, with the general practice of life offices.

If l_{x+n} persons, each aged $x+n$, each contribute A to a fund, the contributions accumulated at rate of interest i to be divided among the survivors at the expiration of m years, the amount which each survivor will receive will be

$$\frac{l_{x+n}(1+i)^m}{l_{x+n+m}}\mathrm{A}=\frac{\mathrm{D}_{x+n}}{\mathrm{D}_{x+n+m}}\mathrm{A}$$

Again, if the same persons each contribute P per annum in advance for m years to a fund, provided they are living, the amount per survivor at the end of the period will be

$$\mathrm{P}\frac{l_{x+n}(1+i)^m+\ .\ .\ .\ +l_{x+m-1}(1+i)}{l_{x+n+m}}$$

or,

$$\mathrm{P}\frac{\mathrm{N}_{x+n-1}-\mathrm{N}_{x+n+m-1}}{\mathrm{D}_{x+n+m}}$$

Similarly, if each of these persons who dies within the period of m years has paid to him the sum S at the end of the year of death, the accumulated amount of these payments at the end of the period will be

$$S\{(l_{x+n}-l_{x+n+1})(1+i)^{m-1}+\ .\ .\ .\ +(l_{x+n+m-1}-l_{x+n+m})\}$$

and therefore the amount per survivor will be

$$S\frac{M_{x+n}-M_{x+n+m}}{D_{x+n+m}}$$

From these formulas it is obvious that the cash surplus to be allotted to a policy at the end of a valuation period of m years, on the assumption that no interim bonuses are declared, will be expressed by the formula

$$\begin{aligned}&({}_nV'_x+{}_nB_xA'_{x+n}+Z)\frac{D_{x+n}}{D_{x+n+m}}\\ &\quad+(\pi'_x+X)\frac{N_{x+n-1}-N_{x+n+m-1}}{D_{x+n+m}}-(1+{}_nB_x)\frac{M_{x+n}-M_{x+n+m}}{D_{x+n+m}}\\ &\qquad-{}_{n+m}V'_x-{}_nB_xA'_{x+n+m}\quad .\ .\ .\ .\ .\quad (2)\end{aligned}$$

Here the commutation symbols are to be calculated by the mortality, and at the rate of interest actually experienced, and the other symbols have the same meanings as before. This formula will be applied only to the first five years of a policy's existence, so that it reduces to

$$Z\frac{D_x}{D_{x+5}}+(\pi'_x+X)\frac{N_{x-1}-N_{x+4}}{D_{x+5}}-\frac{M_x-M_{x+5}}{D_{x+5}}-{}_5V'_x$$

Or, adopting the notation of Mr. Sprague's paper (*J.I.A.*, xxii, 391–441),

$$Z\frac{D_{(x)}}{D_{x+5}}+(\pi'_x+X)\frac{N_{(x)}-N_{x+5}}{D_{x+5}}-\frac{M_{(x)}-M_{x+5}}{D_{x+5}}-{}_5V'_x$$

For valuation periods other than the first, interim bonuses are to be allotted as before. The surplus arising in the first year of any such period on l_{x+n} policies each effected at age x will therefore be

$$\begin{aligned}&l_{x+n}\{X+\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n}\}(1+i)\\ &\quad-(l_{x+n}-l_{x+n+1})(1+{}_nB_x)\\ &\quad-l_{x+n+1}({}_{n+1}V'_x+{}_nB_xA'_{x+n+1})\end{aligned}$$

and therefore the surplus per policy will be

$$\begin{aligned}&\{X+\pi'_x+{}_nV'_x+{}_nB_xA'_{x+n}\}(1+i)\\ &\quad-(1-p_{x+n})(1+{}_nB_x)-p_{x+n}({}_{n+1}V'_x+{}_nB_xA'_{x+n+1})\end{aligned}$$

By expressing π'_x, ${}_nV'_x$, ${}_{n+1}V'_x$, A'_{x+n}, and A'_{x+n+1} in terms of annuities, it is easily seen that this expression may be written in the form

$$X(1+i)+v'(i-i')-\frac{a'_{x+n}(1+i)-p_{x+n}(1+a'_{x+n+1})}{1+a'_x}$$
$$+{}_nB_x\{v'(i-i')-d'[a'_{x+n}(1+i)-p_{x+n}(1+a'_{x+n+1})]\}$$

Let us, for brevity, denote the expression

$$a'_{x+n}(1+i)-p_{x+n}(1+a'_{x+n+1})$$

by $\phi(x,n)$. Then we obtain for the cash surplus on each policy existing at the end of the valuation period

$$X(1+i)\frac{(1+i)^5-1}{i}+v'(i-i')\frac{(1+i)^5-1}{i}$$
$$-\frac{\phi(x,n)(1+i)^4+\phi(x,n+1)(1+i)^3+\ldots+\phi(x,n+4)}{1+a'_x}$$
$$+{}_nB_x\{v'(i-i')\frac{(1+i)^5-1}{i}-d'[\phi(x,n)(1+i)^4+\ldots+\phi(x,n+4)]\}$$
$$\quad\ldots\ldots\ldots\quad(3)$$

a formula which lends itself readily to arithmetical computation.

The function $\phi(x,n)$ is first tabulated, then the function $\phi(x,n)(1+i)^4+\ldots+\phi(x,n+4)$, and the remainder of the process is obvious.

It may be here noticed by way of parenthesis, that if $p_{x+n}=p'_{x+n}$, that is, if the mortality experienced is the same as that calculated upon, then since $p'_{x+n}(1+a'_{x+n+1})=a'_{x+n}(1+i')$, $\phi(x,n)=(i-i')a'_{x+n}$, which suggests an easier method of tabulating formula (1) than that actually adopted.

By the method here indicated, *i.e.*, by using formula (2) for the first five years of insurance, and formula (3) after the first five years, have been formed the annexed tables, A′, B′, and C′. The basis of the valuations is the same as before, namely, H^M 3 per-cent; the experience is taken to be Sprague's Select Mortality Tables, interest 4 per-cent. In the following table are set, side by side, the total bonuses of the hypothetical office—(1) for "experience," H^M 4 per-cent, (2) for "experience," Sprague's Select Mortality Tables, 4 per-cent, the valuation basis in each case being H^M 3 per-cent.

The figures of the following table appear to me to be interesting, though they do not call for any lengthy comment. We may notice that the bonuses arising from surplus premiums are nearly the same for each experience, and this is what we should naturally expect. For Mr. Sprague's Select Mortality Tables the bonuses are, generally speaking, smaller than those for the H^M experience, except for ages at entry 50 and 60; for age at entry 20 the differences are very considerable—in fact, for the former experience no bonus arises until the third quinquennium.

Hypothetical Office—Reversionary Bonus.

Experience H^M 4 per-cent			Experience S. S. M. T. 4 per-cent		
Age	Total Bonus from Surplus Premiums	Total Bonus	Total Bonus from Surplus Premiums	Total Bonus	Age
Age at Entry 20					
25	·93	1·39	·92	− 2·05	25
30	4·94	6·49	4·89	− ·95	30
35	8·80	12·06	8·71	2·97	35
40	12·55	18·13	12·38	7·40	40
45	16·19	24·69	15·96	12·73	45
50	19·77	31·79	19·46	18·63	50
55	23·29	39·41	22·89	24·84	55
60	26·79	47·58	26·30	31·75	60
65	30·31	56·34	29·74	39·29	65
70	33·88	65·69	33·23	47·40	70
75	37·54	75·68	36·82	56·26	75
80	41·31	86·32	40·51	65·61	80
85	45·24	97·66	44·38	75·76	85
Age at Entry 30					
35	1·08	1·61	1·07	2·18	35
40	4·70	6·47	4·68	5·79	40
45	8·19	11·90	8·16	10·45	45
50	11·57	17·91	11·51	15·77	50
55	14·88	24·50	14·76	21·45	55
60	18·14	31·68	17·97	27·93	60
65	21·39	39·47	21·18	35·11	65
70	24·66	47·86	24·39	42·89	70
75	27·99	56·89	27·69	51·49	75
80	31·41	66·57	31·06	60·60	80
85	34·96	76·92	34·57	70·53	85
Age at Entry 40					
45	1·45	2·06	1·45	3·64	45
50	4·99	7·05	4·98	8·03	50
55	8·41	12·71	8·39	12·88	55
60	11·76	19·06	11·72	18·66	60
65	15·08	26·09	15·00	25·20	65
70	18·39	33·77	18·28	32·43	70
75	21·73	42·12	21·61	40·54	75
80	25·15	51·15	24·99	49·17	80
85	28·68	60·85	28·51	58·67	85
Age at Entry 50					
55	1·85	2·59	1·90	5·47	55
60	5·46	7·90	5·51	10·25	60
65	9·00	14·04	9·04	15·98	65
70	12·51	20·96	12·53	22·53	70
75	16·03	28·65	16·04	30·09	75
80	19·62	37·09	19·61	38·26	80
85	23·31	46·25	23·29	47·36	85
Age at Entry 60					
65	2·30	3·22	2·49	7·87	65
70	6·13	9·06	6·33	13·37	70
75	9·96	15·91	10·15	20·15	75
80	13·83	23·66	14·02	27·69	80
85	17·81	32·26	18·00	36·36	85

TABLE A.—*Experience* H^{M} $3\frac{1}{2}$ *per-cent—all Loading spent.*

Age	CASH BONUS			REVERSIONARY BONUS		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
Age at Entry 20						
25	·082	...	·082	·23	·23	25
30	·207	·002	·209	·53	·76	30
35	·348	·008	·356	·83	1·59	35
40	·503	·019	·522	1·11	2·70	40
45	·674	·035	·709	1·37	4·07	45
50	·864	·059	·923	1·63	5·70	50
55	1·066	·090	1·156	1·87	7·57	55
60	1·279	·130	1·409	2·09	9·66	60
65	1·495	·180	1·675	2·31	11·97	65
70	1·703	·239	1·942	2·50	14·47	70
75	1·904	·309	2·213	2·69	17·16	75
80	2·075	·386	2·461	2·86	20·02	80
85	2·217	·469	2·686	3·01	23·03	85
Age at Entry 30						
35	·113	...	·113	·26	·26	35
40	·284	·003	·287	·61	·87	40
45	·473	·011	·484	·94	1·81	45
50	·683	·026	·709	1·25	3·06	50
55	·906	·048	·954	1·54	4·60	55
60	1·141	·079	1·220	1·81	6·41	60
65	1·379	·119	1·498	2·06	8·47	65
70	1·609	·170	1·779	2·29	10·76	70
75	1·831	·230	2·061	2·50	13·26	75
80	2·020	·298	2·318	2·69	15·95	80
85	2·176	·374	2·550	2·86	18·81	85
Age at Entry 40						
45	·157	...	·157	·31	·31	45
50	·399	·004	·403	·71	1·02	50
55	·655	·016	·671	1·08	2·10	55
60	·925	·036	·961	1·43	3·53	60
65	1·198	·066	1·264	1·74	5·27	65
70	1·462	·105	1·567	2·02	7·29	70
75	1·717	·156	1·873	2·27	9·56	75
80	1·934	·215	2·149	2·49	12·05	80
85	2·113	·283	2·396	2·69	14·74	85
Age at Entry 50						
55	·226	...	·226	·37	·37	55
60	·555	·006	·561	·83	1·20	60
65	·889	·022	·911	1·26	2·46	65
70	1·211	·049	1·260	1·62	4·08	70
75	1·521	·087	1·608	1·95	6·03	75
80	1·786	·136	1·922	2·23	8·26	80
85	2·005	·194	2·199	2·47	10·73	85
Age at Entry 60						
65	·330	...	·330	·46	·46	65
70	·757	·009	·766	·99	1·45	70
75	1·169	·031	1·200	1·46	2·91	75
80	1·520	·065	1·585	1·84	4·75	80
85	1·811	·111	1·922	2·16	6·91	85

TABLE A.—*Experience* H^M 4 *per-cent—all Loading spent.*

Age	CASH BONUS			REVERSIONARY BONUS		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
	Age at Entry 20					
25	·165	...	·165	·46	·46	25
30	·417	·009	·426	1·09	1·55	30
35	·701	·034	·735	1·71	3·26	35
40	1·015	·079	1·094	2·32	5·58	40
45	1·361	·148	1·509	2·92	8·50	45
50	1·745	·247	1·992	3·52	12·02	50
55	2·153	·382	2·535	4·10	16·12	55
60	2·583	·558	3·141	4·67	20·79	60
65	3·018	·781	3·799	5·24	26·03	65
70	3·439	1·051	4·490	5·78	31·81	70
75	3·845	1·371	5·216	6·33	38·14	75
80	4·191	1·732	5·923	6·87	45·01	80
85	4·477	2·131	6·608	7·41	52·42	85
	Age at Entry 30					
35	·227	...	·227	·53	·53	35
40	·572	·013	·585	1·24	1·77	40
45	·955	·047	1·002	1·94	3·71	45
50	1·379	·108	1·487	2·63	6·34	50
55	1·829	·201	2·030	3·28	9·62	55
60	2·305	·333	2·638	3·92	13·54	60
65	2·785	·508	3·293	4·54	18·08	65
70	3·250	·730	3·980	5·12	23·20	70
75	3·698	1·000	4·698	5·70	28·90	75
80	4·080	1·313	5·393	6·26	35·16	80
85	4·396	1·664	6·060	6·80	41·96	85
	Age at Entry 40					
45	·317	...	·317	·61	·61	45
50	·805	·018	·823	1·45	2·06	50
55	1·322	·066	1·388	2·24	4·30	55
60	1·867	·149	2·016	3·00	7·30	60
65	2·419	·274	2·693	3·71	11·01	65
70	2·952	·445	3·397	4·37	15·38	70
75	3·466	·663	4·129	5·01	20·39	75
80	3·906	·927	4·833	5·61	26·00	80
85	4·268	1·231	5·499	6·17	32·17	85
	Age at Entry 50					
55	·455	...	·455	·74	·74	55
60	1·120	·026	1·146	1·70	2·44	60
65	1·794	·092	1·886	2·60	5·04	65
70	2·444	·203	2·647	3·41	8·45	70
75	3·072	·364	3·436	4·17	12·62	75
80	3·608	·573	4·181	4·85	17·47	80
85	4·050	·827	4·877	5·47	22·94	85
	Age at Entry 60					
65	·665	...	·665	·92	·92	65
70	1·528	·037	1·565	2·01	2·93	70
75	2·359	·126	2·485	3·02	5·95	75
80	3·070	·270	3·340	3·88	9·83	80
85	3·657	·465	4·122	4·62	14·45	85

TABLE A.—*Experience* H^{M} 4½ *per-cent—all Loading spent.*

Age	CASH BONUS			REVERSIONARY BONUS		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
Age at Entry 20						
25	·250	...	·250	·70	·70	25
30	·631	·021	·652	1·66	2·36	30
35	1·062	·079	1·141	2·66	5·02	35
40	1·536	·183	1·719	3·65	8·67	40
45	2·061	·347	2·408	4·66	13·33	45
50	2·643	·586	3·229	5·70	19·03	50
55	3·261	·915	4·176	6·75	25·78	55
60	3·913	1·353	5·266	7·82	33·60	60
65	4·572	1·912	6·484	8·93	42·53	65
70	5·210	2·602	7·812	10·05	52·58	70
75	5·824	3·434	9·258	11·24	63·82	75
80	6·349	4·392	10·741	12·46	76·28	80
85	6·783	5·472	12·255	13·75	90·03	85
Age at Entry 30						
35	·342	...	·342	·80	·80	35
40	·866	·029	·895	1·90	2·70	40
45	1·445	·108	1·553	3·01	5·71	45
50	2·088	·251	2·339	4·13	9·84	50
55	2·770	·473	3·243	5·24	15·08	55
60	3·490	·791	4·281	6·36	21·44	60
65	4·218	1·219	5·437	7·49	28·93	65
70	4·922	1·769	6·691	8·61	37·54	70
75	5·601	2·451	8·052	9·78	47·32	75
80	6·181	3·256	9·437	10·95	58·27	80
85	6·659	4·179	10·838	12·16	70·43	85
Age at Entry 40						
45	·479	...	·479	·93	·93	45
50	1·217	·041	1·258	2·22	3·15	50
55	2·001	·151	2·152	3·48	6·63	55
60	2·827	·348	3·175	4·72	11·35	60
65	3·663	·645	4·308	5·94	17·29	65
70	4·471	1·057	5·528	7·12	24·41	70
75	5·250	1·593	6·843	8·31	32·72	75
80	5·916	2·251	8·167	9·47	42·19	80
85	6·466	3·025	9·491	10·65	52·84	85
Age at Entry 50						
55	·687	...	·687	1·11	1·11	55
60	1·695	·058	1·753	2·61	3·72	60
65	2·715	·211	2·926	4·03	7·75	65
70	3·702	·474	4·176	5·37	13·12	70
75	4·652	·857	5·509	6·69	19·81	75
80	5·465	1·363	6·828	7·92	27·73	80
85	6·135	1·989	8·124	9·12	36·85	85
Age at Entry 60						
65	1·004	...	1·004	1·38	1·38	65
70	2·312	·085	2·397	3·08	4·46	70
75	3·572	·292	3·864	4·69	9·15	75
80	4·649	·630	5·279	6·12	15·27	80
85	5·538	1·096	6·634	7·44	22·71	85

TABLE A.—*Experience* H^M 5 *per-cent—all Loading spent.*

Age	Cash Bonus			Reversionary Bonus		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
Age at Entry 20						
25	·336	...	·336	·94	·94	25
30	·849	·038	·887	2·26	3·20	30
35	1·429	·144	1·573	3·66	6·86	35
40	2·068	·337	2·405	5·11	11·97	40
45	2·773	·645	3·418	6·62	18·59	45
50	3·558	1·100	4·658	8·23	26·82	50
55	4·390	1·736	6·126	9·90	36·72	55
60	5·267	2·593	7·860	11·68	48·40	60
65	6·155	3·707	9·862	13·59	61·99	65
70	7·014	5·105	12·119	15·60	77·59	70
75	7·842	6·820	14·662	17·80	95·39	75
80	8·549	8·838	17·387	20·17	115·56	80
85	9·134	11·160	20·294	22·77	138·33	85
Age at Entry 30						
35	·460	...	·460	1·07	1·07	35
40	1·165	·053	1·218	2·59	3·66	40
45	1·944	·197	2·141	4·14	7·80	45
50	2·810	·462	3·272	5·78	13·58	50
55	3·729	·879	4·608	7·45	21·03	55
60	4·698	1·485	6·183	9·19	30·22	60
65	5·679	2·314	7·993	11·01	41·23	65
70	6·627	3·395	10·022	12·90	54·13	70
75	7·541	4·758	12·299	14·93	69·06	75
80	8·322	6·399	14·721	17·08	86·14	80
85	8·967	8·319	17·286	19·40	105·54	85
Age at Entry 40						
45	·643	...	·643	1·25	1·25	45
50	1·637	·074	1·711	3·02	4·27	50
55	2·693	·276	2·969	4·80	9·07	55
60	3·805	·640	4·445	6·61	15·68	60
65	4·931	1·200	6·131	8·45	24·13	65
70	6·019	1·987	8·006	10·30	34·43	70
75	7·069	3·026	10·095	12·26	46·69	75
80	7·966	4·325	12·291	14·26	60·95	80
85	8·706	5·886	14·592	16·37	77·32	85
Age at Entry 50						
55	·923	...	·923	1·49	1·49	55
60	2·281	·105	2·386	3·55	5·04	60
65	3·654	·386	4·040	5·57	10·61	65
70	4·982	·873	5·855	7·54	18·15	70
75	6·263	1·595	7·858	9·54	27·69	75
80	7·357	2·565	9·922	11·51	39·20	80
85	8·261	3·785	12·046	13·52	52·72	85
Age at Entry 60						
65	1·349	...	1·349	1·86	1·86	65
70	3·110	·153	3·263	4·20	6·06	70
75	4·808	·533	5·341	6·48	12·54	75
80	6·259	1·162	7·421	8·61	21·15	80
85	7·457	2·043	9·500	10·66	31·81	85

TABLE B.—*Bonuses from Annual Surplus Margin* 10*s.*

Experience H^{M} 3 per-cent Cash Bonus 2·7342			Experience H^{M} 3½ per-cent Cash Bonus from Surplus Margin 2·7751			
Age	Reversionary Bonus		Cash Bonus	Reversionary Bonus		Age
	New	Total		New	Total	
Age at Entry 20						
25	7·63	7·63	2·775	7·75	7·75	25
30	6·97	14·60	2·852	7·27	15·02	30
35	6·37	20·97	2·939	6·84	21·86	35
40	5·81	26·78	3·036	6·45	28·31	40
45	5·29	32·07	3·145	6·09	34·40	45
50	4·83	36·90	3·269	5·77	40·17	50
55	4·42	41·32	3·406	5·51	45·68	55
60	4·06	45·38	3·558	5·29	50·97	60
65	3·77	49·15	3·723	5·13	56·10	65
70	3·52	52·67	3·896	5·01	61·11	70
75	3·32	55·99	4·079	4·95	66·06	75
80	3·17	59·16	4·261	4·94	71·00	80
85	3·07	62·23	4·439	4·98	75·98	85
Age at Entry 30						
35	...	...	2·775	6·46	6·46	35
40	...	...	2·852	6·06	12·52	40
45	...	...	2·939	5·69	18·21	45
50	...	...	3·037	5·36	23·57	50
55	...	...	3·146	5·08	28·65	55
60	...	...	3·266	4·85	33·50	60
65	...	...	3·398	4·68	38·18	65
70	...	...	3·539	4·55	42·73	70
75	...	...	3·687	4·48	47·21	75
80	...	...	3·837	4·45	51·66	80
85	...	...	3·986	4·47	56·13	85
Age at Entry 40						
45	...	...	2·775	5·37	5·37	45
50	...	...	2·853	5·04	10·41	50
55	...	...	2·939	4·75	15·16	55
60	...	...	3·035	4·51	19·67	60
65	...	...	3·141	4·33	24·00	65
70	...	...	3·255	4·19	28·19	70
75	...	...	3·376	4·10	32·29	75
80	...	...	3·501	4·06	36·35	80
85	...	...	3·627	4·07	40·42	85
Age at Entry 50						
55	...	...	2·775	4·48	4·48	55
60	...	...	2·852	4·24	8·72	60
65	...	...	2·937	4·05	12·77	65
70	...	...	3·030	3·90	16·67	70
75	...	...	3·131	3·80	20·47	75
80	...	...	3·235	3·75	24·22	80
85	...	...	3·343	3·75	27·97	85
Age at Entry 60						
65	...	...	2·775	3·82	3·82	65
70	...	...	2·852	3·67	7·49	70
75	...	...	2·935	3·56	11·06	75
80	...	...	3·024	3·51	14·57	80
85	...	...	3·117	3·50	18·07	85

TABLE B.—*Bonuses from Annual Surplus Margin* 10*s.*

Experience H^M 4 per-cent Cash Bonus from Surplus Margin 2·8165				Experience H^M 4½ per-cent Cash Bonus from Surplus Margin 2·8584			
Age	Cash Bonus	Reversionary Bonus		Cash Bonus	Reversionary Bonus		Age
		New	Total		New	Total	
Age at Entry 20							
25	2·817	7·86	7·86	2·858	7·98	7·98	25
30	2·975	7·59	15·45	3·102	7·91	15·89	30
35	3·156	7·35	22·80	3·388	7·89	23·78	35
40	3·366	7·15	29·95	3·727	7·92	31·70	40
45	3·608	6·98	36·93	4·127	7·99	39·69	45
50	3·888	6·87	43·80	4·602	8·13	47·82	50
55	4·207	6·80	50·60	5·158	8·34	56·16	55
60	4·569	6·79	57·39	5·804	8·63	64·79	60
65	4·971	6·85	64·24	6·543	9·02	73·81	65
70	5·410	6·96	71·20	7·371	9·49	83·30	70
75	5·885	7·15	78·35	8·295	10·07	93·37	75
80	6·375	7·39	85·74	9·282	10·77	104·14	80
85	6·875	7·71	93·45	10·326	11·58	115·72	85
Age at Entry 30							
35	2·817	6·56	6·56	2·858	6·65	6·65	35
40	2·975	6·32	12·88	3·102	6·59	13·24	40
45	3·157	6·11	18·99	3·389	6·56	19·80	45
50	3·367	5·95	24·94	3·729	6·58	26·38	50
55	3·608	5·83	30·77	4·127	6·67	33·05	55
60	3·882	5·77	36·54	4·593	6·83	39·88	60
65	4·188	5·77	42·31	5·127	7·07	46·95	65
70	4·525	5·82	48·13	5·730	7·37	54·32	70
75	4·891	5·94	54·07	6·405	7·78	62·10	75
80	5·272	6·12	60·19	7·131	8·27	70·37	80
85	5·666	6·36	66·55	7·905	8·87	79·24	85
Age at Entry 40							
45	2·817	5·45	5·45	2·858	5·53	5·53	45
50	2·975	5·25	10·70	3·101	5·48	11·01	50
55	3·156	5·10	15·80	3·388	5·48	16·49	55
60	3·364	5·00	20·80	3·723	5·53	22·02	60
65	3·598	4·96	25·76	4·111	5·67	27·69	65
70	3·857	4·96	30·72	4·551	5·86	33·55	70
75	4·141	5·03	35·75	5·048	6·13	39·68	75
80	4·440	5·15	40·90	5·588	6·48	46·16	80
85	4·753	5·33	46·23	6·168	6·92	53·08	85
Age at Entry 50							
55	2·817	4·55	4·55	2·858	4·62	4·62	55
60	2·974	4·42	8·97	3·101	4·61	9·23	60
65	3·153	4·35	13·32	3·383	4·66	13·89	65
70	3·354	4·32	17·64	3·708	4·77	18·66	70
75	3·576	4·34	21·98	4·077	4·95	23·61	75
80	3·815	4·43	26·41	4·483	5·20	28·81	80
85	4·066	4·56	30·97	4·925	5·53	34·34	85
Age at Entry 60							
65	2·817	3·88	3·88	2·858	3·94	3·94	65
70	2·973	3·83	7·71	3·100	3·99	7·93	70
75	3·149	3·82	11·53	3 376	4·10	12·03	75
80	3·340	3·87	15·40	3·686	4·28	16·31	80
85	3·546	3·98	19·38	4·028	4·52	20·83	85

TABLE B.—*Bonuses from Annual Surplus Margin* 10*s*.

EXPERIENCE H^M 5 PER-CENT
Cash Bonus from Surplus Margin 2·901

Age	Cash Bonus	Reversionary Bonus		Age
		New	Total	
		Age at Entry 20		
25	2·901	8·10	8·10	25
30	3·233	8·24	16·34	30
35	3·635	8·46	24·80	35
40	4·121	8·76	33·56	40
45	4·710	9·11	42·67	45
50	5·426	9·58	52·25	50
55	6·284	10·16	62·41	55
60	7·310	10·86	73·27	60
65	8·513	11·73	85·00	65
70	9·902	12·74	97·74	70
75	11·495	13·95	111·69	75
80	13·252	15·37	127·06	80
85	15·174	17·03	144·09	85
		Age at Entry 30		
35	2·901	6·75	6·75	35
40	3·233	6·87	13·62	40
45	3·635	7·03	20·65	45
50	4·123	7·28	27·93	50
55	4·710	7·61	35·54	55
60	5·412	8·04	43·58	60
65	6·240	8·60	52·18	65
70	7·200	9·27	61·45	70
75	8·304	10·08	71·53	75
80	9·530	11·06	82·59	80
85	10·878	12·21	94·80	85
		Age at Entry 40		
45	2·901	5·61	5·61	45
50	3·233	5·71	11·32	50
55	3·634	5·87	17·19	55
60	4·116	6·12	23·31	60
65	4·687	6·46	29·77	65
70	5·353	6·89	36·66	70
75	6·124	7·43	44·09	75
80	6·987	8·11	52·20	80
85	7·942	8·91	61·11	85
		Age at Entry 50		
55	2·901	4·69	4·69	55
60	3·232	4·80	9·49	60
65	3·628	5·00	14·49	65
70	4·094	5·27	19·76	70
75	4·638	5·63	25·39	75
80	5·254	6·09	31·48	80
85	5·942	6·67	38·15	85
		Age at Entry 60		
65	2·901	4·00	4·00	65
70	3·230	4·16	8·16	70
75	3·618	4·39	12·55	75
80	4·064	4·71	17·26	80
85	4·568	5·13	22·39	85

TABLE C.—*Bonuses from Single Surplus Margin £1.*

Age	EXPERIENCE H^M 3% 1st Cash Bonus 1·159		EXPERIENCE H^M 3½% 1st Cash Bonus 1·188		EXPERIENCE H^M 4% 1st Cash Bonus 1·217		EXPERIENCE H^M 4½% 1st Cash Bonus 1·246		EXPERIENCE H^M 5% 1st Cash Bonus 1·276	
	Reversionary Bonus		Reversionary Bonus		Reversionary Bonus		Reversionary Bonus		Reversionary Bonus	
	New	Total	New	Total	New	Total	New	Total	New	Total
Age at Entry 20										
25	3·24	3·24	3·32	3·32	3·40	3·40	3·48	3·48	3·56	3·56
30	...	3·24	·08	3·40	·17	3·57	·27	3·75	·37	3·93
35	...	3·24	·09	3·49	·18	3·75	·29	4·04	·41	4·34
40	...	3·24	·09	3·58	·19	3·94	·31	4·35	·45	4·79
45	...	3·24	·09	3·67	·20	4·14	·34	4·69	·50	5·29
50	...	3·24	·09	3·76	·21	4·35	·36	5·05	·55	5·84
55	...	3·24	·10	3·86	·22	4·57	·39	5·44	·61	6·45
60	...	3·24	·10	3·96	·24	4·81	·42	5·86	·68	7·13
65	...	3·24	·10	4·06	·25	5·06	·46	6·32	·75	7·88
70	...	3·24	·10	4·16	·26	5·32	·50	6·82	·84	8·72
75	...	3·24	·11	4·27	·28	5·60	·54	7·36	·93	9·65
80	...	3·24	·11	4·38	·30	5·90	·59	7·95	1·04	10·69
85	...	3·24	·12	4·50	·31	6·21	·64	8·59	1·16	11·85
Age at Entry 30										
35	2·70	2·70	2·76	2·76	2·83	2·83	2·90	2·90	2·97	2·97
40	...	2·70	·07	2·83	·15	2·98	·23	3·13	·31	3·28
45	...	2·70	·07	2·90	·15	3·13	·24	3·37	·34	3·62
50	...	2·70	·07	2·97	·16	3·29	·26	3·63	·38	4·00
55	...	2·70	·08	3·05	·17	3·46	·28	3·91	·42	4·42
60	...	2·70	·08	3·13	·18	3·64	·30	4·21	·46	4·88
65	...	2·70	·08	3·21	·19	3·83	·33	4·54	·52	5·40
70	...	2·70	·08	3·29	·20	4·03	·36	4·90	·57	5·97
75	...	2·70	·09	3·38	·21	4·24	·39	5·29	·64	6·61
80	...	2·70	·09	3·47	·22	4·46	·42	5·71	·71	7·32
85	...	2·70	·09	3·56	·24	4·70	·46	6·17	·79	8·11
Age at Entry 40										
45	2·24	2·24	2·30	2·30	2·35	2·35	2·41	2·41	2·47	2·47
50	...	2·24	·06	2·36	·12	2·47	·19	2·60	·26	2·73
55	...	2·24	·06	2·42	·13	2·60	·20	2·80	·29	3·02
60	...	2·24	·06	2·48	·13	2·73	·22	3·02	·32	3·34
65	...	2·24	·06	2·54	·14	2·87	·24	3·26	·35	3·69
70	...	2·24	·07	2·61	·15	3·02	·26	3·52	·39	4·08
75	...	2·24	·07	2·68	·16	3·18	·28	3·80	·43	4·51
80	...	2·24	·07	2·75	·17	3·35	·30	4·10	·48	4·99
85	...	2·24	·07	2·82	·18	3·53	·33	4·43	·54	5·53
Age at Entry 50										
55	1·87	1·87	1·92	1·92	1·97	1·97	2·01	2·01	2·06	2·06
60	...	1·87	·05	1·97	·10	2·07	·16	2·17	·22	2·28
65	...	1·87	·05	2·02	·11	2·18	·17	2·34	·24	2·52
70	...	1·87	·05	2·07	·11	2·29	·18	2·52	·27	2·79
75	...	1·87	·05	2·12	·12	2·41	·20	2·72	·30	3·09
80	...	1·87	·06	2·18	·13	2·54	·22	2·94	·33	3·42
85	...	1·87	·06	2·24	·13	2·67	·24	3·18	·37	3·79
Age at Entry 60										
65	1·60	1·60	1·64	1·64	1·68	1·68	1·72	1·72	1·76	1·76
70	...	1·60	·04	1·68	·09	1·77	·14	1·86	·19	1·95
75	...	1·60	·04	1·72	·09	1·86	·15	2·01	·21	2·16
80	...	1·60	·04	1·76	·10	1·96	·16	2·17	·23	2·39
85	...	1·60	·05	1·81	·10	2·06	·17	2·34	·26	2·65

TABLE A′.—*Experience Sprague's Select Mortality Tables 4 per-cent: all Loading spent.*

Age	Cash Bonus			Reversionary Bonus		Age
	Policy	Bonus	Total	New	Total	
			Age at Entry 20			
25	−1·062	...	−1·062	−2·97	− 2·97	25
30	−1·098	− ·029	−1·127	−2·87	− 5·84	30
35	·150	− ·107	·043	·10	− 5·74	35
40	·475	− ·118	·357	·76	− 4·98	40
45	1·025	− ·120	·905	1·75	− 3·23	45
50	1·449	− ·087	1·362	2·40	− ·83	50
55	1·746	− ·024	1·722	2·78	1·95	55
60	2·294	·064	2·358	3·50	5·45	60
65	2·779	·196	2·975	4·10	9·55	65
70	3·220	·372	3·592	4·62	14·17	70
75	3·740	·601	4·341	5·27	19·44	75
80	4·020	·861	4·881	5·66	25·10	80
85	4·419	1·179	5·598	6·28	31·38	85
			Age at Entry 30			
35	·475	...	·475	1·11	1·11	35
40	− ·024	·023	− ·001	...	1·11	40
45	·584	·027	·611	1·18	2·29	45
50	1·052	·062	1·114	1·97	4·26	50
55	1·379	·123	1·502	2·43	6·69	55
60	1·985	·218	2·203	3·27	9·96	60
65	2·520	·358	2·878	3·97	13·93	65
70	3·007	·542	3·549	4·57	18·50	70
75	3·582	·784	4·366	5·30	23·80	75
80	3·891	1·053	4·944	5·74	29·54	80
85	4·331	1·386	5·717	6·42	35·96	85
			Age at Entry 40			
45	1·129	...	1·129	2·19	2·19	45
50	·429	·059	·488	·86	3·05	50
55	·805	·088	·893	1·44	4·49	55
60	1·500	·147	1·647	2·45	6·94	60
65	2·115	·249	2·364	3·26	10·20	65
70	2·674	·397	3·071	3·95	14·15	70
75	3·334	·600	3·934	4·78	18·93	75
80	3·688	·838	4·526	5·25	24·18	80
85	4·194	1·135	5·329	5·98	30·16	85
			Age at Entry 50			
55	2·210	...	2·210	3·57	3·57	55
60	·672	·117	·789	1·17	4·74	60
65	1·423	·171	1·594	2·20	6·94	65
70	2·105	·270	2·375	3·06	10·00	70
75	2·910	·424	3·334	4·05	14·05	75
80	3·343	·622	3·965	4·60	18·65	80
85	3·960	·875	4·835	5·42	24·07	85
			Age at Entry 60			
65	3·901	...	3·901	5·38	5·38	65
70	1·077	·209	1·286	1·66	7·04	70
75	2·145	·298	2·443	2·96	10·00	75
80	2·719	·443	3·162	3·67	13·67	80
85	3·536	·642	4·178	4·69	18·36	85

Experience Sprague's Select Mortality Tables, 4 per-cent.

TABLE B′. TABLE C′.

BONUSES FROM ANNUAL SURPLUS MARGIN 10s.				BONUSES FROM SINGLE SURPLUS MARGIN £1		
Age	Cash Bonus	Reversionary Bonus		Reversionary Bonus		Age
		New	Total	New	Total	
Age at Entry 20						
25	2·905	8·11	8·11	3·55	3·55	25
30	2·897	7·39	15·50	·09	3·64	30
35	3·100	7·22	22·72	·16	3·80	35
40	3·282	6·97	29·69	·17	3·97	40
45	3·534	6·84	36·53	·18	4·15	45
50	3·803	6·72	43·25	·20	4·35	50
55	4·071	6·58	49·83	·20	4·55	55
60	4·445	6·60	56·43	·22	4·77	60
65	4·845	6·68	63·11	·24	5·01	65
70	5·271	6·78	69·89	·25	5·26	70
75	5·779	7·02	76·91	·27	5·53	75
80	6·221	7·22	84·13	·28	5·81	80
85	6·766	7·59	91·72	·31	6·12	85
Age at Entry 30						
35	2·892	6·73	6·73	2·94	2·94	35
40	2·954	6·28	13·01	·13	3·07	40
45	3·131	6·06	19·07	·14	3·21	45
50	3·332	5·88	24·95	·15	3·36	50
55	3·540	5·72	30·67	·16	3·52	55
60	3·819	5·68	36·35	·17	3·69	60
65	4·123	5·68	42·03	·18	3·87	65
70	4·452	5·73	47·76	·20	4·07	70
75	4·841	5·88	53·64	·21	4·28	75
80	5·191	6·02	59·66	·22	4·50	80
85	5·617	6·30	65·96	·24	4·74	85
Age at Entry 40						
45	2·910	5·63	5·63	2·47	2·47	45
50	2·969	5·24	10·87	·12	2·59	50
55	3·132	5·06	15·93	·12	2·71	55
60	3·337	4·96	20·89	·13	2·84	60
65	3·568	4·92	25·81	·14	2·98	65
70	3·821	4·92	30·73	·15	3·13	70
75	4·119	5·00	35·73	·16	3·29	75
80	4·398	5·10	40·83	·17	3·46	80
85	4·734	5·31	46·14	·18	3·64	85
Age at Entry 50						
55	2·967	4·79	4·79	2·12	2·12	55
60	2·973	4·42	9·21	·10	2·22	60
65	3·148	4·34	13·55	·11	2·33	65
70	3·344	4·30	17·85	·12	2·45	70
75	3·573	4·34	22·19	·13	2·58	75
80	3·799	4·41	26·60	·13	2·71	80
85	4·065	4·56	31·16	·14	2·85	85
Age at Entry 60						
65	3·139	4·33	4·33	1·95	1·95	65
70	2·985	3·84	8·17	·10	2·05	70
75	3·163	3·84	12·01	·11	2·16	75
80	3·348	3·88	15·89	·11	2·27	80
85	3·563	4·00	19·89	·12	2·39	85

DISCUSSION.

The PRESIDENT (Mr. A. Day) congratulated Mr. Sunderland upon the paper which was his first submitted for public discussion. With reference to the statement that any system of distribution was equitable which the policyholders understood, and to which they gave assent, he demurred to the term "equitable": what was really meant was rather that against it the assured could not have ground for complaint. The ordinary public seemed to show a decided preference for a system of reversionary bonuses which increased at each division of profits, and he noticed the system which Mr. Sunderland had laid before them had a tendency to produce such increasing bonuses.

Mr. H. W. MANLY said the subject was a very old and vexed one, and might be generalized as the equitable distribution of surplus. A rough-and-ready method by which bonuses could be calculated, and which was intelligible to the public, seemed to be all that was desired by the offices. Notwithstanding this, they, as a scientific body, should apply their scientific knowledge to ascertain what would be theoretically equitable under all the varying circumstances and conditions. Mr. Sunderland had set himself the following problem: Given an office valuing by the H^M 3 per-cent Tables, how to ascertain what reversionary bonuses could be added at each quinquennium, assuming that the share of the surplus allotted to each policy was determined by ascertaining the profit accruing, first from surplus loading, and secondly from surplus interest, and that interim bonuses were paid on claims accruing during the interval between the valuation periods on the same basis, supposing that the actual mortality exactly corresponded with the H^M Table, and that the rate of interest realized varied from 3 per-cent up to 5 per-cent. To solve this problem, Mr. Sunderland had calculated a series of tables showing the reversionary bonuses which could be allotted—first, in respect of surplus interest accruing on the pure premiums paid during the interval, and on the reserves, including the reserves for the corresponding reversionary bonus previously allotted; secondly, in respect of the surplus loading of 10*s.* per annum for every £100 assured, and surplus interest on the reserves for corresponding reversionary bonuses; and, thirdly, in respect of a single surplus payment of £1 on an assurance of £100, and interest on the reserves for corresponding reversionary bonuses, when the rate of interest realized was 3, 3½, 4, 4½, and 5 per-cent. Given those tables, he showed how to apply them in a particular case: and if they desired at any time to find what bonuses could be given by that method of distribution when the valuation is made by H^M 3 per-cent, and the experience corresponded either with the H^M Table, or the Select Mortality Table, Mr. Sunderland had given them the means of at once doing it. He had shown them in the hypothetical office what such bonuses were likely to be. Presumably, his "hypothetical office" was a fair average office, and the premiums were fair average premiums, charged by offices of the present day. If they examined the table they would find that taking the age at entry at 30, the bonus for the first quinquennium was 7*s.* per-cent per annum, in the following quinquennium it was £1 per-cent per annum, in the next £1. 4*s.* per-cent, and so it increased until the policy was 40 years old, when it reached about £2 per-cent per annum. His experience was that the public wanted to begin with 2 per-cent, and to have it increased at every succeeding period. That consummation, apparently, was not going to arrive yet, in the light of Mr. Sunderland's figures. He understood that Mr. Sunderland considered this to be an equitable method of distribution. That brought him to the question, what is "equitable"? The object desired was, he assumed, to give every policyholder the full benefit of what he paid, both in respect of his assurance and his share of profits. If they were making 4¼ per-cent or 4½ per-cent on their investments, could it be "equitable" to make a valuation at 3 per-cent? By so doing they were providing large bonuses for future policyholders out of profits made by their predecessors. That could not be equitable. Then was it equitable to debit all the initial expenditure against the accruing profits of the first quinquennium? They incurred the heavy charge on the assumption that they were going to receive an annuity during the assured's life. Would it not be more equitable to distribute the initial expenditure over that period? There was also the question of profits that arose from lapses, surrenders, and so on, which had not been dealt with; but if they attempted to discover in that way what would be an equitable distribution, considering the irregular manner in which the premiums were often loaded, they would be involved in great difficulties. He thought it better to determine first what bonuses were going to be given, and to then accommodate their premiums to meet them. If they constructed their premiums equitably, they could arrive at a fairly equitable distribution. In their treatment of this subject hitherto they had always endeavoured to evolve a uniform system of distribution, which was not desirable. It was better that the public should have various systems set before them.

Mr. George King said the paper only touched the threshold of the subject. He would like to know on what the premiums of the hypothetical office were based, because, so far as he could judge, they were not very suitable premiums, and it was doubtful whether such elaborate tables as had been based upon them would be of very much service. The premiums seemed too low for the early ages and too high for the older, and that was the reason why the bonuses came out so small at some of the younger ages of entry. He (Mr. King) did not think they could so completely distribute the expenses for the various ages at entry as to enable them to say how much was contributed to profits by a policy taken out at the age of 30, and how much by one taken out at the age of 60. There were other difficulties in the way of making an exact calculation from age to age; but, at the same time, the tables given, if taken in a broader way, would be very useful. On the general question of the distribution of surplus, one remarkable thing was that the rate of expenses seemed to go up, while the rate of interest had rather a tendency to come down. The natural result would be that bonuses should also have a tendency to come down: but bonuses seemed to be steadily increasing. This was not entirely because of the lighter mortality, but was due mainly to the tendency to keep larger reserves. Was it right to make such large reserves, and what was to be done with the larger bonus available in consequence of making them? If these larger reserves were not made, if they divided up to the hilt, the result would be practically to give a uniform cash bonus throughout the duration of the policy, and it would not be fair, under those circumstances, to give a uniform, still less an increasing, reversionary bonus. But if they made a much larger reserve by valuing at a low rate of interest, the effect was to keep back part of the surplus accruing from the present policyholders, and to make it a fund for giving bonuses in future. It would not be fair so to keep back the surplus from existing policyholders, in order to give larger bonuses to future members who had not contributed to it; but if they had some system by which those whose bonuses were kept back at the beginning got a larger bonus later on, that difficulty would be overcome. These considerations led to the result that if they made larger reserves than required for safety, they were bound in honour to give a larger bonus to the older policies than to the more recent ones. The instinct of the public for an increasing reversionary bonus seemed to be a true one, provided they made larger reserves to meet it. He had made the following table, founded upon Mr. Sunderland's hypothetical office, by grouping together all the ages at entry:

Mr. Sunderland's Hypothetical Office Reversionary Bonus. All Ages at Entry.

Duration of Assurance	New Bonus per £100	Total Bonus to Date	New Bonus per-cent per annum on Sum Assured	New Bonus per-cent per an. on Sum Assured and previous Bonuses
5	2·34	2·34	·47	·45
10	5·66	8·00	1·13	1·05
15	6·66	14·66	1·33	1·16
20	7·62	22·28	1·52	1·24
25	8·60	30·88	1·72	1·31
30	9·11	39·99	1·82	1·30
35	10·05	50·04	2·01	1·34
40	10·56	60·60	2·11	1·31
45	11·54	72·14	2·31	1·34
50	12·14	84·28	2·43	1·32
55	13·16	97·44	2·63	1·33
60	13·93	111·37	2·79	1·33
65	15·05	126·42	3·01	1·33

The bonuses per-cent per annum on the sum assured alone, in the first quinquennium, thus came out 10*s*., in the second £1. 3*s*., and increased steadily, until at the end of 65 years they reached 3 per-cent per annum. This confirmed figures that he had gathered from various sources as to the effect of valuing at a low rate of interest and making a higher one. A difference of 1 per-cent in the rate of interest would just about keep up a uniform reversionary bonus of £1. 5*s*. per-cent per annum on the sum assured alone. In the assumed case there was a difference of 25*s*. per-cent in the rate of interest. If they took the percentage per annum of the bonuses on existing bonuses, as well as on the sum assured, the figures were still more remarkable. The bonus was the same for the first quinquennium under each plan, in the second it went up to £1. 1*s*., after the third it became practically uniform, varying from £1. 6*s*. 4*d*. up to about £1. 6*s*. 10*d*. Taking the various ages at entry together, they got rid of most of the causes of disturbance. Formerly he did not think they could get equity by distributing surplus as a uniform percentage on the sum assured and existing bonuses, but he had come to a different view, and the figures Mr. Sunderland had placed before them seemed to confirm his more recent opinion. If they, in those cases, gave a uniform cash bonus, or divided in proportion to the loading on their premiums, injustice would be done. It appeared from Mr. Sunderland's figures that even giving a uniform percentage on the sum assured and the previous bonuses gave too much to recent assurances, but that was partly owing to Mr. Sunderland's treatment of expenses. He agreed with Mr. Manly that it was not fair to charge the whole of the initial expenditure against the first bonus of a particular policy. A great deal of the initial expenditure was incurred for the good of all, and therefore a certain proportion might, for bonus purposes, be thrown over the whole business. He (Mr. King) had been apparently advocating the distribution of surplus by way of a uniform percentage of reversionary bonus on sum assured and existing bonuses, but he did not mean that that was necessarily the best, much less the only method of meting out rough justice. There might be other ways equally good, and he urged that they must not only think of the premiums they had charged, but also of their system of valuation, when they proceeded to select a method of distributing surplus.

Mr. C. J. HARVEY remarked that Mr. Sunderland had referred to his (Mr. Harvey's) formula as approximate, and seemed to imply that the one given by himself gave closer results. He would, therefore, reproduce his formula, so that it might at once be seen wherein it differed from that given by Mr. Sunderland. In furtherance of that purpose, the third and fourth parts of his formula were presented in rather a different form to that in which it had been given in the *Journal*. His formula was:

$$\{\mathrm{P}'_x-(\mathrm{P}_x+r+c\mathrm{P}'_x)\}\,(1+i')\,\frac{(1+i')^5-1}{i'} \quad \ldots\ldots \quad (1)$$

$$\mathrm{P}_x(i'-i)\,\frac{(1+i')^5-1}{i'} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\begin{aligned}{}_n\mathrm{V}_x[i'(1+i')^4-i(1+i)^4]+{}_{n+1}\mathrm{V}_x[i'(1+i')^3-i(1+i)^3]&\\ +\ldots+[{}_{n+4}\mathrm{V}_x(i'+i)] \quad \ldots\ldots\ldots\ldots \quad (3)&\end{aligned}$$

$$\begin{aligned}{}_n\mathrm{B}_x[\mathrm{A}_{x+n}[i'(1+i')^4-i(1+i)^4]+\mathrm{A}_{x+n+1}[i'(1+i')^3-i(1+i)^3]&\\ +\ldots+[\mathrm{A}_{x+n+4}(i'-i)] \quad \ldots\ldots\ldots\ldots \quad (4)&\end{aligned}$$

The dashed letters here represented the actual experience, and the undashed letters the assumed experience; whereas the converse was the case with Mr. Sunderland's formula. The four parts of his (Mr. Harvey's) formula gave—1. The profit from the unspent loading; 2. The profit from the surplus interest on the pure premiums; 3. The profit from the surplus interest on the policy-values; 4. The profit from the surplus interest on the bonus-values. Parts (1) and (2) of his formula would be seen to be identical with Mr. Sunderland's, for

$$P'_x-(P_x+r+cP'_x)=X$$

might be variable or constant as we pleased. On comparing parts (3) and (4), however, a difference would be observed, and he was afraid that Mr. Sunderland's reasoning was fallacious. The problem to be solved was a very simple one. Given two companies, one of which realized i' on its investments, and the other i, $i'>i$, and for the sake of simplicity let it be assumed that the funds in each office were equal and remained constant for a quinquennium. Then the office which invested at i' would gain in interest on each unit of its funds

$$i'\frac{(1+i')^5-1}{i'}-i\frac{(1+i)^5-1}{i}$$

over that of the office which only realized i, but according to Mr. Sunderland the gain would only be

$$(i'-i)\frac{(1+i')^5-1}{i'},$$

and he thereby understated the gain in interest by

$$i\left\{\frac{(1+i')^5-1}{i'}-\frac{(1+i)^5-1}{i}\right\}.$$

Mr. Sunderland's formula became, when corrected, identical with his own. The subject of increasing expenditure was a very important one, and too much attention could hardly be given to the effect produced by such expenditure on the bonuses. The method of distributing bonuses was described in the paper he had read to the Institute a few years ago, the object being to find the equity of some of the methods at present in use. He there showed that where there was a small surplus interest of, say, ½ per-cent, a reversionary bonus of a constant percentage on the sums assured for each premium paid during a quinquennium gave results identical with those of his formula, and where the surplus interest was a little higher, the allocation of a reversionary bonus by a percentage on the sum assured and previous bonuses gave results very near the truth indeed.

Mr. Jas. Chisholm said that his impression was that the bonuses had a tendency to be larger about thirty years ago than they were now. Competition between the offices then took the form of giving large bonuses in place of spending large sums of money to get business. The main lesson to be learnt from the paper was shown in the table containing the results worked out on Mr. Sprague's experience. It would be found that those results showed that such experience would not allow an office charging the rates assumed by Mr. Sunderland to give a reversionary compound bonus on the sum assured, and previous bonuses of 30*s*. per-cent, which was the ideal that so many insurance companies aimed at. The paper would have a wholesome effect in strengthening the hands of actuaries in rather laying by further portions of the surpluses in the reserves than in increasing the bonuses. The total bonuses by Mr. Sprague's experience were less for the ages of 20, 30, and 40, than they were by the H^M, while for the ages of 50 and 60 it was the reverse. Was not that caused by the fact that the H^M premium entered into both sets of calculations, and that, as a matter of fact, it was too small a quantity for the lower ages, whereas, for the higher ages, it was too large, and therefore produced too small a bonus at the higher ages by the H^M experience as compared with Mr. Sprague's experience, and too large a bonus at the younger ages. Instead of going into these details of calculation in order to find out what bonus they could afford to give, he agreed with Mr. Manly's opinion, that the simpler way was to equate the premiums to the benefit intended to be given, always bearing in mind that the valuation system must accord with both. He advocated the doing away with the bonus system altogether. Its origin was that the premiums at first charged were based upon the only known data to the actuaries of the day—the Northampton, and then the Carlisle Table,—and were loaded with a very large margin, really more, as it has turned out, than was sufficient to meet the necessities of the case. Now that they had attained to a more accurate scientific knowledge of what the proper premium ought to be, there was no reason for making these large additions. They must have some margin for safety, but nothing like that included in the old premiums. Offices started within the last thirty years brought out lower rates of premium than the older offices for "with-profit" policies, and there was now a movement in favour of deferred bonuses, and the meaning of that was that the same tendency was in progress —cutting down profit margins, and approximating profit rates to non-profit rates by deferring the payment of the bonus as long as possible.

Mr. G. H. RYAN said that no subject of greater practical importance was ever discussed at their meetings. The treatment of it in Mr. Sunderland's paper was unavoidably partial and incomplete. Although the title of the paper would cover a wider compass, the author had dealt only with the bonuses arising from surplus premiums and surplus interest on whole-life policies. The mode of distribution of bonuses on whole-life policies had in many cases been fixed by deed, or by practice and precedent long established. But it was not so with regard to other forms of assurance which had more lately sprung into existence, notably the endowment assurance system. The methods employed by different companies in distributing profits upon endowment assurance policies were extraordinarily diverse, and anyone taking up the broad question of distribution of surplus might advantageously direct his attention to the important subject of endowment assurance bonuses. He agreed with Mr. Manly that questions of bonus could not be regulated by hard and fast adherence to rules and regulations of mathematical equity. The subject was far too general and practical in its nature for that. Were bonuses to be allotted according to such fixed principles as Mr. Sunderland had set out, they would be constantly having conflicts arising between the true and false bases of valuation. Nevertheless, such enquiries as Mr. Sunderland's were of very great value, because they prevented offices from breaking adrift entirely from considerations of scientific fitness. They also appeared to have a real practical value as indicating how they could preserve equity, not merely among policies of different ages, but among the different classes of members. It was, in his opinion, not a matter of so much concern whether they gave to an entrant, at the several stages of his policy, the exact equivalent of his share of the surplus, as whether they gave to endowment assurance policyholders and other members of special classes of assurance their fair and proper shares. The tendency of the present time was, no doubt, to give too large bonuses to recent entrants in regard to whole-life policies; and Mr. Sunderland's figures fully confirmed this view. There were many reasons for that which he need not dwell upon. He considered the hypothetical office table given in the paper possessed more value than the other tables, which appeared to be merely specimen or test tables framed upon arbitrary bases with regard to the expenses and interest-yield of offices. The hypothetical office table, however, took premiums which might be regarded as fairly normal, and the assumption that had been made with regard to expense was not out of the way. Hence, the results were important as showing the probable working of an average company. With regard to the formula used for interpolation (p. 363), he suggested that Mr. Sunderland should give the general formula of which it was a particular instance. This, he thought, was as follows:

$$V_x = V_0 + x a_1 - \frac{x(1-x)}{2} b_0,$$

and would be found in the *Journal* (xi, p. 69), in Mr. Woolhouse's paper on Interpolation.

Mr. A. W. SUNDERLAND, in reply, said it was possible he had not made the drift of his paper quite clear. He gathered from Mr. Manly's remarks that he considered the hypothetical office was put forward as one of the most important parts of it. It, however, was merely an illustration of the tables he had introduced, and was quite subsidiary to the main object of the paper, which was to show how, from the three tables, A, B, and C, any ordinary office which experienced the H^M mortality and valued by H^M 3 per-cent, might calculate a bonus table to fit its own case. He did not advocate this hypothetical office as being typical, and the office premiums had, in fact, been taken to a great extent at random. It was not worth while going to any considerable trouble to calculate a table of premiums, since no table could be formed which would satisfy every-one. Each office had its own table of premiums, and if it experienced the H^M mortality, and valued by H^M 3 per-cent, could, from the A, B, and C tables, without difficulty form its own bonus table in the way he had illustrated. He had selected one of the most difficult cases, namely, where it was necessary to interpolate between the tables for four different rates of interest. In a great many cases, of course, this interpolation would be unnecessary. He could not agree that to form the 4¼ per-cent tables a simple interpolation between those at 4 per-cent and those at 4½ per-cent would be sufficient. He was able to state that that course would introduce considerable errors at the higher policy ages. He could not appreciate the force of

Mr. Manly's objections to a 3 per-cent valuation. It was, in his opinion, very desirable, in the interests of the offices, that a low rate of interest should be used in the valuations. The rate of interest was at present declining, and there seemed to him a good prospect of this decline continuing in the future. Policyholders reaped the advantage of a low valuation-rate of interest, both in increased security and in future profits. Nor could he see how it was practicable to carry out with equity Mr. Manly's suggestion of a bonus distribution on a predetermined plan, the premiums being calculated to provide these bonuses, since it was extremely improbable that the hypotheses made in calculating the table of premiums would accord with future experience. Mr. Harvey appeared to consider that the formulas in the paper were approximations. They were, however, under the hypotheses made, not approximately, but exactly, correct. The question of the distribution of profits among endowment assurances had not been overlooked by him, but it was quite beyond the scope of the present paper.

**Jubilee Address to the Queen.*

TO THE QUEEN'S MOST EXCELLENT MAJESTY.

MAY IT PLEASE YOUR MAJESTY,

WE, the President, Council, and Members of the Institute of Actuaries, Incorporated by your Royal Charter, beg leave humbly to approach your Majesty with our most respectful and hearty congratulations on the auspicious occasion of the Jubilee of your Majesty's prosperous and beneficent reign.

In common with the rest of your Majesty's subjects, we recognize with profound satisfaction the growth, during the last fifty years, of National greatness and all that tends to the consolidation of Empire; and we may be permitted to express the confident belief that nothing in the history of your Majesty's reign will be more memorable, or be found to have done more to strengthen the State, than the progress which, under your Majesty's fostering care, has been made in the development of Institutions tending to create and encourage provident habits. Among these are included Savings Banks, Building Societies, and the various forms of co-operation by aid of which small means are accumulated or economized; Friendly Societies providing against sickness and the wants of old age, with their wide-reaching ramifications and subtle influences for good;

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* For reference to this Address, see the account of the Proceedings at the Annual General Meeting, given on p. 408.

and, lastly, those more prominent and, in respect of their magnitude, more important Institutions—the Life Assurance Societies of the Kingdom—the guidance of which is our special privilege and concern.

In the recent grant to the Institute of Actuaries of a Royal Charter, by the terms of which there is confided to us the duty of extending and improving the data and methods of actuarial science, we gratefully acknowledge the increased dignity given to our calling, and rejoice to see a new proof of the anxious solicitude which your Majesty has ever shown, not only for the extension and development, but also for the wise direction on sound principles, of Provident Institutions calculated to promote the welfare of your people.

And, finally, we humbly pray that your Majesty's reign may be long continued, becoming day by day more famous as that in which individual thrift has so conspicuously increased the National prosperity.

Given under the Common Seal of the said Institute,
June 14th, 1887.

ARCHIBALD DAY, *President.*

Seal.

GEORGE S. CRISFORD, } *Honorary*
T. E. YOUNG, } *Secretaries.*

SYLLABUS OF PRIZES OFFERED BY MR. SPRAGUE

FOR ESSAYS UPON THE FOLLOWING SUBJECT.

On the Equitable Distribution of the Profits of a Mutual Life Office, when it is a condition of the contract between the Members that no Policy shall receive any share of the Profits until after the lapse of a certain time, which may be either a fixed term of years for all Policies, or may depend upon the age at entry, or upon the Premium paid.

The Council of the Institute of Actuaries have the pleasure to announce that Mr. Sprague has kindly placed at their disposal the sum of £70, which sum is to be applied in giving two Prizes, namely, £50 to the author of the best Essay upon this subject, and £20 to the author of the second best Essay, or is to be otherwise divided between two or more Essayists, if and as the Adjudicators may recommend.

Without restricting Candidates in their treatment of the subject, it is suggested that particular attention should be given to the arrangements that must be made in order to secure that justice shall be done as between entrants of the same age who join the office at different times; but Candidates are not debarred from considering what scale of distribution should be adopted for the purpose of doing justice as between entrants of different ages who join at the same time.

CONDITIONS OF THE COMPETITION.

1. That the Competition be open to all Members of the Institute.
2. That the Essays be sent in to the President of the Institute before or on 4 May 1888.

(*If Essays are written in a foreign language, they should be accompanied by a literal English translation.*)

3. That the names of the Competitors be sent in under seal, with a Motto corresponding to one to be prefixed to the Essay; the Motto and Essay not to be in the handwriting of the Competitor.
4. That the Essay or Essays to which a Prize may be awarded be the property of the Institute.
5. That a Prize or Prizes be only awarded if the Adjudicators shall consider an Essay or Essays to be worthy of the distinction.
6. That unsuccessful Essays be returned, on application at the Institute, with the corresponding envelopes unopened.

Note on the Word "Actuary."

[From *A New English Dictionary on Historical Principles, founded mainly on the Materials collected by the Philological Society.* Edited by J. A. H. Murray, LL.D. (Oxford, at the Clarendon Press, 1884.)]

Actuary: (adaptation of Latin actuāri-us, an amanuensis, a keeper of accounts, formed on actus, act).

1. A registrar or clerk, a notary: an officer appointed to write down the acts or proceedings of a court. Still used in the Convocation of the Province of Canterbury.

1553. Fox, *A. & M.* in Cobbett's *State Trials*, I, 628: "Requiring also the copies as well of the articles as of his protestation, of the Actuaries."—1658. Bramhall, *Consecr. Bishops*, iii, 30: "The same publick Notary who was principall Actuary both at Cardinall Pole's consecration and Archbishop Parker's."—1667. Chamberlayne, *St. Grt. Brit.*, I, II, viii (1743), 73: "To this Court (of Arches) belongeth an Actuary, a Registrar, and a Beadle. The office of the Actuary is to attend the Court, set down the Judge's decrees."—

1717. Blount, *Law Dict.*: "*Actuary* (actuarius) is the Scribe that registers the acts and Constitutions of the Convocation."—1879. *Whitaker's Alman.*, 155, *Conv. of Prov. of Cant.*: "(Officers) Vicar-General, Registrar, Actuary."

†2. The managing Secretary or Accountant of a public Company. *Obs.*

1804. W. Taylor, in *Ann. Rev.*, II, 238: "The Managers and Actuaries of our public Companies."

3. An official in an Insurance office, whose duty it is to compile statistical tables of mortality, and estimate therefrom the necessary rates of premium, &c.; or one whose profession it is to solve, for Insurance Companies or the public, all monetary questions that involve a consideration of the separate or combined effect of Interest and Probability, in connection with the duration of human life, the average proportion of losses due to fire or other accidents, &c.

1849. Macaulay, *Hist. Eng.*, I, 283: "An Actuary of eminent skill subjected the ancient parochial registers of baptisms, marriages and burials, to all the tests which the modern improvements in statistical science enabled him to apply."—1859. *Q. Rev.*, No. 211, 75: "Many Actuaries acknowledge the soundness of that basis for life assurance and annuity calculations."

NOTES.

(i) *Obsolete senses*, like obsolete words, have † prefixed.

(ii) "*Obs.*" means obsolete.

(iii) The *quotations* illustrate the forms and uses of the word, showing the age of the word generally and of its various senses particularly, the earliest, and, in obsolete words or senses, the latest, known instances of its occurrence being always quoted. (The years named are those in which the stated uses of the word occurred.)

(iv) The sections 1, 2, and 3, show the several stages in the history of the use of the word.

CORRESPONDENCE.

FRIENDLY SOCIETY LEVIES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—The following problem is sometimes met with in the valuations of small Friendly Societies, namely: Let there be m members in a society, and let there be upon the death of each member a levy of 1 paid by each remaining member. It is required to find the value of all the levies.

The only published solutions that I know of were given by Mr. G. F. Hardy in a letter to the *Insurance Record* of 10 Sept. 1880. He there demonstrated the three formulas

$$W = \frac{m^2}{2}\bar{A}_{xx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$W = \frac{m(m-1)}{2}\bar{A}_{xx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$W = m^2\left\{\frac{a_{x:x-1}}{p_{x-1}} - a_{xx}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where all the members are assumed to be of one age, x, and where the symbol W is used to represent the value of the levies.

Mr. Hardy in his letter also showed that formulas 1 and 3 give only approximate solutions, since they contemplate the possibility of a fraction of an individual dying, and the remaining fraction paying the proportionate part of a levy; but that formula 2 is rigidly accurate.

For Mr. Hardy's solutions your readers may be referred to his letter. My present object is to give an exceedingly simple and elegant demonstration of formula 2, which has been supplied to me by Mr. Hubert Ansell.

Let each of the m members be supposed to have effected several assurances, namely, one upon the life of each remaining member jointly with his own, but so that there are no duplicate pairs. Then it will happen that on the death of every member, since he is assured jointly with every other member of the Society, there will become payable as many sums of 1 as there are surviving members; and this is the same sum as would be paid if a levy of 1 were made upon each surviving member. Therefore the present value of the levies must be the same as the present value of the assurances supposed to have been effected, namely, $\frac{m(m-1)}{2}\bar{A}_{xx}$; the number of assurances being the number of combinations that may be made of m things taken two at a time, or $\frac{m(m-1)}{2}$.

Perhaps Mr. Ansell's solution, as given above, may be made a little clearer by means of a slight modification. Instead of assuming *joint-life* assurances, let each member effect a *contingent* assurance on the life of each other member against his own life, by means of which to pay his share of the levies as they arise. The value of the levies will then be exactly equal to the value of all the contingent assurances. The value of each contingent assurance is $\bar{A}^1_{xx}$, which is equal to $\frac{1}{2}\bar{A}_{xx}$. There are m members, each of whom effects $m-1$ contingent assurances, so that there are $m(m-1)$ such assurances at the outset, and therefore their aggregate value is $\frac{m(m-1)}{2}\bar{A}_{xx}$, as before.

When all the members are not of one age, an average age must be found, and this may conveniently be done by means of Mr. Makeham's formula for the law of human mortality.

Theoretically, formula 2 is the correct solution of the problem, but in practice it is not entirely satisfactory. If members secede from the Society, or if new members join, the value of the levies, even as regards the original members who remain, is changed; and there is no practical method by which account can be taken of these causes of disturbance.

I am, Sir,
Your obedient servant,
GEORGE KING.

92, *Cheapside, London, E.C.*
25 *May* 1887.

[We hope to place before our readers, in our next issue, a communication from Mr. G. F. Hardy on the above subject.—Ed. *J.I.A.*]

INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—You have on two occasions, namely in January 1877 and January 1882, given tables showing the amounts of the Issue Insurances in force according to the latest returns; and it may interest your readers to see the following table, in which the information is brought down to the present time. The figures are, in all cases, taken from the five Blue-Books containing the Board of Trade returns of the years 1881 to 1885 inclusive, except in the case of the *Caledonian* and the *Sovereign* Offices, which made no return during those years, in consequence of their investigations being made at intervals of seven and six years respectively. In these cases, therefore, I have used the figures given in the valuation returns to 31 December 1885, which will appear in the Blue-Book of 1886.

There has been a very great development of this class of business during the period embraced by the returns, as will be at once seen from the following summary.

Date of Table	Number of Policies	Net Sums Assured	Net Premiums Received	Average Premium per-cent
		£	£	
1877	261	875,558	62,238	6·75
1882	383	1,264,166	97,495	7·51
1887	539	1,696,747	116,704	6·82

The general result is that the number and amount of the transactions have about doubled in the course of ten years. The average premium is now almost exactly the same as it was in 1877, but in 1882 it was somewhat higher.

I am, Sir,
Your obedient servant,
T. B. SPRAGUE.

Edinburgh,
10 *March* 1887.

Name	Date of Valuation	Number of Policies	Net Amounts, deducting Reassurances		Gross Amounts			
			Sums Insured	Premiums Received	Sums Insured	Premiums Received	Reserve	Aver Premi per-c
			£	£	£	£	£	
Guardian . . .	31 Dec. 1884	107	289,153	17,538	394,854	23,865	19,174	6·0
Equity and Law .	31 Dec. 1884	80	256,566	17,677	333,609	23,469	23,469	7·0
Eagle . . .	31 Dec. 1882	30	194,535	21,947	357,130	32,810	12,047	9·1
Law Union . .	30 Nov. 1884	66	124,021	7,042	179,521	9,762	9,762	5·4
North British & Merc.	31 Dec. 1880	16	74,200	4,332	83,200	5,232	5,925	6·2
Legal and General .	31 Dec. 1881	16	65,450	3,357	69,650	3,540	1,772	5·0
Norwich Union .	30 June 1881	14	62,850	4,227	62,850	4,227	1,589	6·7
Scottish Equitable .	1 Mar. 1883	23	61,063	4,497	70,413	5,236	4,975	7·4
Law . . .	31 Dec. 1884	19	57,709	3,761	92,709	5,861	5,861	6·3
Standard . . .	15 Nov. 1880	14	55,920	4,254	85,920	5,829	3,766	6·7
Reliance . . .	31 Dec. 1882	10	42,230	2,962	44,230	3,130	2,687	7·0
National . . .	31 Dec. 1884	12	36,900	1,343	43,200	1,605	1,605	3·7
Rock . . .	19 Aug. 1882	6	34,500	3,318	34,500	3,318	3,318	9·6
London Assurance .	31 Dec. 1880	11	34,462	2,600	40,462	2,999	2,849	7·4
Pelican . . .	31 Dec. 1880	6	34,015	2,343	34,015	2,343	2,343	6·8
Union . . .	30 June 1882	7	26,500	1,706	26,500	1,706	1,125	6·4
Scottish Union & Nat.	31 Dec. 1884	7	25,308	1,217	35,308	1,667	1,200	4·7
Caledonian . .	31 Dec. 1885	9	23,350	983	37,850	1,292	969	3·4
Alliance . . .	31 Dec. 1883	3	18,000	894	18,000	894	894	4·9
Imperial . . .	31 Jan. 1881	9	17,765	1,087	17,765	1,087	1,087	6·1
Clerical, Med. & Gen.	30 June 1881	4	17,000	1,061	17,000	1,061	1,061	6·2
Royal Exchange .	31 Dec. 1880	3	16,200	393	16,200	393	314	2·4
Edinburgh . .	31 Mar. 1885	5	13,425	455	13,425	455	409	3·3
Universal . . .	31 Dec. 1882	5	12,400	1,247	22,000	1,473	1,686	6·7
Northern . . .	31 Dec. 1880	8	12,400	782	15,900	1,003	1,003	6·3
Crown . . .	25 Mar. 1885	4	11,973	671	52,933	3,682	3,314	6·9
Liv. & London & Globe	31 Dec. 1883	3	11,000	599	11,000	599	320	5·4
City of Glasgow .	20 Jan. 1884	6	10,500	614	11,500	667	667	5·8
Commercial Union .	31 Dec. 1882	6	9,940	583	36,220	4,722	4,486	13·0
General . . .	31 Dec. 1882	4	9,300	535	11,300	661	331	5·8
University . .	1 May 1885	4	8,800	679	8,800	679	679	7·7
West of England .	31 Dec. 1882	7	7,450	468	7,450	468	474	6·2
Scottish Imperial .	31 Dec. 1880	3	5,500	163	5,500	163	120	2·9
Sovereign . . .	31 Dec. 1885	2	5,000	284	5,000	284	142	5·6
Westminster . .	31 Dec. 1881	4	4,362	453	4,362	453	411	10·3
Patriotic . . .	31 July 1884	2	4,000	480	4,000	480	275	12·0
British Empire .	31 Dec. 1884	2	1,500	106	1,500	106	50	7·0
Atlas . . .	25 Dec. 1884	1	1,000	45	1,000	45	4	4·5
Scottish Metropolitan	20 May 1882	1	500	1·3	500	1·3	·8	·2
39 Companies .	...	539	1,696,747	116,704·3	2,307,276	157,267·8	122,163·8	6·8

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, 1887.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE.
(PART I.)

Examiners.—W. SUTTON, Esq., M.A.; F. W. WHITE, Esq.; E. WOODS, Esq.

First Paper.

ARITHMETIC AND ALGEBRA.

1. Find the value to 7 decimal places of

$$\frac{3+\dfrac{4}{6\frac{1}{4}}+8\frac{3}{8}}{4+\dfrac{5}{7\frac{1}{5}}+\dfrac{6\frac{3}{4}}{7\frac{1}{9}}}$$

Express 2 furlongs + 81 poles + 11 yards + 8 feet + 4 inches as a decimal fraction of a mile.

2. A person invests £999. 10*s*. in the 3 per Cents. at 101¼. When the 3 per Cents. have fallen to 99⅛ he transfers his capital into the 2½ per Cents. at 79. Find the alteration in the annual income from the investment.

3. State and prove the rule for the extraction of the cube root.
What is the cube root of 128024064?

4. State and prove the rule for finding the G. C. M. of two or more algebraical expressions.

What is the G. C. M. of $a^3+a^2b-ab^2-b^3$, $a^3-2a^2b-ab^2+2b^3$, and $a^3-3ab^2+2b^3$?

5. Show that a quadratic equation of one unknown cannot have more than two roots.

6. Solve the following equations:

$$ax^2-a^2(x+b^2)=ab(x-ab) \quad . \quad . \quad . \quad . \quad (1)$$

$$\left.\begin{aligned} x^4y^3-x^3y^4&=n^3 \\ x^2y-xy^2&=n \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\left.\begin{aligned} x^{\frac{3}{2}}+y^{\frac{2}{3}}&=3x \\ x^{\frac{1}{2}}+y^{\frac{1}{3}}&=x \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

7. If three quantities are in A. P., find under what circumstances the second will have to the first a greater ratio than the third has to the second.

8. The Arithmetic mean of two numbers exceeds the Geometric mean by 13, and the Geometric mean exceeds the Harmonic mean by 12. Find the numbers.

9. Define *Permutations* and *Combinations*, and given n things find the number of each when the n things are taken r at a time.

Twelve balls can be arranged in *three* separate parcels, of which the first parcel always contains *three* balls, the second always *four*, and the third always *five*. In how many ways can this be done?

10. Find the numerically greatest term in the expansion of $(a+x)^n$, where n is a positive integer.

What will be the numerically greatest term of $(3a+7b)^7$?

BOOK-KEEPING.

11. Describe the two methods of book-keeping known as single and double entry respectively.

What are their respective merits?

12. Enumerate the principal books necessary for book-keeping by double entry.

State clearly the distinguishing features of a Cash Account, a Profit and Loss Account, a Statement of Receipts and Expenditure, a Trading Account, and a Balance Sheet.

Second Paper.

EUCLID (Books I to IV, Definition of Book V, and Book VI).

1. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

2. The opposite angles of any quadrilateral figure described in a circle are together equal to two right angles.

3. What is Euclid's definition of *ratio?* What are similar rectilineal figures?

In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle and to one another.

ARITHMETIC AND ALGEBRA.

4. Given that the roots of the equation $x^3-7x^2+14x-8=0$ are in geometric progression; find them.

5. Write down (1) the general term, (2) the sum to n terms, (3) the sum *ad infinitum* of the series

$$\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+\frac{1}{4.6}+\&c.$$

6. Define a *logarithm*, and find the connection between the Napierian and the Common logarithms of a given number.

Show that $\epsilon^x=1+x+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+\&c.$

7. What is the rule for determining the logarithm of a quotient? Given log $1·9328 = 0·286187$, obtain the logarithm of $(1·9328)^{\frac{1}{3}} \times (1·9328)^{\frac{1}{6}} \times (1·9328)^{-1}$.

8. What is meant by the probability of an event happening, and how is this denoted mathematically?

If p denote the probability of an event happening, what interpretation is to be assigned to the cases where $p=0$, $p=1$, and $p=\frac{1}{2}$?

9. Five balls are drawn from a bag of 12 balls, of which 3 are known to be marked ones. What is the probability that all the 3 marked ones are among the 5 drawn?

10. The probability of the happening of an event in one trial being known, find the probability of its happening at least r times in n trials.

Find the chance that a person with two dice will throw aces at least four times in six trials.

FINITE DIFFERENCES.

11. Obtain a formula for expressing u_{x+m} in terms of u_x, and the successive finite differences Δu_x, $\Delta^2 u_x$, &c.

$$\text{If } \left.\begin{aligned} u_1&=4 \\ u_2&=30 \\ u_3&=120 \\ u_4&=340 \\ u_5&=780 \end{aligned}\right\} \text{ and } \Delta^4 u_x \text{ is constant, find an algebraical expression for } u_x.$$

12. Show that

$$\Delta^n u_x = u_{x+n} - \frac{n}{1} u_{x+n-1} + \frac{n.n-1}{1.2} u_{x+n-2} - \&c.$$

Hence show that, second differences being constant

$$\Delta_{x+2} - 2\Delta_{x+1} + \Delta_x = 0.$$

It is optional on the part of the Candidate to answer one or more of the following questions; but due weight will be given to answers sent in.

PLANE CO-ORDINATE GEOMETRY.

13. Find the equation to a straight line, using rectangular co-ordinates, which cuts the axes at $x=a$ and $y=b$ respectively.

What straight line has for its equation $x=a$?

14. Find under what conditions the general equation

$$Ax^2 + Bxy + Cx^2 + Dx + Ey + F = 0$$

represents the equation to a circle.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW.
(PART II.)

Examiners.—T. G. ACKLAND, Esq.; G. TODD, Esq., M.A.; F. B. WYATT, Esq.

First Paper.

1. Obtain a formula for determining approximately the rate of interest in an annuity-certain.

2. Show how the several payments of an annuity-certain may be divided into principal and interest, and demonstrate that the amount of principal remaining outstanding at the end of any year is equal to the present value of the annuity for the remaining term.

3. How would you proceed to verify the columns of a printed table representing the present values and amounts of sums and of annuities-certain?

4. Define "rate of mortality", "force of mortality", "central death rate", and give expressions showing approximately the relations between these three functions. Explain generally the practical application and use of the "force of mortality" in life assurance investigations.

5. Obtain the probability that out of three lives, (x), (y), (z),

(1) One at least will fail in the nth year.

(2) Not more than two will fail in the nth year.

(3) The three lives will fail in the following order: one before the nth year, one in the nth year, one after the nth year.

6. Find the annual premium for an annuity payable to (x) in the event of his surviving (y), the premiums paid to be returned without interest in the event of (x) dying before (y).

7. State a general formula for the value of an annuity on a single life or combination of lives, payable m times a year, and apply it to determine the value of a continuous reversionary annuity.

8. Prove, and give a verbal explanation of the following formula:

$$_nV_x = \frac{P_x(N_{x-1}-N_{x+n-1})-(M_x-M_{x+n})}{D_{x+n}}.$$

9. Deduce a formula for the continuous construction of a table of single premiums for survivorship assurances on (x) against (y).

10. Find the value of the second, and every third succeeding presentation to a living, assuming all the nominees to be of the same age on presentation.

11. Prove that the amount of a free policy, which can be granted in lieu of a whole-life policy effected at age x which has been n years in force, may be expressed by the formula $\left(1-\frac{P_x}{P_{x+n}}\right)$. Give a verbal

explanation of this formula, and show to what other classes of assurance it is applicable.

12. Given $P_{25}=\cdot01521$, $P_{42}=\cdot02654$, $a_{42}=15\cdot5679$, find the value of a whole-life policy for £1,500 effected at age 25, which has been 17 years in force, and to which a reversionary bonus of £383 is attached.

The following questions are optional on the part of the Candidate.

13. Define a "differential coefficient", and find the differential coefficient of the quotient of two functions.

Differentiate (1) x^n (from first principles).

(2) e^{x^n}

(3) $\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

14. Obtain the criteria for maxima and minima points in a curve, and exemplify their significance by reference to a mortality curve.

Second Paper.

15. State clearly the meaning of the term "force of discount", and obtain a formula exhibiting its relation to the effective rate of interest.

16. State a general formula for the value of an annuity-certain for n years, whose successive payments are $u_1, u_2, u_3, \ldots\ldots u_n$, and explain how the values of annuities of the figurate numbers can be employed in determining such values.

17. A bond for £1,000, bearing interest at 3 per-cent for 20 years, is to be sold. What can a purchaser give to realize 5 per-cent from his investment (1) supposing the bond to be repayable at par at the end of 20 years, (2) supposing the bond to be repayable in 20 equal annual instalments? Given $a_{\overline{20|}}$ at 5 per-cent $=12\cdot4622$.

18. State a general formula for the class of benefits the single premiums for which can be derived from Orchard's Tables, and explain it verbally.

19. If $(1+a_x)=m\,{}_{n-1|}a_x$, find the relation between P_x and ${}_nP_x$.

20. Find an approximate value for the single premium for an assurance upon a life (x) payable at the instant of death.

21. Describe the construction of the D and N columns for joint lives proposed by Professor De Morgan, and applied by Dr. Farr to "English Life Table No. 3", and point out in what respects it differs from that previously used. Write down in commutation symbols the single premium for an ordinary survivorship assurance by both methods.

22. Prove that the single premium for an assurance payable on a life (x) attaining the age of $(x+n)$ or dying previously, may be

expressed by the formula $\frac{1-i\,{}_{n-1}a_x}{1+i}$, and show that this is equal to $\frac{M_x-M_{x+n}+D_{x+n}}{D_x}$.

23. Obtain the value of an annuity payable so long as two at least out of three lives are in existence. Obtain this value also by general reasoning.

24. Explain the principles upon which Mr. Woolhouse has constructed his Improved Theory of Assurances, and state briefly the advantages of the method.

25. Prove that the value, after n years, of an endowment assurance policy effected on a life (x) payable at $(x+n+m)$ or previous death, may be expressed by either of the following formulas:

$$\frac{P_{x+n.\overline{m}|}-P_{x.\overline{n+m}|}}{P_{x+n.\overline{m}|}+d}, \qquad \frac{A_{x+n.\overline{m}|}-A_{x.\overline{n+m}|}}{1-A_{x.\overline{n+m}|}}.$$

26. A policy effected on a life aged x at entry, has been n years in force. It is proposed to substitute therefor a policy on an approved life now aged y. Determine the future annual premium for such substituted policy.

27. An annuity of £100 per annum, payable until the death of the last survivor of three lives, A, B, and C, aged respectively, 20, 30, and 40, is to be divided equally between A and B during their joint lives, afterwards between the survivor and C, if living, and ultimately is payable to the last survivor. Find the value of A's interest.

Given $a_{20}=20{\cdot}2246$ $\qquad a_{20.30}=16{\cdot}1739$
$a_{30}=18{\cdot}4156$ $\qquad a_{30.40}=13{\cdot}9872$
$a_{40}=16{\cdot}1026$ $\qquad a_{20.40}=14{\cdot}5274$
$a_{47}=14{\cdot}1097$ $\qquad a_{20.47}=12{\cdot}9502$
$a_{48}=13{\cdot}8064$ $\qquad a_{20.48}=12{\cdot}7015$

The following question is optional on the part of the Candidate.

28. Explain the meaning of, and the necessity for, the introduction of a constant in the process of integration.

Integrate (1) $\int\frac{dx}{x}$

(2) $\int\frac{x^{n-1}}{a+bx^n}\cdot dx$

(3) $\int a^x.dx$

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW.
(PART III.)

Examiners—H. W. MANLY, Esq.; JAS. CHISHOLM, Esq.; W. T. GRAY, Esq.

First Paper.

1. Explain the theory of a pure-premium valuation.

2. Describe the method of determining from the Registers of an Assurance Company, the relative numbers of living and dying at each year of age.

Alternative Questions.

3. Give a short account of the construction and graduation of the Carlisle Table; and state the reason for supposing that the second enumeration of the inhabitants of Carlisle was not obtained in such a detailed manner as the first.

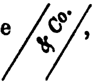
4. What were the materials used, and the methods pursued, in constructing the English Life Table No. 3?

5. Give Mr. J. A. Higham's formulas for the adjustment of mortality tables; and explain wherein they differ from Mr. Woolhouse's.

6. A Life Office, 30 years of age, has been transacting a steady business since its commencement: give a rough idea of the relative reserves required by valuations made by (1) Combined H^M and $H^{M(5)}$ Tables at 3 per-cent interest; (2) H^M 3 per-cent; (3) H^M 4 per-cent; (4) Carlisle 3 per-cent.

7. Explain the distinction between a Warranty and a Representation.

8. Explain the application and principle of *tacking*, when there are a series of mortgages on an estate.

9. What is the effect of simply crossing a cheque , and adding the words "not negociable"?

10. How is the issue of Bank of England notes regulated by the Act of 1844? What restrictions exist against the issue of notes by Banks other than the Bank of England?

11. A, aged 30 next birthday, is entitled to $\frac{1}{6}$ of £30,000 Consols on the death of B, aged 60 last birthday, provided he survives B: you are requested to give your opinion as to the market value of A's interest.*

12. A, aged 60, and B, aged 40, have successive life interests in a sum of £10,000 Consols, over which they have a power of appointment

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* The Candidate will be supplied with any book or books of Tables out of the Library that he may apply for in writing to the Examiners, but he will not be allowed to retain any book more than 15 minutes.

which they can only jointly exercise. They now determine to exercise this power and divide the fund. Give in symbols their respective shares, and state the table of mortality and rate of interest you would employ.

It is optional on the part of the Candidate to answer one or both of the following questions, but due weight will be given to answers sent in.

13. Give a strict verbal definition of the term *force of mortality*.

Assuming $\mu_x = -\frac{1}{l_x}\cdot\frac{dl_x}{d_x}$, find l_x in terms of μ_x.

How can you tell, by mere inspection, that there is a mistake in the following approximate formula for the value of μ_x—

$$\mu_x = \frac{8(d_{x-1}+d_x)-(d_{x-2}+d_{x+1})}{12l_x}$$

What is the correct expression?

14. Show that $\mu_{xy} = \mu_x + \mu_y$.
What is the value of $\mu_{\overline{xy}}$?

Second Paper.

15. How would you proceed to find the strain upon the funds of a company caused by the death claims of the past year.

16. If the premium required for an extra risk is accurately represented by an addition of £1 to the net annual premium for the assurance of £100 on a single life at all ages, what extra charge should be made for Joint-Life Assurances, (*a*) when one life only is exposed to the extra risk, (*b*) when both are exposed to the extra risk?

Alternative Questions.

17. Company A has lost all its capital, and its assets are insufficient to meet its liabilities in full. Company B has agreed to take over the business of Company A on a reduction of contracts. Describe the procedure by which the transfer must be carried out, and explain on what basis you would advise the reduction of contracts to be made.

18. Explain the difference between the Married Women's Property Acts, 1870 and 1882, so far as they relate to Life Assurance Policies. How would you propose to deal with the following case:—A husband who effected a policy for the benefit of his wife and children is unable to continue his payments, his wife is living and the children are minors—

(1) if the policy was issued before the Act of 1882?

(2) if issued after 1882?

19. What are the sources of Revenue of the British Government? Give in round numbers the amount raised from each source.

20. Explain the following terms, which are used in the weekly accounts of the Bank of England:—"Rest", "Public Deposits", "Government Securities", and on both sides "Other Securities."

Alternative Questions.

21. It is desired to value a Friendly Society, giving benefits in sickness and at death, by means of tables founded on its own experience, making allowance for secessions. What particulars would you ask for, and in what form? How would you deal with negative values in such a valuation? Give your reasons.

22. A large old-established Institution has a Pension Fund to provide pensions to its officials after the age of 60, when they must retire. Every official on his entrance must be under the age of 21, and must pass the medical officer as a person of sound constitution. One of the conditions of his appointment is that he shall subscribe a fixed percentage of his salary towards the Pension Fund. The pension is a function of the average salary received during the whole period of his service. An official, on leaving the service, receives no compensation for his contributions to the Pension Fund. How would you proceed to value such a fund? State whether you would consider negative values admissible, and give your reasons.

23. A, aged x, is entitled to a reversionary life interest on the death, without issue, of B, aged y. A is desirous of borrowing on his life interest by way of a reversionary charge, but wishes to have the option of redeeming the charge at any time within five years. Investigate a formula for the charge, and also for the redemption money; and state what mortality tables and rates of interest you would use.

24. An estate consists of £10,000 Consols; Railway Debenture Stocks of the present market value of £10,000; and £20,000 share in a business, which, as far as can be ascertained, is solvent and in a flourishing condition, the firm having to pay out the share at the rate of £1,000 a year, with interest on the balance at the rate of 6 per-cent per annum. Subject to two annuities of £100 each to two ladies, E and F, aged 65 and 70 respectively, the income of the estate is left to A, a female aged 65, for life; and at her death the estate is to be divided equally between B and C absolutely. B is bankrupt, and the trustee in bankruptcy proposes to put his share up for sale. You are requested to give your opinion as to the present market value of B's interest.*

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* The Candidate will be supplied with any book or books of Tables out of the Library that he may apply for in writing to the Examiners, but he will not be allowed to retain any book more than 20 minutes.

N.B.—Where alternative questions are set, the Candidate is required to answer one only. No additional credit will be given if both are answered.

It is optional on the part of the Candidate to answer the following question, but due weight will be given to the answer sent in.

25. Show that $$\bar{A}^{1}_{xy}=\mu_x\bar{a}_{xy}-\frac{d\bar{a}_{xy}}{dx},$$

and hence deduce the approximate expression for the value of the annual premium corresponding to this single premium $\bar{A}^{1}_{xy}$, viz.:

$$\mu_x+\frac{1}{2}\frac{(a_{x-1.y}-a_{x+1.y})-\mu_x}{a_{xy}+1}.$$

PROCEEDINGS OF THE INSTITUTE.—SESSION 1886–7.

First Ordinary Meeting, 29 *November* 1886.

The President (Mr. A. DAY) in the Chair.

The following gentleman was elected a Fellow of the Institute:

Tennant, John Bell.

The following gentlemen were elected Associates of the Institute:

Buckley, Thomas John Wesley.
Chatham, James.

The President delivered an inaugural address.

Second Ordinary Meeting, 20 *December* 1886.

The President (Mr. A. DAY) in the Chair.

The following gentlemen were elected Fellows of the Institute:

Cockburn, Henry Robertson.
Gunn, Niel Ballingal.

The following gentlemen were elected Associates of the Institute:

Fox, Morris.
Owen, Evan Frederick.

Mr. H. W. Manly, Vice-President, read a paper on "The American Tontine and Mutual Assessment Schemes."

The following gentlemen took part in the discussion:—Messrs. Adler, Ansell, Bailey, Justican, G. King, Newbatt, Ryan, Searle, Straker, and Wyatt.

Third Ordinary Meeting, 31 *January* 1887.

The President (Mr. A. DAY) in the Chair.

The under-mentioned gentleman was elected an Associate of the Institute:

Richardson, Josephus Hargreaves.

Mr. G. H. Ryan read a paper on "The Method of estimating Expected Deaths and Expected Claims; with observations upon certain modes of deducing the numbers exposed to the risk of death."

The following gentlemen took part in the discussion:—Messrs. Adler, Ansell, Bailey, Burridge, C. D. Higham, G. King, Manly, Newbatt, Rothery, and Todd.

Fourth Ordinary Meeting, 28 *February* 1887.

The President (Mr. A. Day) in the Chair.

Mr. Geo. King read a paper on "The Numerical Calculation of the Values of Complex Benefits, by means of Formulas of Approximate Summation."

The following gentlemen took part in the discussion:—Messrs. Ackland, Bumsted, G. F. Hardy, Ryan, Sunderland, and Sutton.

Mr. King further read a letter, dated 27 Feb., from Mr. T. B. Sprague, bearing upon the subject of the paper.

Fifth Ordinary Meeting, 28 *March* 1887.

The President (Mr. A. Day) in the Chair.

Mr. C. D. Higham read a paper entitled "A Proposal for the Amendment of the Law relating to the Assignment of Policies of Life Assurance."

The following gentlemen took part in the discussion:—Messrs. Adler, Bailey, F. Bell, Burridge, C. E. Brown, A. Hendriks, Humphreys, G. E. Humphreys, Matthews, Saunders, Whitcher, Whittall, and the President.

Sixth Ordinary Meeting, 25 *April* 1887.

The President (Mr. A. Day) in the Chair.

Mr. A. W. Sunderland read a paper on "Bonuses arising from Surplus Premiums and Surplus Interest."

The following gentlemen took part in the discussion:—Messrs. Chisholm, Harvey, G. King, Manly, and Ryan.

The Fortieth Annual General Meeting, 4 *June* 1887.

The President (Mr. A. Day) in the Chair.

The Proceedings at the Annual General Meeting will be found on p. 408.

The following is the Report of the Council for the year:

"The Council of the Institute of Actuaries have the pleasure of reporting, at the close of the third year since the date of its incorporation and the thirty-ninth since its foundation, that the prosperity of the Institute has been fully maintained.

"The number of members on the register on the 31st March, 1887, was 484. The following schedule exhibits in detail the record of membership during the year:

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
I. Number of Members in each class on the 31st March 1886 .	3	152	253	24	9	441
II. Withdrawals by						
(1) Death . .	1	3	1	...	...	24
(2) Resignation .	...	1	7	5	...	
(3) Default in payment of Subscriptions .	...	1	3	2	...	
	2	147	242	17	9	417
III. Additions to Membership . . .	...	...	6	61	...	67
	2	147	248	78	9	484
IV. Transfers						
(1) By Examination:						
from Associates	...	...	4	...	...	...
to Fellows .	...	4	...	...	...	...
(2) By Election:						
from Associates	...	...	3	...	...	...
to Fellows .	...	3	...	...	...	...
V. Number of Members in each class on the 31st March 1887 .	2	154	241	78	9	484

"The accounts for the year, which have been duly audited, and which are now submitted to the members, show that the total income was £1,227. 18*s*. 8*d*. as compared with £1,148. 14*s*. 9*d*. for last year, or an increase of £79. 3*s*. 11*d*.,—the annual subscriptions inclusive of entrance fees being £969. 3*s*. 0*d*., against £865. 4*s*. 0*d*. in 1886: on the other hand the expenditure has increased, the amount being £1,552. 3*s*. 5*d*., as compared with £802. 11*s*. 4*d*. For the increase of expenditure the members will in some measure have been prepared by the Report of the Council to the last annual meeting, which pointed to £194. 18*s*. 11*d*. having been paid in connection with the Charter and Bye-Laws since the closing of the year's account, and to the heavy expenditure which would immediately be due in respect of the *Text-Book*, Part II, as nothing up to that date had been paid on this account. The present Statement contains £343. 12*s*. 5*d*. paid in respect of the Bye-Laws and Charter, and £274. 5*s*. 9*d*. on account of the *Text-Book*. The members will understand that under this head of expenditure considerable further sums will require to be paid before the work is finally completed.

"It will be seen that the total funds of the Society have suffered diminution to the extent of £324. 4*s*. 9*d*., but they still amount to the satisfactory sum of £3,033. 0*s*. 8*d*.

"The Council regret that though considerable progress has been made with the preparation of Part II of the *Text-Book*, Mr. King has been unable, through pressure of other duties, to complete the work up to the present time.

"The Council have been gratified by the increased attendance at the sessional meetings. At the first meeting held on the 29th November 1886, the President (Mr. Archibald Day) delivered an inaugural address; and the following papers have been read and discussed during the session:—

"20 *December* 1886—'On the American Tontine and Mutual Assessment Schemes'—by Mr. H. W. Manly.

"31 *January* 1887—'On the Method of estimating Expected Deaths and Expected Claims, with observations upon certain modes of deducing the number exposed to the risk of death'—by Mr. G. H. Ryan.

"28 *February* 1887—'On the Numerical Calculation of the Values of Complex Benefits by means of Formulas of Approximate Summation'—by Mr. G. King.

"28 *March* 1887—'A Proposal for the Amendment of the Law relating to the Assignment of Policies of Life Assurance'—by Mr. C. D. Higham.

"25 *April* 1887—'On Bonuses arising from Surplus Premiums and Surplus Interest'—by Mr. A. W. Sunderland.

"The examinations were held on the 29th and 30th April, when the unprecedented number of sixty-four candidates presented themselves. The examiners reported that thirty-nine candidates appeared for examination in the First Part, of whom seventeen passed, namely:

Allen, A. G.
Ansell, G. F.
Cooper, E. J.
Coote, E. C.
Davies, G. W. McC.
Fellows, R. H.
Fraser, D. C.
Gamble, A. F. M.
Gordon, J. C.
Holliday, J.
Lidstone, G. J.
Phelps, W. P.
Reeve, C. E.
Renton, D.
Stahlschmidt, L.
Thiselton, H. C.
Watt, G.

"In the Second Part there were sixteen candidates, of whom eleven passed, namely:

Bearman, H.
Bridgman, A. H.
Cross, R.
Faulks, J. E.
Hemming, A. G.
Hose, F. E. A.
Marks, G.
Molyneux, A. E.
Mullins, B. W.
Newman, P. L.
Warner, S. G.

"In the Third Part nine candidates presented themselves, of whom four passed, namely:

Byers, F. M. T.
Kyd, T.
Lemon, W. K.
Nightingale, H. E.

"In all cases the names of the successful candidates have been placed in alphabetical order.

"Other examinations have been held at various times during the session in connection with the admission of Students.

"The classes for the instruction of members in the subjects connected with Parts I and II of the examinations have been well attended, and the results have been satisfactory. The Council have determined that these classes shall be continued.

"It is with great regret that the Council record the deaths of Mr. J. Hill Williams and Mr. Peter Gray. The former rendered valuable services to the profession as one of the honorary secretaries of the Institute in connection with obtaining and arranging the experience contributed by the companies on which the Institute Tables are founded; and subsequently, as President, he maintained, by his character and capacity, the honourable traditions associated with the Presidential chair. It would be superfluous to enumerate the important benefits conferred on the profession by Mr. Gray, as his publications have formed part of the necessary professional training of every actuary; but the Council may specially call to remembrance that as Mr. Williams largely assisted in laying the foundations of the Institute Tables, Mr. Gray materially laboured in raising the superstructure by his valued supervision of the calculations based thereon.

"In consequence of the death of Mr. Hill Williams the Council, at a special meeting in pursuance of the Bye-laws, elected Mr. Henry Harben to fill the vacancy in the Council.

"Considering that the time had arrived when the Messenger Legacy Fund might again be utilized, the Council issued, in the early part of the session, a syllabus of conditions for a prize essay competition upon the subject of Friendly Societies, with the offer of two prizes of £50 and £25. The subject being one of increasing importance, it is hoped that the competition will result in some valuable addition to our knowledge.

"The Council have also the pleasure to announce that Mr. Sprague has generously offered, through the medium of the Institute, a sum of £70 to be applied as prizes for essays on a certain method of Distribution of Profits—and that the syllabus will immediately be issued containing the conditions of competition.

"It may be mentioned that a copy of the Charter and Bye-Laws has been sent to every member of the Institute, and that a permanent Register is in course of completion which will furnish an authentic record and evidence of membership.

"In conclusion, the Council refer with satisfaction to the important step which they have taken in relinquishing the lease of the rooms which they held from the Statistical Society in Adelphi Terrace, and in removing the offices of the Institute to the commodious premises of Staple Inn Hall with its two adjacent rooms. The Institute has incurred increased pecuniary responsibility in so doing, but the Council are bound to recognize the liberal spirit in which they have been met by the Directors of the Prudential Assurance Company—the present owners of this ancient property—and they trust that they have at last secured a permanent and suitable home for the Institute, corresponding to its higher dignity and its ever-increasing duties; although it is not without a feeling of regret that they sever a tie of so many years' duration as that which has existed between themselves and so distinguished and kindred an Association as the Statistical Society.

"ARCHIBALD DAY,
"*President.*"

Income and Expenditure for the Year ending 31 *March* 1887.

Dr.

1886-7.		£ s. d.	£ s. d.
Amount of Funds, 31 March 1886—			
Messenger Legacy Fund		305 19 3	
Brown Prize Fund		241 3 11	
General Fund		2,810 2 3	
			3,357 5 5
Annual Subscriptions—			
Town Fellows		277 4 0	
Country ,,		81 18 0	
Fellows elected since Issue of Bye-Laws:			
(1) Entrance Fees	£15 15 0	38 17 0	
(2) Subscriptions	23 2 0		
Town Associates		287 14 0	
Country ,,		105 0 0	
Associates elected since Issue of Bye-Laws:			
(1) Entrance Fees	£10 10 0	23 2 0	
(2) Subscriptions	12 12 0		
Students:			
(1) Entrance Fees	£63 0 0	155 8 0	
(2) Subscriptions	92 8 0		
			969 3 0
Dividends			92 16 0
"Mortality Experience" Sales			13 4 11
Journal Sales			136 4 0
Text-Book, Part I, Sales			15 18 0
Logarithm Card Sales			0 12 9
			£4,585 4 1

Cr.

1886-7.	£ s. d.	£ s. d.
Rent	150 0 0	
Salaries	179 0 0	
Text-Book	274 5 9	
Journal	330 8 1	
Examinations	69 8 0	
Stationery	62 9 0	
Bye-Laws and Charter	343 12 5	
Library	17 5 7	
Meetings	53 7 6	
House Expenses	32 7 5	
Stamps	23 10 0	
Fire Insurance, 15*s.*; Income Tax, £5. 6*s.* 8*d.*	6 1 8	
Assistance	5 5 0	
Bank Charges	0 5 9	
Miscellaneous	4 17 3	
		1,552 3 5
Amount of Funds, 31 March 1887—		
Messenger Legacy Fund	312 1 9	
Brown Prize Fund	248 18 7	
General Fund	2,472 0 4	
		3,033 0 8
		£4,585 4 1

Examined and found correct,

(Signed) ALFRED B. ADLARD,
T. H. ADEY,
CHAS. WINDETT, } *Auditors.*

14 *April* 1887.

Balance Sheet, 31 *March* 1887.

Dr.

	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1*s.* 10*d.* Consols), cost	203 17 8	
Unappropriated Dividends	108 4 1	
		312 1 9
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200 0 0	
Unappropriated Dividends	48 18 7	
		248 18 7
General Fund		2,472 0 4
		£3,033 0 8

Cr.

		£ s. d.
Consols, £1,600, cost		1,498 19 3
Metropolitan Railway 4 per-cent Debenture Stock, £1,200, cost		1,225 17 3
Cash at London and Westminster Bank		272 9 9
Cash in hand, including Subscriptions		7 7 5
Arrears of Subscription:—		
Town Fellows	£6 6 0	
Country ,,	8 8 0	
Town Associates	2 2 0	
Country ,,	3 3 0	
Do. under new Bye-Laws	8 8 0	
		28 7 0
		£3,033 0 8

Examined and found correct,

(Signed) ALFRED B. ADLARD,
T. H. ADEY,
CHAS. WINDETT, } *Auditors.*

14 *April* 1887.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the Members was held at Staple Inn Hall, Holborn, on Saturday last, the President, Mr. Archibald Day, in the chair.

The PRESIDENT, in moving that an address be presented to the Queen (the text of the Address will be found on p. 386) said that, considering that the Institute had been re-constructed under a Charter direct from the Crown, and that it was now representative of the whole united actuarial profession, they should, in common with other learned and scientific societies, profess their loyalty to the Crown and their congratulations to the Queen on the occasion of the jubilee of her reign. Connected as the profession was with the vast provident institutions of the country, they must rejoice in the development which had taken place in them during the fifty years of the reign of our gracious and beloved Sovereign. In the first year of the reign building societies received legislative sanction, and during its course savings banks, which were established just prior to it, had multiplied and prospered, and had become a department of Government in connection with the Post Office, the deposits amounting to nearly £100,000,000; co-operative societies had been founded in vast numbers; friendly societies had multiplied throughout the length and breadth of the land; the funds of the life assurance companies had enormously increased (probably more than trebled); and some societies which devoted their operations mainly to industrial business had advanced by giant strides. It had been Her Majesty's happiness to witness this vast increase in the evidence of the national prosperity; and the nation had not been slow to appreciate the warm interest, and the fostering care, which she had exhibited in the welfare of the provident classes. He trusted that the members of this Institute might be permitted humbly to address Her Majesty, expressing their thankfulness that she had not only been permitted in her lifetime to see the fruit of her Royal support, but to be assured by her people of their appreciation and gratitude.

Mr. B. NEWBATT, in seconding the resolution, rejoiced to find that the address had received that cordial welcome which those who promoted it had no difficulty in predicting for it. It was not necessary that they should be politicians to join heartily in the language of this address. All, as good citizens, were willing to join in a tribute of homage and respect to a gracious and exalted lady, and an act of self-congratulation on the splendid achievements and the unsurpassed renown of the last fifty years.

The motion was then put and carried amid applause.

The Report of the Council (given on p. 403) having been read,

The PRESIDENT, in moving its adoption, said he might assume a tone of congratulation upon the results of the year and the record which had been read. The subject of the greatest congratulation was that mentioned in the last sentence of the report—that they were here assembled in Staple Inn Hall, which the Prudential Assurance Company had leased to them as tenants. One of the first duties that came upon the Council last session was to consider that the term of their lease with the Statistical Society would expire at Midsummer, and whether they had accommodation enough at

Adelphi Terrace to justify them in continuing there for another term. The committee appointed to consider this subject decided that the accommodation was insufficient. Search was made in various directions for other rooms without much success; but eventually negotiations were concluded for renting Staple Inn Hall from the Prudential Assurance Company, who had treated them very liberally. It was a real matter of regret—perhaps rather sentimental than otherwise—that they should have to separate from the Statistical Society. The Institute had been with them as joint or sub-tenants since 1852; and as far back as 1848, the year in which the Institute was founded, a proposition was made to the Statistical Society that they should share the accommodation which they then possessed. He also congratulated the members on the increase in the membership of the Institute. They had now 484 members, there being an increase of 43 during the year. The increase in the number of students was 54; the increase in the number of Fellows, 2; while there had been a decrease amongst the Associates of 12. There would no doubt be a further increase in the number of students. With regard to the Fellows, he should be content if they maintained the number at 150 or 152, and hoped the Associates would not fall off materially, so that they might be able to reckon on a constituency of at least 500. That was a matter for satisfaction and congratulation. The increase in the income of the Institute was not very considerable this year, but the subscriptions and entrance fees amounted to £969, as against £865 last year. He thought the figures very satisfactory—say, in round numbers, 500 members and £1,000 income a-year from subscriptions. The expenditure had increased very materially. It was a significant fact that in the year of his presidency more money had been spent by the Institute than on any previous occasion. One of the principal items of expense was £344, the balance of the expenses in connection with the Charter and Bye-Laws, for which there was no remedy. Then the sum of £274 had been paid on account of the *Text-Book*. That work had been in hand some time, but they had not hitherto paid any money away for it; and the £274 was not a very serious item, seeing that they expected, when they had spent still more money upon it, to get some return by sales of the book to the members and to the public generally. These two items—£344 and £274—made up £618, and might fairly be chargeable to capital account. The capital had diminished since last year, and there must be anticipated further heavy expenses in connection with removal, furniture, &c. The *Journal* expenses had increased by £87. This was due to the fact that the cost of four numbers was included in the payment, as against that of three numbers included in last year's accounts. It was a matter of congratulation that the *Journal* had, by the energy of the honorary secretaries, aided by the assistant editor, been brought up to date—the April number having been issued in the month of April. He felt reasonably satisfied that they were in a position with the income they had to live within their means. The papers which had been read during the session were partly practical, partly scientific, and well up to the usual standard. The meetings had been extremely well attended, and the debates well sustained. Coming to the examinations, the rules had been the same as were adopted last year. It was a matter for further congratulation that a larger number of members came up for examination, and a larger proportion of the candidates who came up passed. The success of candidates meant that they had exercised a vast amount of self-denial and perseverance; and he hoped they would feel rewarded by the fact of their success, and that they would also be rewarded in other ways. The report referred to the death of Mr. Hill Williams and Mr. Peter Gray. They all knew their merits. He had had occasion in public to mention the loss they had sustained by their removal, and did not think he need refer further to the subject, except to say that they all felt when they lost old friends—and especially such old friends as Mr. Hill Williams and Mr. Gray—how few there were left of those who originally established the Institute. Two other Fellows of the Institute had died during the year—Mr. Linford and Mr. Moncrieff Wilson, who both died in harness. They were very active and good men of business. Their respective offices regretted their loss very much, and so did the Council, for they were both of them good men and true. Respecting the Messenger prize essays, at the beginning of this year the Council issued a syllabus of prizes for essays on the subject of "Friendly Societies", and they hoped that several members were engaged on this competition. It would redound to their personal credit, and add to the information which the Institute possessed, and to the knowledge of the profession generally. This was known before to-day, but it was not known that Mr. Sprague had, of his great kindness and generosity, in his love of the Institute, and his desire for the education of its rising members, offered a prize for an essay on a subject referred to in the report, which was more actuarial than historical, and which might well employ those who had recently passed their examinations, and were fresh upon such matters. He would assure them that writing essays on any definite subject was excellent practice. He then moved that the report and accounts be adopted.

Mr. W. SUTTON, in seconding the adoption of the report, said he did so with great pleasure, and remarked that at the present time he was between his twentieth and twenty-first year of membership of the Institute. As a matter of curiosity he had referred back to the proceedings at the annual meeting of the Institute in June 1866—just twenty-one years ago. At that meeting Mr. Bailey, who had done such yeoman's service for so many years, was in the chair as one of the Vice-Presidents, Mr. Jellicoe being at the time the President of the Institute. Mr. Hill Williams, whose loss they mourned to-day, was one of the Honorary Secretaries, and their worthy President was his colleague. The annual report on that occasion dealt with two matters, one of which had now become historical, namely, the preparation of the Institute of Actuaries' Tables. That was a great and useful undertaking, and it was then only in its first stage. The Council were evidently anxious to see their way to meet the inevitable expenditure. Mr. Bailey, in his capacity as chairman of that meeting, made some extremely suitable remarks on the examinations, and to that subject he (Mr. Sutton) proposed hereafter to refer. As regards the financial state of the Institute and its membership, they would see that the Institute had every reason to be satisfied with the progress it had made. At the end of March 1866, the Institute had 95 Fellows, as against 154 now; there were 125 Associates, as against 241—in each case the number being nearly doubled—and they had now, in addition to the 154 and

241, a large class of 78 students; so that in twenty-one years the Institute had, as regards membership, more than doubled its number; and he might, perhaps, say that, in addition to the number being greater, their quality had not, on the whole, deteriorated. With regard to finance, the Members had not much to complain of. In 1866 the total funds were "estimated at £1,300." The phrase was somewhat curious. In those days the Institute was obliged to put the best face on things. Accordingly, in the 1866 balance sheet, there appeared the usual elastic item, which in this case took the form of "Value of books in the library, £381. 11*s*. 2*d*." They now had a fund of £3,000, exclusive of any elastic items, and the Institute was greatly to be congratulated on that satisfactory state of things. The other matter which he would like to mention was in regard to the examinations. Mr. Bailey, twenty-one years ago, made some excellent remarks on what he playfully called the Scylla and Charybdis of the examinations. Similar remarks applied with almost as much force now, especially with regard to the interval which took place between one set of examinations and the next —a period of twelve months. For young men between the ages of twenty and twenty-eight this was undoubtedly a long period, and personally he should be very glad to see that interval shortened. He would even go so far as to say that probably the Council would, at an early period, endeavour to reconsider the matter. But there were practical difficulties involved. For instance, the younger members had recently had no examination fees to pay; and he, for one, had been extremely pleased to see that what might be described as somewhat in the nature of taxes on the acquisition of knowledge had been done away with. But if they instituted examinations at shorter periods than twelve months, they would have to incur additional expense. If, therefore, the Council should see their way to shorten the interval between the examinations, the younger members must not be surprised if they found some sort of examination fee again instituted by the Council.

The resolution was at once unanimously agreed to.

ELECTION OF OFFICERS.

A ballot was then taken for the election of President, Vice-Presidents, Council, and Officers for the ensuing year. Mr. J. W. MILLER and Mr. STANLEY DAY were appointed Scrutineers, and reported that the following list had been unanimously adopted:

President.

ARCHIBALD DAY.

Vice-Presidents.

ALEXANDER J. FINLAISON, C.B.
HENRY WILLIAM MANLY.
BENJAMIN NEWBATT.
WILLIAM SUTTON, M.A.

Council.

Marcus Nathan Adler, M.A.
Arthur Hutcheson Bailey.
George William Berridge.
*Thomas G. C. Browne.
*Henry Cockburn.
Thomas Homans Cooke.
George Stephen Crisford.
Archibald Day.
David Deuchar.
Edward Docker, M.A.
Alexander J. Finlaison, C.B.
Richard Charles Fisher.
John Ralph Grimes.
Henry Harben.
Augustus Hendriks.
Charles Daniel Higham.
George Humphreys, M.A.
Henry William Manly.
*Thomas Marr.
*Frank McGedy.
Benjamin Newbatt.
*Gerald Hemmington Ryan.
Thomas Bond Sprague, M.A.
Thomas Young Strachan.
William Sutton, M.A.
James Terry.
Spencer C. Thomson, B.A.
George Todd, M.A.
Frank Bertrand Wyatt.
Thomas Emley Young, B.A.

* New Members of Council.

Treasurer.

George Humphreys, M.A.

Honorary Secretaries.

George Stephen Crisford.
Thomas Emley Young, B.A.

On the motion of Mr. H. J. Rothery, seconded by Mr. E. Justican, Mr. T. H. Adey, Mr. A. H. Bridgman, and Mr. W. G. Glennie, were elected auditors for the ensuing year.

Mr. W. Hughes proposed, and Mr. J. G. Priestley seconded, a vote of thanks to the President, Vice-Presidents, Council, and Officers of the Institute for their services during the past year.

The resolution was cordially adopted.

The President, Mr. H. W. Manly (on behalf of the Council), and Mr. T. E. Young (on behalf of the Hon. Secretaries), responded.

On the motion of Mr. G. R. Jellicoe, seconded by Mr. W. R. Dovey, a vote of thanks was presented to the retiring Auditors, Messrs. Adey, Adlard, and Rusher.

The meeting then adjourned till the 28th of November.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Mortality Experience of the Marine and General Mutual Life Assurance Society in respect of the Lives of Mariners, from 1852 *to* 1879.

By the courtesy of the Directors and officials of the Marine and General Mutual Life Assurance Society, we are enabled to place before our readers several valuable tables of experience showing the mortality to which a large body of seafaring men, who have been members of the Society, has been subjected.

The experience extends over a period of about twenty-seven years—from 1852 to 1879. The change that has come over the shipping trade during that time is remarkable. Wooden sailing vessels have been replaced by iron steamers of gigantic size and enormous power. The opening of the Suez Canal has afforded a speedier, but less healthy, high road to the East, and in other respects the changes have been none the less complete and revolutionary. The lives under observation in the Society's experience were officers and other sea employés in the service of a few leading steamship companies trading to different parts of the world, whose vessels were invariably among the best of their day. Some of these companies have for long periods carried Her Majesty's mails—a circumstance that ensures conditions, as regards the class of vessel and the character of the officers, which must operate favourably upon the mortality experience of their sea-going employés. In number, the deaths observed throughout the period to which the statistics relate were 898, and 35,439 years of exposure were passed through (*see* Table 1). Both in respect to the time embraced in the experience's span, and the number of facts passed in review, the observations would therefore seem to have a claim to being considered trustworthy as a measure of the special hazards of a mariner's occupation.

The data have been sub-divided into several classes, which may be explained as follows:

Class A includes captains, officers, surgeons and pursers.

Class B includes engineers, stewards and petty officers.

Eastern Trades take account of mariners voyaging to India, China and Australia, whether through the Suez Canal and Red Sea, or round the Cape of Good Hope. In earlier years, all the vessels on which any of the persons under observation served went round the Cape of Good Hope; but this is now the exception rather than the rule.

Western Trades comprise mariners trading, for the most part, to and from the West Indies, and the north-east coast of Brazil.

It is understood that a few cases of mariners engaged in the Mediterranean trade are included in the general class of Eastern Trades; and that a small number of officers in the North Atlantic lines are grouped in the Western Trades' class. In both instances the effect upon the main body of observations is quite unimportant.

The life tables used as standards of comparison in testing the experience of the Marine and General Mutual Life Assurance Society are the Carlisle, Northampton, H^M, and two special "extra-risk" tables. The first represents the effect of adding an extra premium of 1 to the normal H^M premium for 100, throughout the whole range of life. This table will be found in the *Journal* (xxiv, 21, "On the Subject of Extra Risks", by Mr. G. H. Ryan) where its characteristics are fully examined. The other table is precisely similar in form, and was specially computed for the purposes of the present investigation, by adding 1½ to the normal premium for 100 at all ages. It will be clear that if certain classes of risks were found to exist which showed an experience in complete accord with these tables at the various groups of ages, the additional hazards incurred would exactly require the extra premiums of 1 and 1½ over and above the ordinary premium for each 100 assured. Though practically self-evident, this is an important fact to consider in view of certain results brought out in the subjoined tables of experience.

Reviewing, now, these results, we see that the expected and actual deaths may be brought into juxtaposition in the following manner, the totals for all ages alone being regarded:

Number of Actual Deaths to each Expected Death by the following Tables. (*Actual Deaths*, 898.)

TABLE 1, Total Experience, Classes A and B	By Carlisle Table . . .	2·22
	„ Northampton Table . .	1·26
	„ H^M Table . . .	2·52
	„ Hypo. 1 per-cent Table .	1·10
	„ Hypo. 1½ „ „ .	·88
	„ Hypo. 1⅕ „ „ .	1·00 (approximately)

Broadly speaking, therefore, mariners, as a class, require an additional premium (according to the experience of this Society) of about 1·2 per-cent per annum.

Number of Actual Deaths to each Expected Death by the following Tables. (*Actual Deaths*, 362.)

TABLE 2, Class A	By Carlisle Table . . .	2·00
	„ Northampton Table . .	1·14
	„ H^M Table . . .	2·24
	„ Hypo. 1 per-cent Table .	1·00
	„ Hypo. 1½ „ „ .	·79

An additional premium of 1 per-cent per annum would consequently appear to be necessary to meet the risks of captains and officers of the mercantile marine. The deaths in the Hypothetical 1 per-cent Table occur, however, somewhat earlier than in the Society's experience, notwithstanding that the total of the expected deaths coincides so remarkably with that of the actual deaths. By charging a net extra premium of 1 per-cent, a small margin would probably result, which might be regarded as a provision for fluctuations, expenses, and the increased liability of the office in respect of the reversionary bonus-additions.

Number of Actual Deaths to each Expected Death by the following Tables. (*Actual Deaths*, 536.)

TABLE 2, Class B	By Carlisle Table . . .	2·40
	„ Northampton Table . .	1·36
	„ H^M Table . . .	2·74
	„ Hypo. 1 per-cent Table .	1·19
	„ Hypo. 1½ „ „ .	·94

The Hypothetical 1½ per-cent Table appears virtually to coincide with the Society's experience in regard to engineers, stewards, and petty officers, though here again it will be observed that the tabular deaths occur at a slightly earlier period than the actual deaths. As before, we may say that, according to the teachings of the facts before us, the extra charge of 1½ per-cent to members of this class would leave a small margin for miscellaneous purposes.

It will not be necessary for us to analyze the succeeding tables with equal fulness. The division of the data into the two classes of Eastern and Western Trades, while, doubtless, important in the special case of the Marine and General Office, would not be of practical significance to the generality of insurance companies. We may, however, roughly summarize the results in the undernoted form:

Table	Class	Extra Premium required according to Experience of the Marine and General
1.	All Classes	1·2 per 100 assured.
2.	Class A	1·0
	Class B	1·35
3.	Eastern Trades (all Classes)	1·15
4.	„ (Class A)	·8
	„ (Class B)	1·35
5.	Western Trades (all Classes)	1·45
6.	„ (Class A)	1·15
	„ (Class B)	1·7

Investigations into class mortality have always possessed a great interest for actuaries, and the practical importance of such enquiries is daily becoming greater. In life assurance matters, the tendency to cut down extra premiums in all directions is one of the most striking features of the times. Hence it is the more essential that the path of safety should be marked out and illumined by the beacons of past experience. The tables, relating to the working of the Marine and General for upwards of 25 years, are, we imagine, a unique collection of accurate observations bearing upon the mortality of mariners; and we do not doubt that they will be considered by readers of the *Journal* to form a valuable contribution to our present store of information regarding class mortality.

G. H. R.

TABLE 1.

Total Mariners' Experience. All Classes.

Actual and Expected Deaths.

CLASSES A AND B.

Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^M^ Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
20–24	3,375·	57	23·9	51·5	22·6	63·9	83·0
25–29	8,074·5	158	66·7	131·6	56·4	154·8	204·6
30–34	7,574·5	164	76·8	134·1	61·2	157·5	201·7
35–39	5,969·	158	65·3	115·8	56·3	133·3	169·6
40–44	4,393·5	137	61·8	97·5	47·4	107·2	133·0
45–49	2,938·	86	42·5	74·1	39·7	79·5	97·8
50–54	1,733·	72	26·0	52·4	30·2	54·2	65·5
55–59	820·	35	17·5	29·2	19·4	30·8	36·3
60–64	365·	18	13·4	15·7	12·3	17·4	19·7
65–69	139·5	6	6·1	7·4	6·8	8·7	9·7
70–74	48·	5	3·1	3·6	3·5	4·0	4·3
75–81	9·	2	·9	·9	·9	1·1	1·2
All Ages	35,439·	898	404·0	713·8	356·7	812·4	1,026·4

TABLE 2.

Total Mariners' Experience. In separate Classes.

Actual and Expected Deaths.

CLASS A.

Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^M^ Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
20–24	2,091·	35	14·6	31·3	13·9	39·5	51·1
25–29	2,773·	34	22·9	45·2	19·5	53·1	70·3
30–34	2,820·5	52	28·6	49·8	22·9	58·7	75·3
35–39	2,612·	63	28·5	50·7	24·6	58·2	74·4
40–44	2,210·5	71	31·1	49·1	24·0	53·2	67·0
45–49	1,569·	40	22·7	39·6	21·3	42·6	52·3
50–54	869·5	41	13·0	26·3	15·2	27·2	32·7
55–59	394·	13	8·5	14·0	9·3	14·7	17·6
60–64	168·5	8	6·2	7·2	5·7	8·0	9·2
65–69	60·5	3	2·5	3·2	3·0	3·7	4·2
70–74	20·5	1	1·4	1·6	1·4	1·7	1·9
75–79	5·	1	·5	·5	·5	·5	·6
All Ages	15,594·	362	180·5	318·5	161·3	361·1	456·6
CLASS B.							
20–24	1,284·	22	9·3	20·2	8·7	24·4	31·9
25–29	5,301·5	124	43·8	86·4	36·9	101·7	134·3
30–34	4,754·	112	48·2	84·3	38·3	98·8	126·4
35–39	3,357·	95	36·8	65·1	31·7	75·1	95·2
40–44	2,183·	66	30·7	48·4	23·4	54·0	66·0
45–49	1,369·	46	19·8	34·5	18·4	36·9	45·5
50–54	863·5	31	13·0	26·1	15·0	27·0	32·8
55–59	426·	22	9·0	15·2	10·1	16·1	18·7
60–64	196·5	10	7·2	8·5	6·6	9·4	10·5
65–69	79·	3	3·6	4·2	3·8	5·0	5·5
70–74	27·5	4	1·7	2·0	2·1	2·3	2·4
75–79	4·	1	·4	·4	·4	·6	·6
All Ages	19,845·	536	223·5	395·3	195·4	451·3	569·8

Table 3.

Mariners' Experience.—Eastern Trades. All Classes.

Actual and Expected Deaths.

Classes A and B.							
Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^M Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
20–24	3,311·5	49	23·3	50·1	22·5	63·4	81·6
25–29	5,835·5	101	47·8	94·9	40·3	111·5	147·8
30–34	4,719·	104	48·0	83·7	38·2	96·8	125·6
35–39	3,298·	91	36·0	63·9	31·3	75·2	94·4
40–44	2,130·	62	30·0	47·3	22·8	50·9	64·1
45–49	1,277·	41	18·5	32·0	17·5	34·0	42·8
50–54	711·	39	10·6	21·5	12·5	22·1	26·7
55–59	388·5	18	7·6	13·9	9·3	14·6	17·5
60–64	184·	11	6·7	7·9	6·4	9·0	10·3
65–69	75·5	2	3·4	4·0	3·8	4·9	5·3
70–74	24·5	4	1·5	1·8	1·8	2·2	2·4
75–79	7·	2	·7	·7	·8	·9	1·0
All Ages	21,961·5	524	234·1	421·7	207·2	485·5	619·5

Table 4.

Mariners' Experience—Eastern Trades. In separate Classes.

Actual and Expected Deaths.

Class A.							
Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^M Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
20–24	1,546·5	21	7·5	16·2	10·5	29·6	38·1
25–29	2,346·	28	19·6	38·8	16·2	44·8	59·5
30–34	1,720·	28	18·7	32·5	13·9	35·3	45·8
35–39	1,212·5	28	14·3	25·2	11·5	27·6	34·6
40–44	819·5	22	12·5	19·7	8·8	19·6	24·7
45–49	522·	14	8·5	14·6	7·2	13·9	17·5
50–54	272·5	16	4·7	9·5	4·8	8·5	10·2
55–59	150·5	6	3·6	5·9	3·6	5·7	6·8
60–64	69·5	6	3·1	3·6	2·4	3·4	3·9
65–69	26·	...	1·2	1·5	1·3	1·7	1·8
70–74	12·	2	1·1	1·2	·9	1·1	1·2
75–79	5·	1	·6	·6	·6	·6	·7
All Ages	8,702·	172	95·4	169·3	81·7	191·8	244·8
Class B.							
20–24	1,765·	28	15·8	33·9	12·0	33·8	43·5
25–29	3,489·5	73	28·2	56·1	24·1	66·7	88·3
30–34	2,999·	76	29·3	51·2	24·3	61·5	79·8
35–39	2,085·5	63	21·7	38·7	19·8	47·6	59·8
40–44	1,310·5	40	17·5	27·6	14·0	31·3	39·4
45–49	755·	27	10·0	17·4	10·3	20·1	25·3
50–54	438·5	23	5·9	12·0	7·7	13·6	16·5
55–59	238·	12	4·0	8·0	5·7	8·9	10·7
60–64	114·5	5	3·6	4·3	4·0	5·6	6·4
65–69	49·5	2	2·2	2·5	2·5	3·2	3·5
70–74	12·5	2	·4	·6	·9	1·1	1·2
75–79	2·	1	·1	·1	·2	·3	·3
All Ages	13,259·5	352	138·7	252·4	125·5	293·7	374·7

TABLE 5.

Mariners' Experience—Western Trades. All Classes.

Actual and Expected Deaths.

Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^{M} Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
CLASSES A AND B.							
20–24	1,422·	33	10·0	21·5	9·6	27·1	34·9
25–29	1,701·5	39	13·8	27·7	11·7	32·5	43·0
30–34	1,140·5	27	13·4	20·0	9·2	23·4	30·3
35–39	618·5	20	6·7	12·0	5·9	14·0	17·8
40–44	307·	15	4·3	6·9	3·3	7·4	9·2
45–49	133·	2	2·0	3·4	1·8	3·6	4·5
50–54	60·	3	1·0	1·8	1·1	1·9	2·3
55–59	42·5	2	1·0	1·4	1·0	1·5	1·9
60–64	15·	1	·6	·6	·5	·7	·8
65–69	2·	...	·2	·2	·2	·2	·2
70–74	...	...	...	...	...	...	...
75–79	...	...	...	...	...	...	...
All Ages	5,442·	142	53·0	95·5	44·3	112·3	144·9

TABLE 6.

Mariners' Experience—Western Trades. In separate Classes.

Actual and Expected Deaths.

Age	Exposed	Actual Deaths	Expected Deaths				
			Carlisle Table	Northampton Table	H^{M} Table	Hypo. Table, 1 per-cent	Hypo. Table, 1½ per-cent
CLASS A.							
20–24	839·5	19	4·4	9·7	5·7	16·0	20·6
25–29	724·	12	6·5	12·9	5·0	13·8	18·3
30–34	422·	7	4·8	8·2	3·4	8·7	11·2
35–39	193·5	7	2·6	4·5	1·9	4·3	5·6
40–44	104·	3	1·7	2·6	1·1	2·5	3·1
45–49	65·	1	1·1	1·8	·9	1·8	2·2
50–54	41·	3	·5	1·4	·8	1·3	1·6
55–59	30·	1	·6	1·0	·7	1·1	1·3
60–64	15·	1	·6	·6	·5	·7	·8
65–69	2·	...	·2	·2	·2	·2	·2
70–74	...	...	...	...	...	...	...
75–79	...	...	...	...	...	...	...
All Ages	2,436·	54	23·0	42·9	20·2	50·4	64·9
CLASS B.							
20–24	582·5	14	5·6	11·8	3·9	11·1	14·3
25–29	977·5	27	7·3	14·8	6·7	18·7	24·7
30–34	718·5	20	8·6	11·8	5·8	14·7	19·1
35–39	425·	13	4·1	7·5	4·0	9·7	12·2
40–44	203·	12	2·6	4·3	2·2	4·9	6·1
45–49	68·	1	·9	1·6	·9	1·8	2·3
50–54	19·	...	·5	·4	·3	·6	·7
55–59	12·5	1	·4	·4	·3	·4	·6
60–64	...	...	...	...	...	...	...
65–69	...	..	...	...	...	...	...
70–74	...	...	...	...	...	...	...
75–79	...	...	...	...	...	...	...
All Ages	3,006·	88	30·0	52·6	24·1	61·9	80·0

Observations respecting portions of Mr. Sprague's paper on The Graphic Method, &c. (*J.I.A.*, xxvi, 77). *By* W. S. B. WOOLHOUSE, F.R.A.S., &c.

MY attention has recently been drawn, first to a general discussion upon a paper by Mr. Sprague on what he calls "The Graphic Method of Adjusting Mortality Tables", and afterwards to the paper itself, printed in the *Journal* (xxvi, 77). In the perusal of this paper I am much surprised to find that Mr. Sprague should have been unfortunately led to advance such a diatribe of erroneous statements in disparagement of the method I originally devised for the purpose of adjusting the mortality tables of the Institute. As these gratuitous and inconsiderate statements, if left unnoticed, are calculated to be grossly misleading as to the soundness of the mathematical principles upon which my method is founded, I shall here, on behalf of students and others who may be interested in the subject, specially describe the rationale of the process as before presented in a less connected form, in my two papers; and add thereto such further explanation as may seem to be required.

According to the method prescribed in my first paper, and since adhered to by myself, the number living at each age in the mortality table is formally adopted as the most manageable and suitable element for final adjustment; at ages beyond the limiting age of the table it is at once conveniently, as well as accurately, put down as zero, a practical facility that cannot be over-estimated; and it has also this essential advantage, that precisely the same aggregate tabular mortality or decrement must necessarily be finally retained between all points of actual coincidence, in whatever way the intermediate numbers may be modified. A neutralization of errors to a certain extent has also necessarily taken place in deducing the numbers living at each age from the observed numbers at risk and the corresponding deaths.

The tabular values to be primarily adjusted are therefore, the NUMBERS LIVING at successive years of age as deduced, without any adjustment, from the original observations.

If we begin at the first age in the table, and extract the numbers living at quinquennial intervals of age, we can, by the formula for interpolation, with central differences, determine all the values for the intervening ages, and so obtain a complete series of values, constituting a distinct Table A.

Again, if we begin with the second age in the table, and, as before, extract the numbers living at quinquennial intervals, we can, by interpolation, obtain another complete series of values, constituting, say, Table B; and so on. By proceeding in this manner, all the changes will be exhausted on the completion of five interpolated sets, namely, Tables A, B, C, D, E, because if this operation were carried on any further, the previous work would evidently recur. And as the progression of the values to be adjusted is disturbed and distorted by errors of observation and other incidental imperfections, the five interpolated tables just described will slightly differ from each other throughout the whole period. So that, for every stated age, these tables supply five separate and independent values of the number living, each of them drawn from the observed facts by a simple and direct process; and as these values are, in each case, five independent determinations of the same identical quantity, by taking the average or arithmetical mean, the adjusted number living at the stated age is finally obtained. Moreover, it is easily shown by the mathematical theory of probabilities that the probable errors of the interpolated values are severally less than those of the original values from which they are immediately derived. For an interpolated value of l is determined thus:

$$l = \frac{z(z^2-25)}{50}\left(\frac{l_{z-5}}{z-5} - 2\frac{l_z}{z} + \frac{l_{z+5}}{z+5}\right).$$

Let ϵ_{z-5}, ϵ_z, ϵ_{z+5} denote the probable errors of l_{z-5}; l_z, l_{z+5} respectively. Then, the squared probable error of a sum being the sum of the squared probable errors of its components, the square of the probable error of the interpolated value of l is

$$\frac{z^2(z^2-25)^2}{2500}\left\{\left(\frac{\epsilon_{z-5}}{z-5}\right)^2 + 4\left(\frac{\epsilon_z}{z}\right)^2 + \left(\frac{\epsilon_{z+5}}{z+5}\right)^2\right\}.$$

Now, as ϵ_{z-5}, ϵ_z, ϵ_{z+5} appertain to a limited portion of the mortality table, we may fairly assume their squares to have a common average value ϵ^2; and the square of the probable error of the interpolated value of l becomes then

$$= \frac{z^2(z^2-25)^2}{2500}\left\{\frac{1}{(z-5)^2} + \frac{4}{z^2} + \frac{1}{(z+5)^2}\right\}\epsilon^2$$

$$= \left(1 - \frac{6z^2(25-z^2)}{2500}\right)\epsilon^2.$$

Hence, again, by the theory of probabilities, the square of the probable error of the arithmetical mean of the five values of l, that is the square of the probable error of the adjusted value (l) is

$$=\frac{1}{5^2}\Sigma\left(1-\frac{6z^2(25-z^2)}{2500}\right)\epsilon^2$$

$$(\text{for } z=-2,\ -1,\ 0,\ +1,\ +2)$$

$$=\cdot 1793\epsilon^2.$$

It is thus ascertained that the resulting adjusted table is more accurate and trustworthy than would have resulted from an ordinary combination of five times the experience, and that the probable errors of the observations are thereby considerably reduced. For this reason the resulting adjusted numbers living will in general observe such a satisfactory progression as to justify their being given to an extra digit.

The determination of each adjusted value (l) involves operations equivalent to the following:

$$\left.\begin{array}{lll}\gamma_1=l_{-1}+l_{+1}, & \gamma_3=l_{-3}+l_{+3}, & \gamma_6=l_{-6}+l_{+6}\\ \gamma_2=l_{-2}+l_{+2}, & \gamma_4=l_{-4}+l_{+4}, & \gamma_7=l_{-7}+l_{+7}\end{array}\right\} \quad . \quad . \quad (1)$$

$$\left.\begin{array}{ll}f=\gamma_1-\gamma_3, & h=\gamma_6-\gamma_3\\ g=\gamma_2-\gamma_3, & k=\gamma_7-\gamma_4\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$5(l)=l+\gamma_1+\gamma_2-\cdot 04(f+4g+2h+3k) \quad . \quad . \quad . \quad (3)$$

Otherwise, using d for l, the adjusted decrements may be obtained, which process will be attended by the convenience of smaller numbers.

The adjusted numbers (l) on being differenced to second differences will test the value of the original data and show the accuracy of the work. At one or two exceptional places, however, there may yet remain some slight traces of irregularity, but they are so few and so trivial as to be readily amended by mere inspection of the differences. A repetition of the process of adjustment for this purpose would be objectionable, inasmuch as it would introduce terms of the series that are too remote to be admitted as having any legitimate influence on the result.

In the foregoing method of adjustment every individual element of the data supplied by the experience has its proper influence in determining the several results, and there does not exist anything of an arbitrary nature in the process. No extraneous condition or restriction is placed upon the quantities, which are freely permitted to manifest and assert their own law.

The foundation of the method, theoretically considered, is the obvious condition that the true series of numbers living, assuming all errors to be eliminated therefrom, must, in their progression, manifest the existence of some kind of unknown law. That is without assigning any form whatever to this law, it is merely premised that the numbers living as shown by a perfectly adjusted mortality table must, within certain limits, be practically susceptible of interpolation for intermediate ages, and there is no assumption whatever beyond this universal principle of calculation.

Thus the entire process is neither arbitrary nor empirical, and does not in any way interfere with the organic relations that exist amongst the true values, but operates exclusively upon the incidental imperfections, that is to say, its efficacy is wholly expended on the neutralization of the small positive and negative portions of the data which constitute the errors of observation.

We may here give another simple illustration of these general principles. The unadjusted numbers living may properly be regarded as consisting of two components, namely (1) the true value, and (2) the error with its proper algebraic sign prefixed. Thus the values required to be adjusted are made up of two columns, under the heads

True values of *l*	(±) Corresponding errors

We may evidently consider the process to be applied to these columns separately; then, if the several results be combined so as to make a third column, that column will exhibit our adjusted values (*l*). Now it is known that the operation of interpolation will not affect or disturb the first column, because the true values follow a fixed law, and are known to be practically adapted to the purposes of interpolation. Every experienced computer knows this. On the other hand, when the same operation is applied to the second column, there will evidently ensue a breaking up of the errors into smaller values, in consequence of the mixture of opposite algebraic signs, since positive and negative errors are equally probable both as to sign and amount. This will account for the smallness of the probable error before ascertained in respect of the adjusted result (*l*). And thus it is mathematically proved that, with suitable data, a proper application of the method must necessarily lead to satisfactory results, and can never have a tendency to distort the law which the facts may follow, as Mr. Sprague so repeatedly alleges.

It should here be clearly understood that the finally adjusted table must not be regarded as exhibiting the true values of *l*, but only as a table of values in which the probable errors have been made as small as possible.

I beg also to state, that the method stands alone as systematically based on rational principles, directed primarily to the elimination of error, and as regarding smoothness of progression only as an adjunct, conveying a secondary advantage of some practical value. This smoothness of progression is, no doubt, very desirable, and, as a matter of course, will be convenient for all ulterior calculations made in a series, as exhibiting in the progressions of the results a continual check against inaccuracy, as well as obviating the occurrence of incongruous results. But the predominant object sought to be accomplished has been to reduce the probable errors of the unadjusted quantities down to a minimum, and to finally obtain the most reliable and accurate results that the original observations are capable of yielding; and the method cannot fail to be successful in every case in which the numbers supply sufficient data to be capable of adjustment.

It should also be distinctly noted that the method was specially designed for the adjustment of the NUMBER LIVING at each year of age, and was never put forward by myself for the adjustment of the probability of dying in the year, a function that is not sufficiently amenable to interpolation according to the intervals chosen. However, when the numbers living are adjusted, of course, a complete knowledge of everything else immediately follows.

After the foregoing explanations, I shall only have occasion to make one or two passing remarks on Mr. Sprague's statements.

Mr. Sprague gives various tables showing comparisons of results of adjustment, some of which are said to have been obtained by means of my formula. But these last are adjustments of the probabilities of dying to which my formula was never intended to be applied. They are, therefore, no more than results of the misapplication of my formula, and are misrepresentations of the working of the method. The rather high-handed criticisms in which Mr. Sprague is so freely disposed to indulge are therefore simply amusing. I should have much liked Mr. Sprague to have adjusted the Female Government Annuitants' Table properly by my method, that is, in the same manner that the Institute Tables were done, and applying the adjustment process to the numbers living at each age. He would then have been enabled to make fair comparisons proving the unfailing efficacy of the method. On page 86 Mr. Sprague observes, that as the irregularities of the observed facts do not follow any law, or occur at fixed intervals, it is obvious that they cannot be neutralized and got rid of by any definite mode of procedure, such as quinquennial grouping; and he adds that this argument, when fully considered, seems to be conclusive against all such methods of graduation as Mr. Woolhouse's. If Mr. Sprague can really perceive any resemblance to argument in this statement, I can only say that his logic is very original.

Mr. Sprague transforms my formula into one involving the differences of the function instead of its successive values, and gets for the adjusted function

$$U_x = u_x \ldots - 5\cdot4\Delta^4 u_x + \ldots$$

on which he remarks that if the fourth differences are constant, the formula introduces a constant error, $-5\cdot4\Delta^4 u_x$, into the graduated series. This is undoubtedly true, but it is of no consequence whatever in respect of the method, as the assumption of constant fourth differences will necessarily take a series of numbers very far out of the range of all mortality tables. If Mr. Sprague had carried the expression a little further, say to fifth and sixth differences, he would have startled himself with still more alarming coefficients!

Mr. Sprague afterwards assumes the original series to follow an exponential law $u_x = ca^x$, and shows that the formula will then give a new series, in which the terms bear a constant ratio to those of the original series. But it is found that the constant multiplier is precisely unity for all algebraic functions of the third degree, and this is indeed all that is required to establish the practical accuracy of my method.

Mr. Sprague proceeds to investigate the most general form the function can take, so as to cause the multiplier to be exactly unity. This he proposes to do by solving the linear equation, which he calls an equation of differences, the general solution of which depends upon the several roots of a corresponding reciprocal algebraic equation. His final general solution of the equation of differences as put down in the *Journal* is nearly right, but not quite. However, as it is practically useless it is not worth correcting.

I have but little to say about the graphic method of graduation which Mr. Sprague so strenuously advocates. I am compelled to object to it as arbitrary, empirical, plodding, and unscientific; but I am not disposed to denounce it altogether, as some operators, in common with Mr. Sprague, may have an excusable partiality for it.

Progress of Life Assurance throughout the World, from 1859 *to* 1883. *By* Marco Besso.

[Translated and abridged from *Le Moniteur des Assurances.*]

THE important and laborious investigation of which we give the results in the following pages is the last serious contribution to statistical knowledge by Mons. Marco Besso, a Fellow of the Institute of Actuaries of some years' standing, and an insurance expert of wide experience and great repute throughout the continent. The paper was at first published in part in the *Bulletin de l'Institute de Statistique,* and a translation subsequently appeared in the German Annual Magazine of Herr Ehrensweig. To enable the statistical tables and the general conclusions drawn from them to be made available for reference, in full, the author placed his original manuscript at the disposal of the *Moniteur des Assurances* of Paris, in whose columns a verbatim reproduction appeared. We should state that a literal translation of Mons. Marco Besso's paper has been courteously supplied us by Mons. Baillot, the London representative of the *Moniteur,* from which we have made full extracts.

It remains but to add that, at our suggestion, enquiries were made of Mons. Marco Besso concerning the sources from which he derived his statistics; and the following is the reply received to this communication:

[*Translation of Mr. Besso's Letter.*]

Assicurazioni Generali.

Trieste, 5*th July,* 1887.

The sources from which I obtained my statistics are very numerous. For France they were derived from the *Moniteur des Assurances,* for Germany in the publications of the *Gotha Company,* for England from the *Post Magazine* and from the *Review,* for the United States from the yearly reports of the country, for Austria-Hungary from the annual report of Mr. Ehrensweig. For the other countries I consulted the reports of the various companies and availed myself of the figures supplied by my friends and colleagues. As the figures given by me go back to 1883, it might perhaps be right, in order to have complete statistics, to continue them as far as 1886, but I regret that I cannot at present carry forward the work.

Ed. *J.I.A.*

This investigation has been prepared at the request of the Paris Statistical Society, to celebrate the twenty-fifth anniversary of its existence.

If there is any branch of commercial business to which the science of statistics has been useful, it is certainly that of assurance. To statistics this business is indebted for a scientific basis, without which it would be nothing but gambling. It might be said with reason that the civilization of a country is proved by the lesser or greater adoption of life assurance. It will not, therefore, be amiss to follow, by the aid of statistics, the progress accomplished by life assurance during the last quarter of a century.

We do not believe that in 1859 any hope could have been entertained that life assurance would increase, within a quarter of a century, to the extent that will be shown (see Table No. II). Yet its present proportions are small compared with the requirements of society, if we take into account the millions of persons not yet insured.

The first steps in the onward march were the most difficult. The business was imperfectly understood, and had to fight its way, encountering everywhere inveterate prejudices. It had to rely on itself and on its own experience to acquire the knowledge necessary to its favorable and regular working. Now the impulsion has been given, and the fundamental principles have been tested; but it is yet necessary to introduce such improvements as will render the benefits of assurance accessible to all.

On the continent, life assurance had made but little headway in 1859. The whole amount assured from the origin of the business—namely, from about the beginning of the century—was not then equal even to the sums at present assured in one single year. The policies in force on the continent in 1859, representing 745 millions of francs, were scarcely more than one-half of the new policies issued in 1883, namely, 1,233 millions of francs. And yet it can be truly stated that the number of persons now assured on the continent is not equal to five out of one thousand of the population (see Table No. V).

Much therefore remains to be done; and we may remark, that the greater number of individuals now assured belongs to the middle class, a fact easily ascertained by examining the average sum insured in the principal countries (see Table VI).

It being proved, from the reports of the companies, that assurance policies for very large sums are rather rare, it follows that the extreme points in the scale of the amounts assured on each life are not far apart, or in other words, that the inferior degree in the scale is yet rather high. In other words again, the classes which stand in the greatest need of life assurance, or the working classes, are the very ones which at present avail themselves of it to the smallest extent. It is only in England, and only there during the last few years, that the working classes have largely availed themselves of life assurance, thanks to the propaganda of certain companies which have taken up this special branch of the business. The average amount assured is getting lower in England, in America, and partly also over the continent. This proves that the principles of the business are growing in favour more and more with the people; and the improvements constantly being brought about in its working cannot fail to win the daily-increasing favour of the public.

Life assurance was carried on in 1859 only by proprietary companies, or by free mutual societies. It was at a later period, in England in 1865 and in France in 1868, that an attempt was made by Government to introduce the system of assurance by the State, notably for the benefit of the working classes. This trial has not proved successful: the number of policies issued in England by the Government up to the present time would not satisfy any private company, and the result is the same in France.

Private societies, both on the continent and in England, have likewise tried to introduce industrial assurance. It is only in England that, after a very precarious beginning, this branch of life assurance has spread to a large extent, and has even attained, during the last few years, proportions truly colossal. We have thought it desirable to separate the results of ordinary assurance from those of *Industrial Business,* because this last branch comprises, in reality, a different order of transactions. The truth of this assertion is sufficiently proved by the fact that the average amount of each assurance, which in England is of about £500 for ordinary assurances, does not reach £10 in the Industrial Business. The number of persons thus assured is over six millions, or nearly 1-6th of the entire population of the United Kingdom.

Similar assurances are granted by the State in France, but with some restrictions not freely accepted by the public, notably a probationary period of two years. Altogether the total amount of popular assurances now in force in the United Kingdom is 1,683 millions of francs (£67,320,000): the total amount of assurances by the State reaches only 10 millions of francs, namely, 1-168th of the assurances in force in the private companies.

Denmark, however, where a State Assurance Scheme is also established, forms an exception. This scheme, initiated in the first half of the present century and reorganized in 1871, works admirably well, and is a serious rival to the private companies, owing to the low scale of premiums (at the age of 30 the annual premium is only 2·04 per 100) and to the very favourable conditions of assurance. The amount assured by the State in Denmark represented at the end of 1883 the sum of 53,284,000 francs, or 26⅔ millions for every million of inhabitants, while in England the State assures only 277,000 francs for every million of inhabitants (proportion 1:100) and in France only 27,000 francs (proportion 1:1000).

In the tables given hereafter life annuities have been omitted because those transactions belong to the category of provident assurances only in a selfish sense.

For every kind of comparison and deduction to be drawn from the figures in the following tables, we refer the reader to the tables themselves, namely:

I. Number of life assurance companies in existence at the end of each year.
II. New sums assured annually.
III. Total sums assured in force at the end of each year.
IV. Assurance Funds at the end of each year.
V. Approximate number of policies in force.
VI. Average sum assured on each life.

But it is necessary to state that the figures given in the tables do not represent the assurances in force in each country respectively. There are countries in which life assurance is in a great measure worked by foreign companies, and there are others in which companies find the greatest portion of their business abroad. Thus, a number of French companies reap a large harvest in Belgium, in Holland, in Switzerland, and in Italy; while German companies work the same countries, besides Scandinavia, Russia, and Austria-Hungary. The Trieste companies obtain important business in Italy, and the English companies are at work more or less in every part of the continent and in several colonies. Yet, as a natural supposition, it may be assumed that the assurances completed by foreign companies in France, Germany, and Austria-Hungary are about equal to the assurances granted abroad by the companies of those three countries. As for Russia and Italy, as well as states of secondary importance, there is no doubt that the assurances in force in those countries are much above the figures given in the tables; in other words, the number of lives and the amount of capital assured are of much greater importance than would appear according to our statistics. Italy, for instance, is credited in our tables with scarcely one-half of the assurances really in force, and it is very much the same with Russia.

We must state likewise that, notwithstanding all our researches, we have been unable to obtain all the particulars necessary to make this investigation complete. The figures to which the greatest reliance can be given are those referring to assurances in France, Germany, Austria-Hungary, and Switzerland. But the various English statistics we have consulted do not fully agree. The statistics for Belgium and Holland are somewhat incomplete, and for the other countries, such as Asia, Africa, and South America, we have been unable to obtain reliable statistics; yet we believe that we are not very far from the truth when we affirm that the life assurance companies of those countries have not, so far, secured business of any importance.

The tables do not therefore profess to supply exact statistics as to the situation of life assurance in every country. Our aim has been rather to demonstrate the ever-increasing importance of this provident business, and to draw towards it the attention of economists, who, as a general rule, do not sufficiently study the question, in view of the public interests involved in it.

Our purpose, in giving the very respectable figures of the State Life Assurance in Denmark in opposition to the microscopic figures of State Life Assurance in France and in England, was to prove that the want of success of the State Institution in those two countries is not to be ascribed to the want of popularity of life assurance itself, but to the working systems adopted by them.

The field in which life assurance can spread its wonderful advantages is so vast that private companies need not fear any decrease in their operations in consequence of the introduction of State Assurance. In fact, assurance by the State and by private companies should work together and bring us every day nearer to the greatest of all desiderata: that the life of every father throughout the world should be insured in favour of his wife and children.

Development of Life Assurance from 1859 to 1883.

1.—*Number of Companies in existence at the end of each year.*

Years	Continent														Great Britain and Ireland	United States	Canada	Australia	Other Countries	Full Total
	Germany	Austria-Hungary	Belgium	Denmark	Spain and Portugal	France	Italy	Norway	Netherlands	Country of the Balkans	Russia	Sweden	Switzerland	Total						
1859	19	6	2	...	...	7	1	...	5	...	1	1	2	44	78	14	...	...	...	136
1860	19	8	2	...	...	7	1	...	6	...	1	1	2	47	78	17	...	...	...	142
1861	22	8	2	...	...	7	1	1	6	...	1	1	2	51	80	17	...	...	...	148
1862	22	8	2	...	...	7	2	1	7	...	1	1	2	53	81	18	...	...	...	152
1863	22	8	2	...	...	7	2	1	9	...	1	1	2	55	82	22	...	...	...	159
1864	23	10	2	...	1	8	2	1	9	...	1	1	2	60	84	27	...	...	...	171
1865	24	10	2	...	1	9	2	1	9	...	1	1	3	63	85	30	...	...	...	178
1866	25	11	2	...	1	10	2	1	10	...	1	1	3	67	87	39	...	...	...	193
1867	27	14	2	...	1	10	2	1	11	...	1	2	3	74	87	43	...	...	...	204
1868	28	15	2	...	1	10	2	1	12	...	2	2	3	78	88	55	...	...	...	221
1869	28	18	2	...	1	10	2	1	12	...	2	2	3	81	89	70	...	...	...	240
1870	28	19	2	...	1	10	2	1	12	...	2	2	4	83	91	71	...	...	...	245
1871	29	19	2	...	1	10	2	1	12	...	2	2	4	84	110	68	...	...	...	262
1872	32	20	2	1	1	11	2	1	12	1	2	3	4	92	118	59	...	...	...	269
1873	32	24	2	2	1	12	2	1	13	2	3	4	4	102	120	56	...	...	...	278
1874	38	23	2	2	1	12	2	2	14	2	3	4	4	109	120	50	...	...	...	279
1875	39	21	2	2	1	13	2	2	14	2	3	4	4	109	114	45	...	...	...	268
1876	38	20	2	2	1	13	2	2	15	2	3	4	4	108	108	38	...	...	...	254
1877	38	19	2	2	1	14	2	2	15	2	3	4	4	108	109	34	...	...	...	251
1878	38	19	2	2	1	15	2	2	16	2	3	4	4	110	108	34	...	...	...	252
1879	39	21	2	2	2	15	2	2	16	2	3	4	4	114	108	34	...	...	...	256
1880	39	22	2	2	2	19	3	2	16	2	3	4	4	120	107	34	...	...	...	261
1881	38	22	2	2	3	21	3	2	16	2	4	4	4	123	106	30	...	...	...	259
1882	38	21	3	2	3	23	3	2	17	2	4	4	4	126	107	30	...	...	...	263
1883	38	20	3	2	3	26	3	2	17	2	4	4	4	128	104	30	9	10	...	281

Development of Lif

II.—*New Assurances completed i*

Years.	Continent											
	Germany	Austria-Hungary	Belgium	Denmark		Spain and Portugal	France		Italy	Norway	Nether-lands	The Balkans
				Ord. Assur.	State Assur.		Ord. Assur.	State Assur.				
1859	44.6	2.9	5	...	...	...	42.0	...	1.5	...	4.1	...
1860	50.9	13.2	5	...	...	...	44.3	...	1.3	...	4.3	...
1861	63.4	28.0	5	...	...	...	46.7	...	1.7	...	5.7	...
1862	79.1	40.5	5	...	...	...	60.0	...	1.1	...	5.6	...
1863	115.1	26.9	5	...	...	...	72.7	...	1.2	...	6.1	...
1864	130.3	44.3	5	...	...	...	106.9	...	1.2	...	6.5	...
1865	157.7	53.1	5	...	...	...	134.3	...	1.7	...	6.8	...
1866	140.3	45.5	5	...	...	...	172.7	...	2.1	0.7	7.2	...
1867	178.4	40.3	6	...	...	...	145.4	...	3.5	0.8	7.6	...
1868	199.5	48.7	6	...	...	...	198.6	...	3.8	0.9	8.4	...
1869	226.5	57.7	6	...	...	...	201.8	0.10	4.0	1.0	8.2	...
1870	147.9	69.7	6	...	...	...	141.4	0.10	4.2	1.0	8.9	...
1871	167.2	68.8	6	...	4.4	...	89.0	0.10	3.5	1.2	9.5	...
1872	224.1	76.9	6	...	4.2	...	170.0	0.10	3.8	1.3	9.1	...
1873	264.0	163.1	8	...	3.5	...	187.0	0.10	3.9	1.4	10.1	...
1874	295.0	142.3	8	5.9	3.9	...	237.1	0.10	4.8	1.5	11.8	...
1875	303.5	130.1	8	6.9	3.7	...	254.6	0.10	5.3	1.6	12.1	...
1876	310.0	108.2	8	5.5	3.7	...	284.8	0.10	6.4	1.7	13.1	...
1877	285.6	82.2	8	5.4	4.1	...	278.4	0.10	7.2	1.8	14.2	...
1878	269.2	92.8	10	3.9	3.4	...	315.1	0.10	6.4	2.0	15.5	0.6
1879	268.3	105.6	10	3.0	3.5	...	337.1	0.11	7.6	2.1	16.7	0.6
1880	280.5	165.1	10	4.2	3.7	1.0	455.6	0.15	8.6	2.3	18.5	0.6
1881	290.2	176.7	12	4.6	3.9	1.0	556.4	0.20	17.9	2.9	20.4	0.6
1882	312.5	197.6	12	4.8	3.9	2.0	589.9	...	16.4	2.9	23.6	0.6
1883	322.5	199.2	12	6.0	3.4	2.5	519.0	...	13.8	3.0	25.1	0.6
	5,126.3	2,179.4	182	50.2	49.5	6.5	5,639.8	1.46	132.9	30.1	279.1	3.6

III.—*Aggregate Sums Assured in force a*

Years	Continent											
	Germany	Austria-Hungary	Belgium	Denmark		Spain and Portugal	France		Italy	Norway	Nether-lands	The Balkans
				Ord. Assur.	State Assur.		Ord. Assur.	State Assur.				
1859	363	114	18	...	...	...	194	...	2	...	9	...
1860	396	130	21	...	...	...	230	...	2	...	12	...
1861	438	164	25	...	...	...	253	...	3	...	18	...
1862	501	211	30	...	...	...	388	...	3	...	20	...
1863	581	238	35	...	...	...	358	...	4	...	31	...
1864	672	266	40	...	...	...	421	...	5	...	44	...
1865	781	298	41	...	...	...	572	...	8	1	51	...
1866	848	307	42	...	...	...	590	...	10	2	54	...
1867	957	290	43	...	...	...	639	...	11	2	57	...
1868	1,079	327	44	...	...	...	798	...	12	3	60	...
1869	1,211	386	45	...	...	...	919	...	12	3	62	...
1870	1,263	438	46	...	...	...	1,008	0,25	16	4	66	...
1871	1,346	495	47	...	28	...	973	0,30	18	5	72	...
1872	1,495	536	48	...	31	...	1,008	0,35	19	5	76	...
1873	1,677	752	49	4	34	...	1,110	0,40	21	6	81	...
1874	1,859	762	50	11	36	...	1,248	0,45	24	6	84	...
1875	2,037	756	51	17	38	...	1,368	0,50	26	7	88	...
1876	2,197	754	52	22	40	...	1,507	0,55	28	8	94	...
1877	2,311	650	54	25	42	...	1,625	0,60	30	8	98	...
1878	2,417	681	56	27	44	...	1,779	0,65	32	9	102	0.5
1879	2,534	703	58	30	45	...	1,955	0,70	34	9	104	1
1880	2,666	796	60	34	48	0.3	2,183	0,80	36	11	108	1
1881	2,798	876	64	40	50	1.0	2,487	0,90	52	13	117	2
1882	2,951	933	70	45	52	2.0	2,760	0,90	61	16	132	2
1883	3,120	998	76	51	58	2.8	2,969	1,00	72	18	144	2

SURANCE FROM 1859 TO 1883.

h Year (in Millions of Francs).

Continent				Great Britain & Ireland			United States	Canada	Australia	Other Countries	Full Total
Russia	Sweden	Switzer-land	Total	Ordinary Assurance	Industrial Assurance	State Assur-ance					
2.0	2.4	3.0	107.5	324.3	...	...	156.7	...	...	...	588.5
2.6	2.1	2.9	126.6	342.7	...	...	192.2	...	...	...	661.5
2.5	2.0	4.1	159.1	368.5	...	...	134.9	...	...	...	662.5
2.5	2.0	8.2	204.0	375.4	20.0	...	234.7	...	...	...	834.1
2.7	1.9	7.6	238.7	385.5	40.0	...	485.0	...	...	...	1,149.2
2.7	2.6	8.1	307.6	397.3	60.0	...	841.3	...	...	...	1,606.2
3.0	2.5	7.8	371.9	413.5	80.0	1.02	1,325.3	...	...	...	2,191.72
4.7	2.7	7.2	387.6	422.4	100.0	1.18	2,184.4	...	...	...	3,095.58
3.2	4.8	7.5	397.5	426.8	120.0	0.67	2,546.7	...	...	...	3,491.67
3.1	6.4	9.2	484.9	435.4	140.0	0.67	3,130.1	...	...	...	4,191.07
3.9	7.1	14.1	530.3	526.5	160.0	0.82	3,319.7	5.8	...	...	4,543.12
5.6	7.5	15.6	407.9	577.0	188.99	0.78	3,174.5	7.9	...	...	4,357.07
6.8	8.4	16.2	381.1	526.0	218.61	0.69	2,638.7	13.1	...	...	3,778.20
7.6	13.9	21.4	537.4	438.1	238.12	0.40	2,645.6	26.4	...	...	3,886.02
1.7	16.3	20.0	689.5	505.3	280.20	0.23	2,508.3	23.0	...	...	4,006.73
4.6	21.5	19.3	765.8	586.3	306.46	0.54	1,899.7	26.3	...	...	3,585.10
6.6	16.2	20.0	778.7	524.4	399.03	0.80	1,616.1	25.4	...	...	3,344.43
16.9	14.2	17.1	789.7	581.6	377.54	0.57	1,256.4	27.3	...	...	3,033.11
16.8	15.6	14.0	733.4	605.5	499.22	0.84	962.4	28.6	...	...	2,830.26
22.3	15.5	13.2	769.8	584.2	515.20	0.49	845.1	27.7	60	...	2,802.49
22.0	15.1	15.6	807.31	559.0	513.18	0.47	910.6	30.6	65	...	2,886.16
26.1	17.1	17.0	1,010.45	521.6	532.46	0.51	1,272.2	37.7	70	...	3,444.92
30.6	16.1	19.2	1,152.78	574.5	583.50	0.52	1,201.9	55.79	80	...	3,648.79
38.2	24.4	19.0	1,247.8	618.6	597.78	0.58	1,390.6	61.79	90	...	4,007.25
76.6	34.4	14.7	1,232.8	620.7	639.78	0.59	1,663.6	60.6	100	...	4,318.07
45.3	272.7	322.0	14,620.64	12,241.1	6,610.07	12.97	38,537.0	457.98	465	...	72,944.76

he end of each Year (in Millions of Francs).

Continent				Great Britain & Ireland			United States	Canada	Australia	Other Countries	Full Total
Russia	Sweden	Switzer-land	Total	Ordinary Assurance	Industrial Assurance	State Assur-ance					
28	13	4	745	4,000	...	...	764	...	...	...	5,509
29	14	7	841	4,250	..	...	884	...	...	...	5,975
30	15	11	957	4,500	...	...	887	...	...	...	6,344
30	15	19	1,217	4,750	...	...	993	...	...	...	6,960
30	16	25	1,318	5,000	...	...	1,445	...	...	...	7,763
31	16	32	1,527	5,268	...	...	2,137	...	...	...	8,932
31	17	43	1,843	5,523	...	1	3,137	...	...	...	10,504
33	18	55	1,959	5,791	...	2	4,672	...	...	...	12,424
33	22	67	2,121	6,047	...	3	6,273	...	...	...	14,464
35	27	83	2,468	6,325	...	3	8,657	...	...	...	17,453
39	30	100	2,807	6,725	...	4	9,918	...	...	...	19,454
47	34	110	3,032.25	7,274	205	5	10,929	...	...	...	21,445.25
51	39	119	3,193.30	8,282	296	5	11,348	...	...	...	23,124.30
58	46	134	3,456.35	8,608	308	6	11,420	...	...	...	23,798.35
66	59	147	4,006.40	8,817	351	7	11,265	...	...	...	24,446.40
77	76	158	4,391.45	9,056	421	7	10,785	...	...	...	24,660.45
88	85	167	4,728.50	9,329	654	7	10,379	...	...	...	25,097.50
97	93	172	5,056.55	9,664	685	8	9,344	...	...	...	24,757.55
104	100	177	5,224.60	9,860	734	8	8,403	...	...	...	24,229.60
119	109	179	5,556.10	10,125	871	8	7,957	...	...	...	24,517.10
131	116	182	5,902.70	10,375	1,008	9	7,776	170	650	...	25,890.70
147	126	190	6,407.10	10,500	1,141	9	7,970	189	700	...	26,916.10
159	142	199	7,000.90	10,625	1,327	9	8,491	230	750	...	28,432.90
232	155	203	7,514.90	10,750	1,484	10	8,843	269	800	...	29,770.90

Development of Life

IV.—*Assurance Funds at the end of*

Years	Continent											
	Germany	Austria-Hungary	Belgium	Denmark		Spain and Portugal	France		Italy	Norway	Nether-lands.	The Balkans
				Ord. Assur.	State Assur.		Ord. Assur.	State Assur.				
1859	74.0	10.8	1.5	...	...	...	70.0	...	0.3	...	0.9	..
1860	85.0	13.8	2.0	...	...	...	80.0	...	0.3	...	1.9	...
1861	96.6	15.0	3.0	...	...	...	90.0	...	0.3	...	3.2	...
1862	104.7	16.3	4.0	...	...	...	100.0	...	0.4	...	4.3	..
1863	128.8	21.4	5.0	...	...	...	110.0	...	0.4	...	5.2	...
1864	136.5	21.7	6.0	..	...	...	120.0	...	0.4	...	5.4	...
1865	143.7	23.9	7.0	...	...	...	130.0	...	0.5	0.3	5.8	...
1866	153.9	32.4	8.0	...	...	...	140.0	...	0.8	0.3	6.2	..
1867	169.5	27.5	9.0	...	...	...	160.0	...	1.0	0.4	6.5	...
1868	189.0	32.5	9.0	...	...	...	180.0	...	1.3	0.4	7.1	...
1869	192.0	42.3	9.5	...	...	...	200.0	...	1.5	0.5	7.6	...
1870	195.0	43.9	9.5	...	...	...	219.0	...	1.7	0.6	7.9	...
1871	197.9	52.1	10.0	...	...	...	236.0	...	1.9	0.6	8.2	...
1872	211.3	57.8	10.0	...	...	...	265.0	...	2.2	0.7	8.6	.
1873	234.9	92.9	11.0	0.1	...	...	274.0	...	2.5	0.8	9.5	...
1874	258.9	101.2	11.5	0.2	...	...	306.1	...	2.8	0.8	10.7	...
1875	284.5	108.8	12.0	0.6	...	...	325.6	...	2.9	0.9	12.3	...
1876	311.8	115.9	12.5	0.8	...	...	370.0	...	3.2	1.3	14.5	..
1877	339.8	111.3	13.0	1.2	...	...	405.0	...	3.3	1.4	16.7	...
1878	367.9	116.9	13.5	1.5	...	...	461.0	...	4.7	1.5	16.6	..
1879	400.6	127.3	14.0	2.0	...	...	515.0	...	4.8	1.6	18.3	...
1880	437.5	134.6	15.0	3.2	8.6	0.1	584.3	...	5.1	1.8	19.6	...
1881	477.8	143.0	16.0	4.0	8.9	0.1	631.1	...	6.0	1.7	20.8	0.5
1882	520.4	154.8	17.0	4.6	9.3	0.1	699.3	...	6.7	2.1	21.6	0.6
1883	542.5	162.9	18.0	6.1	9.7	0.1	714.2	...	7.7	2.6	22.3	0.7
...	6,254.5	1,781.0	247.0	24.3	36.5	0.4	7,385.6	...	63.7	20.3	261.7	1.8

V.—*Approximate Number of Policies*

Years	Continent											
	Germany	Austria-Hungary	Belgium	Denmark		Spain and Portugal	France		Italy	Norway	Nether-lands	The Balkans
				Ord. Assur.	State Assur.		Ord. Assur.	State Assur.				
1859	87,073	23,838	2,500	...	...	...	20,568	...	200	...	835	...
1860	95,405	35,626	3,000	...	...	...	22,326	...	250	...	1,234	...
1861	106,550	49,708	3,500	...	...	...	23,137	...	300	...	3,563	...
1862	123,246	57,218	4,000	...	...	...	27,853	...	350	...	4,248	...
1863	144,501	64,683	4,000	...	...	...	34,469	...	700	...	5,638	...
1864	173,105	70,209	4,500	...	...	...	43,085	...	900	...	3,632	...
1865	210,227	72,001	4,500	...	...	...	51,701	...	1,200	190	12,313	...
1866	231,059	75,068	4,500	...	...	...	60,317	...	1,500	280	15,678	...
1867	267,721	78,890	5,000	...	...	...	68,933	...	1,800	376	18,621	...
1868	307,692	84,907	5,500	...	...	...	82,403	...	2,100	447	21,563	..
1869	352,019	115,806	6,000	...	...	...	92,060	...	2,600	523	24,678	...
1870	362,250	128,446	6,500	...	...	...	95,000	...	2,800	664	28,583	.
1871	381,036	133,837	[illegible],500	...	14,181	...	97,481	...	3,000	750	32,478	...
1872	412,295	177,200	7,000	...	15,065	...	99,632	...	3,200	906	36,478	...
1873	451,694	171,450	7,500	1,019	15,796	...	109,820	...	3,500	990	39,583	...
1874	490,392	176,425	8,000	1,793	16,537	...	121,200	...	3,700	1,050	42,683	...
1875	523,842	201,244	8,500	2,639	16,736	...	134,525	...	3,900	1,120	45,348	...
1876	544,275	217,419	8,500	3,458	16,773	...	160,700	...	4,200	1,190	47,638	..
1877	554,512	235,495	9,000	3,888	17,832	...	177,300	...	4,500	1,280	51,533	...
1878	569,283	248,185	9,000	4,164	18,306	...	183,200	..	4,800	1,314	55,112	300
1879	586,789	249,255	9,500	4,838	18,518	...	193,673	2,500	5,491	1,520	58,599	350
1880	608,648	281,636	9,500	5,834	19,091	635	207,660	2,800	5,655	1,780	59,263	400
1881	627,127	299,481	10,000	6,365	19,462	935	235,551	3,200	7,181	2,254	64,208	418
1882	646,697	274,187	11,000	7,577	19,815	1,234	251,686	3,400	7,986	2,622	65,370	441
1883	671,023	306,275	12,000	9,889	19,956	2,105	257,166	3,600	8,629	3,041	[illegible]	[illegible]

SSURANCE FROM 1859 TO 1883.

ch Year (in Millions of Francs).

Continent				Great Britain & Ireland			United States	Canada	Australia	Other Countries	Full Total
Russia	Sweden	Switzer-land	Total	Ordinary Assurance	Industrial Assurance	State Assur.					
5.3	0.6	0.5	163.9	1,212.0	...	...	83.5	...	...	...	1,459.4
5.7	0.8	0.8	190.3	1,302.0	...	...	92.7	...	...	...	1,585.0
6.0	1.0	1.2	216.3	1,392.0	...	...	98.7	...	...	...	1,707.0
6.1	1.1	1.7	238.6	1,482.0	...	...	128.5	...	...	...	1,849.1
6.4	1.3	2.5	281.0	1,572.0	...	...	154.7	...	...	...	2,007.7
6.7	1.5	3.1	301.3	1,630.3	...	...	187.5	...	...	...	2,119.1
6.9	1.7	4.5	324.3	1,655.3	...	...	250.2	...	...	...	2,229.8
7.1	1.9	5.3	355.9	1,775.7	...	...	354.2	...	...	...	2,485.8
7.2	2.1	6.4	389.6	1,895.7	...	...	478.4	...	...	...	2,763.7
7.5	2.2	7.6	436.2	1,935.7	...	...	730.4	...	...	...	3,102.3
7.9	2.7	8.9	472.9	1,976.7	...	...	977.0	...	...	...	3,426.6
9.4	3.1	10.6	500.7	2,155.1	7.9	...	1,194.3	...	...	...	3,858.0
9.4	3.7	12.5	532.3	2,232.2	9.8	...	1,374.6	...	...	...	4,148.9
9.9	4.5	14.7	584.7	2,309.4	10.8	...	1,556.9	...	...	...	4,461.8
10.7	5.4	17.0	658.8	2,393.0	11.5	...	1,682.4	...	...	...	4,745.7
11.6	6.7	19.3	729.8	2,479.4	13.6	...	1,803.3	...	...	...	5,026.1
12·6	8.1	21.5	789.8	2,562.2	16.5	...	1,848.6	...	...	...	5,217.1
13.8	9.6	23.5	876.9	2,661.9	19.8	...	1,869.9	...	...	...	5,428.5
15.0	9.6	25.9	942.2	2,752.8	24.4	...	1,808.0	...	...	...	5,527.4
16.4	12·5	28.3	1,040.8	2,819.2	29.5	...	1,833.8	...	...	...	5,723.3
18.1	15.6	30.3	1,147.6	2,920.0	36.5	...	1,845.5	...	...	...	5,949.6
20.2	15.9	32.8	1,278.7	3,027.9	44.2	...	1,921.4	...	...	...	6,272.2
22.4	20.3	35.0	1,387.6	3,108.3	54.6	...	1,927.8	...	...	...	6,478.3
25.2	20.7	36.8	1,519.2	3,265.9	68.1	...	2,018.4	145.4	...	...	7,017.0
28.1	22.9	39.6	1,577.4	3,283.1	74.9	...	2,230.5	150.0	36.3	...	7,352.2
295.6	175.5	390.3	16,937.2	55,799.8	422.1	...	28,451.2	295.4	36.3	...	101,942.0

in force at the end of each Year.

Continent				Great Britain & Ireland			United States	Canada	Australia	Other Countries	Full Total
Russia	Sweden	Switzer-land	Total	Ordinary Assurance	Industrial Assurance	State Assur.					
2,127	1,535	1,112	139,788	274,568	...	...	49,608	...	...	...	463,964
2,157	1,668	1,986	163,652	298,735	...	...	56,046	...	...	...	518,433
2,208	1,779	2,887	193,632	328,456	...	...	57,202	...	...	...	579,290
2,234	1,879	4,320	225,348	354,218	...	...	65,252	...	...	...	644,818
2,260	1,914	5,668	263,833	374,215	...	...	98,095	...	...	...	736,143
2,273	2,549	7,735	312,988	392,842	...	...	146,729	...	...	...	852,559
2,295	3,184	10,021	367,532	435,628	...	547	209,392	...	...	...	1,013,099
2,429	3,819	11,679	406,329	476,358	...	1,168	305,390	...	...	...	1,189,245
2,567	4,454	14,540	462,902	523,468	...	1,532	401,140	...	...	...	1,389,042
2,559	5,089	18,177	530,437	567,834	...	1,882	537,594	...	...	...	1,637,747
2,839	5,724	21,353	623,602	632,318	...	2,304	656,572	...	...	...	1,914,796
3,331	6,359	21,827	655,760	692,849	...	2,689	747,807	...	...	...	2,099,105
3,574	6,994	23,136	702,967	741,837	1,063,316	3,047	785,360	...	...	...	3,296,527
3,950	7,629	25,356	788,711	763,131	1,244,105	3,804	804,444	...	...	...	3,604,195
4,425	8,899	27,414	842,095	784,064	1,549,563	4,200	817,081	...	...	...	3,997,003
5,031	9,534	29,241	905,556	805,067	1,946,700	4,478	799,534	...	...	...	4,461,335
4,104	10,169	30,686	982,713	821,317	2,589,914	4,848	774,625	...	...	...	5,173,417
4,475	10,804	31,657	1,051,089	842,268	3,057,970	5,118	706,179	...	...	...	5,662,624
4,739	11,439	32,118	1,103,666	866,400	3,639,042	5,511	633,096	...	...	...	6,247,715
5,427	12,074	32,430	1,143,595	885,607	4,088,732	5,740	612,843	...	...	...	6,736,517
8,183	12,709	33,155	1,185,083	904,271	4,396,656	5,966	595,486	20,310	...	...	7,107,769
8,873	13,344	34,255	1,259,374	908,353	4,659,509	6,224	608,681	22,553	...	...	7,464,694
13,025	15,000	35,584	1,339,785	913,634	5,084,519	6,524	627,265	27,624	...	...	7,999,351
16,100	18,300	36,330	1,362,745	935,293	5,659,278	6,819	661,458	31,569	105,248	...	8,762,410
19,021	19,315	37,376	1,438,498	958,066	6,263,658	7,210	705,659	34,692	118,585	...	9,526,368

DEVELOPMENT OF LIFE

VI.—*Average Sum*

Years.	Continent										
	Germany	Austria-Hungary	Belgium	Denmark		Spain and Portugal	France		Italy	Norway	Nether-lands
				Ord. Assur.	State Assur.		Ord. Assur.	State Assur.			
	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.
1859	4,171	3,820	7,000	...	...	...	10,461	...	...	...	4,500
1860	4,152	3,652	7,000	...	...	...	10,314	...	...	...	5,000
1861	4,118	3,306	7,140	...	...	...	10,886	...	...	...	5,169
1862	4,071	3,687	7,500	...	...	...	13,945	...	...	...	4,789
1863	4,019	3,678	8,750	...	...	...	13,831	...	6,365	...	5,413
1864	3,879	3,782	8,888	...	...	...	9,752	...	5,915	...	5,047
1865	3,714	4,133	9,111	...	...	...	9,868	...	6,305	4,984	4,141
1866	3,669	4,084	9,333	...	...	...	9,781	...	6,533	5,567	3,433
1867	3,573	3,671	8,600	...	...	...	9,025	...	5,870	5,772	3,037
1868	3,506	3,850	8,000	...	...	...	9,684	...	5,508	6,222	2,761
1869	3,439	3,415	7,500	...	...	...	9,984	...	4,748	6,489	2,527
1870	3,486	3,410	7,076	...	...	...	10,607	...	5,599	6,033	2,297
1871	3,533	3,696	6,714	...	2,003	...	9,989	...	5,965	6,156	2,204
1872	3,625	3,027	6,857	...	2,087	...	10,114	...	6,086	5,772	2,094
1873	3,713	4,380	6,533	4,277	2,135	...	11,014	...	6,123	5,900	2,035
1874	3,790	4,317	6,250	6,317	2,182	...	10,253	...	6,369	6,145	1,963
1875	3,888	3,754	6,000	6,359	2,272	...	10,168	...	6,587	6,307	1,946
1876	4,037	3,467	6,127	6,399	2,391	...	9,377	...	6,562	6,450	1,971
1877	4,167	2,761	6,000	6,458	2,372	...	9,164	...	6,584	6,397	1,908
1878	4,246	2,743	6,222	6,503	2,402	...	9,708	...	6,599	5,522	1,845
1879	4,317	2,818	6,101	6,680	2,457	...	10,945	280	6,178	4,013	1,778
1880	4,380	2,826	6,315	6,877	2,445	531	10,512	285	6,388	4,513	1,825
1881	4,462	2,925	6,400	6,284	2,557	1,009	10,553	281	7,187	5,278	1,824
1882	4,565	3,410	6,454	5,909	2,611	1,612	10,666	264	7,689	5,946	2,023
1883	4,649	3,259	6,333	5,138	2,670	1,114	11,267	278	8,299	5,771	2,092

History of Life Assurance in the United Kingdom. By CORNELIUS WALFORD, F.I.A.

(*Concluded from* p. 315.)

6.—LIFE ASSURANCE—1844 TO 1870.

The earliest event of the year 1844 was the presentation to Parliament of the "First Report" of the Committee on Joint-Stock Companies, appointed in 1841. The report set out by stating that, as a good deal of the information affecting persons was in the nature of *ex parte* statements, caution would be required in promulgating it: hence much of the evidence collected would be stated summarily. I shall confine myself here to the passages bearing upon Insurance Associations.

"The Bubble Companies which were the immediate occasion of this enquiry, being concerned in effecting assurances and granting annuities, the attention of your Committee was naturally directed to Companies and Societies engaged in that most important class of

ASSURANCE FROM 1859 TO 1883.

Assured on each Life.

CONTINENT				GREAT BRITAIN AND IRELAND			United States	Canada	Australia	Other Countries
The Balkans	Russia	Sweden	Switzer-land	Ordinary Assurance	Industrial Assurance	State Assur-ance				
Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.	Frcs.
...	13,084	8,380	3,478	14,568	...	...	15,402	...	...	...
...	12,826	8,461	3,437	14,309	...	...	15,772	...	...	...
...	13,388	8,311	3,721	13,700	...	...	15,506	...	...	...
...	13,378	8,131	4,287	13,409	...	...	15,224	...	...	...
...	13,367	8,108	4,427	13,361	...	...	14,734	...	...	...
...	13,434	6,465	4,158	13,410	...	...	14,563	...	...	...
...	13,468	5,509	4,339	12,679	...	1,818	14,986	...	...	...
...	12,956	4,793	4,732	12,158	...	1,806	15,297	...	...	...
...	13,622	4,970	4,619	11,552	...	1,762	15,638	...	...	...
...	13,626	5,220	4,545	11,139	...	1,731	16,102	...	...	...
...	13,988	5,178	4,234	10,634	...	1,702	15,105	...	...	...
...	14,370	5,289	5,048	10,502	...	1,689	14,641	...	...	...
...	14,575	5,511	5,130	11,163	276	1,675	14,449	...	...	...
...	14,873	5,971	5,288	11,266	248	1,596	14,195	...	...	...
...	15,349	6,619	5,360	11,372	226	1,572	13,786	...	...	...
...	21,373	7,978	5,401	11,247	216	1,537	13,492	...	...	...
...	21,582	8,405	5,426	11,357	252	1,511	13,398	...	...	...
...	21,620	8,623	5,440	11,473	223	1,480	13,274	...	...	...
...	22,024	8,762	5,507	11,382	202	1,470	13,271	...	...	...
...	21,912	9,001	5,533	11,432	210	1,456	12,983	...	...	...
...	16,067	9,139	5,502	11,473	228	1,430	13,058	8,370	...	...
...	16,544	9,407	5,550	11,559	245	1,410	13,094	8,336	...	...
4,884	12,210	9,466	5,591	11,629	261	1,403	13,542	8,326	...	...
4,956	14,419	8,458	6,057	11,494	263	1,408	13,369	8,521	7,601	...
5,021	14,832	9,303	5,968	11,611	268	1,373	13,573	8,532	7,168	...

undertakings. They believe that, generally, they are deserving of the highest credit on account of the respectability of their management and the soundness of their schemes; but instances of abuse which have occurred justify and demand enquiry. Accordingly, your Committee have taken some evidence on the subject, and for the purpose of obtaining further and more complete information, they issued, at the close of the last session, a list of queries, in answer to which they have received returns from a considerable number of those Companies.

"Your Committee propose, as soon as the information to be supplied by the answers to these queries can be properly collected, to prosecute their enquiries in this direction, with a view, on the one hand, to suggest such practical facilities and encouragements as it may be in the power of the Legislature to afford to these Companies for the furtherance of the system of Life Assurance, so important to the well-being of the Nation at large; and on the other, to prescribe such reasonable precautions as may be requisite to ensure the faithful observances, on the part of such Company, of the extensive obligations into which they enter with the public."

The Report then proceeds to detail the proceedings it had unravelled regarding "Seven Insurance, Annuity and Loan Companies, and Three Mining and General Companies"—but it is not necessary to our present history to follow the narration there given. The legislation which followed was directed to prevent the recurrence of the fraudulent practices hitherto resorted to.

Joint-Stock Companies Registration Act.—As an immediate consequence of the preceding Report, there was enacted the Joint-Stock Companies Registration Act, 7 & 8 Victoria, chapter 110. The preamble recited that it was expedient to make provision for the due Registration of Joint-Stock Companies, and after complete Registration, to invest such Companies with the qualities and incidents of incorporation, with some modifications; *and to prevent the establishment of any Companies which should not be constituted and regulated according to the provisions of this Act.*

It defined a Joint-Stock Company to be:

> Every partnership whereof the capital is divided into Shares, and so as to be transferable without the express consent of all the co-partners:
>
> Every Assurance Company or Association for the purpose of assurance or insurance of lives against any contingency involving the duration of human life, or against the risk of loss or damage to ships at sea, or on voyage, or to their cargoes, or for granting or purchasing Annuities on lives:
>
> Also any Institution enrolled under any of the Acts relating to *Friendly Societies*, which Institutions shall make Assurance on Lives or against any contingency involving the duration of human life, to an extent upon one life, or for any person to an amount exceeding £200:
>
> Whether such Company, Society, or Institution shall be Joint-Stock Companies, or Mutual Assurance Societies, or both:
>
> Every partnership which at its formation, or by subsequent admission (except any admission subsequent on Devolution or other act at law), shall consist of more than 25 members.

The Act was not to apply to existing Companies except as specially therein provided; or to incorporated Companies at all. Its two leading features were:

1. *Provisional Registration.*—This was a preliminary step. Before proceeding to make public, whether by way of prospectus, handbill, or advertizement, any intention or proposal to form any Company for any purpose within the meaning of this Act, certain particulars concerning the same were to be returned to the Registry Office to be formed for the purpose of this Act.
2. *Complete Registration.*—No Company was actually to commence business until this second process had been attained, under severe penalties.

A copy of every balance sheet must be registered.

This was the first general Act under which Insurance Associations could gain a common legal constitution. Under its provisions many hundreds of Insurance Companies were founded, some of which are still in existence. Under its supposed sanction, too,—that is, after compliance with all its requirements,—many fraudulent insurance projects were launched upon the world; and mischiefs far greater in extent, because more numerous, followed than had preceded it. The measure therefore constitutes an epoch in Insurance History. (*See* 1853.)

One of the modes adopted in practice of limiting the liabilities of Shareholders in Companies founded under this Act, was by introducing into the Policies issued by Assurance Associations a proviso that the property and effects of the Company for the time being should alone be liable for any moneys payable under that contract—a clause which had probably been devised for the protection of the members in Mutual Associations at an earlier period; and which did, in fact, afford protection against all but simple contract debts.

The new Act was in force for the two last months only of the year 1844. Some 14 Life Offices—several of them combining other branches of business—were founded, but only one or two under its provisions. It is from the events of the next year therefore that we must look for any indication of the effects of the measure upon Insurance interests. The first Company registered under this Act was the *Monetary* Life office.

Number of Life Assurance Offices in existence.—It is material to note that, of the Life Assurance Associations, the existence of which I have already noted, there were in existence at the end of 1844 some 80; but this number includes several that were little more than Friendly Societies. Mr. Lewis Pocock enumerated at the earlier part of the year but 70. The *Handbook of Life Assurers*, 1842, had enumerated 82.

During the year 1845 there were projected in all 92 Assurance Companies, of which 39 were completely registered and the remaining 53 but provisionally so. Of this total 20 of the completely registered Companies undertook Life business, and 27 of those provisionally registered were designed to carry on Life business. Of the total Companies of the year, no less than 47, or more than one-half of the whole progeny, had Life business in contemplation. *Of these Companies not one is existing at the moment of writing!*

In 1846 there were 32 Insurance Associations completely registered, and 20 provisionally registered—total 52. Of these, 24 contemplated Life business.

In 1847 a Bill was introduced to Parliament, having the following preamble: "Insurance on lives having proved very "advantageous to the families of persons effecting the same, and "for other purposes, it is desirable to encourage Insurance, and "with that view to secure as much as possible that the claims on "account of such policies shall not be defeated by objections on "the part of Insurance Companies." This attempt to interfere with the rights of contract between the Companies and their policyholders, was most properly promptly rejected by the House of Commons. Opportunity may be here taken to remark upon the small amount of litigation which has resulted from the hundreds of thousands of Life Assurance contracts which have been entered into. Those who have had opportunities of judging have sometimes felt that a more determined front to resist frauds might, with advantage, have been assumed. But, on the other hand, in all business dealings litigation should be avoided when possible: the results are too uncertain to justify the outlay of money and the waste of time and energy involved.

Founding the Institute of Actuaries.—But the really important event of the year 1847, as affecting Life Assurance interests, was the founding of the Institute of Actuaries. The first Article of the Constitution of this body tells its purposes in plain language:

> "The Institute of Actuaries of Great Britain and Ireland is an Association founded for the purpose of elevating the attainments and status, and promoting the general efficiency, of all who are engaged in occupations connected with the pursuits of an Actuary; and for the extension and improvement of the data and methods of the science, which has its origin in the application of the doctrine of probabilities

to the affairs of life, and from which life assurance, annuity, reversionary interest, and other analogous institutions derive their principle of operation. It embraces as its peculiar province of inquiry all monetary questions involving a consideration of the separate or combined effects of interest and probabilities."

Limits of Foreign Travel and Residence.—The establishment of the *Colonial* Life Assurance Company in Edinburgh in 1847, by the late Mr. W. T. Thomson, marks an important epoch in the practice of Life Assurance. Hitherto, no settled practice regarding foreign residence and travel had been agreed upon. All was capricious and arbitrary. But an attempt was now made to place "extra risks" upon a rational as also an economic basis.

Amongst the 15 Life Offices founded in 1848 there were three which claim especial notice in any history of Life Assurance. These, taken alphabetically, are:

1. The *Gresham* Society (a proprietary Office) which, starting on lines not entirely original (as is shown under date 1824), and trying the experiment of insuring lives so far below the standard of health as not to be accepted at ordinary rates (with results not deemed satisfactory), accepted the dictum of critics, that there were more Offices than could find business in the United Kingdom, and opened for itself connections on the Continent of Europe.

2. The *Indisputable* Life, which came into existence on the plea that the contract of Life Assurance was of an almost sacred character—so sacred, that the policy once issued, and the premium regularly paid, no tribunal should intervene to prevent the absolute payment of the sum stipulated. Pamphlets were written and circulated in tens of thousands inculcating these views, and entirely ignoring the fact that the trust reposed in Directors and Managers implies that the honest policyholders must be protected from the frauds of the dishonest ones; that the funds are to be administered for the benefit of the members generally, and no preference be shown to dishonesty and craft. By the irony of fate, one of the first claims made upon this hypervirtuous Company involved questions which could alone be adjudicated in a Court of Law; and so it may be said that the theory of indisputability so set up, speedily revenged itself upon its propounders.

3. The *Prudential* Life Office. This Company had on 31 December 1883 in its Industrial Branch alone, more than 5 millions of policies existing, upon which during the year it had received over 2½ millions sterling in premiums, and had paid nearly one million in claims to 100,000 families.

This year, too, there was printed, by order of Parliament, the first series of the "Balance Sheets" rendered by the Companies, registered under the Act of 1844. It will be more reasonable not to dwell upon these now, but to wait until the next issue in 1852.

The following are estimates of Life Assurance business in force at the dates named:

	Millions sterling
1849—Sum Assured in *English* Offices (*vide* Thomson's *Proof Sheets*)*	150,000,000
1852—Sum Assured in *Scotch* Offices (Thomson)	34,000,000
1856—*United Kingdom* (Thomson's *Proof Sheets*)	200,000,000

Mr. F. Hendriks made the further comparative estimate of the sums insured in the three principal countries of Europe in 1851, namely,

	£		£	s.	d.
Great Britain & Ireland	150,000,000	Giving per head of the population	5	8	6
Germany	8,000,000	" "	0	2	6
France	1,000,000	" "	0	0	6

Annual premiums payable under these contracts in Great Britain, £4,500,000.

I may explain that I have already given, and shall continue to give, the estimates regarding the amount, &c., of Life Assurance in force, as they arise in point of date. The facts can then be estimated in regard to all the surrounding and concurrent circumstances.

The Assurance Magazine.—The year 1851 is notably characterized by one main event, namely, *the establishment of the*

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* Mr. Samuel Brown adopted a statement based upon the return of life-policy stamps issued in the year 1849, showing the approximate amounts of the policies so issued:

Stamp Value			Number of Policies	Average Sum insured	Aggregate Sum insured
£	s.	d.		£	£
0	2	6	1,417	40	56,680
0	5	0	7,661	80	612,880
1	0	0	10,627	300	3,188,100
2	0	0	4,455	750	3,341,250
3	0	0	3,153	1,500	4,729,500
4	0	0	334	3,500	1,169,000
5	0	0	253	5,500	1,391,500
			27,900	519	14,488,910

The total value of stamps sold in the year being £33,689. 7*s*. 6*d*., the aggregate of claims paid that year by the British offices was estimated at £2,500,000. The total number of policies in force he estimated to be 300,000, on 225,000 lives—giving 4 policies to every 3 lives insured. The average amount of each policy he estimated at £666. 13*s*. 4*d*.

Assurance Magazine, the design of which was to supply the members of the Insurance and Actuarial professions with information on topics having a special interest for them; also as a medium for the interchange of ideas on questions of theoretical or practical interest.

The *Journal* has now reached its 24th Vol., and stands as a proud monument to its originators and conductors; and a most excellent recently published index to the first twenty volumes adds greatly to their value.

The Insurance Controversy.—In 1852 there broke over the Insurance world that great storm, now known historically as the "Insurance Controversy." Mr. Robert Christie, of Edinburgh, addressed the then President of the Board of Trade, as the self-constituted champion of Insurance interests, in effect of the older Insurance Offices, and demanded "*A thorough scrutiny and* " *investigation into the affairs and responsibility of every Life* " *and Annuity Institution in the United Kingdom, with a view* " *to such enactments as shall protect extensive public interests* " *from the alarming prospective evils of fraud and of ignorance.*" This appeal was supported by a condensed statement of the results of the trading of 25 of the new offices from the date of their promotion, up to dates respectively given in a return made to Parliament, from which the figures were drawn, and which, in short, showed that these 25 Offices had an average of about 3½ years' trading, received in the aggregate on account of paid-up Capital, Life Premiums, and Annuity purchase-money £745,010, and had disbursed in expenses of management £287,339, and paid in claims and annuities £87,989—leaving but £369,700 available to meet engagements under Insurance contracts in force. In other words they had expended about 50 per-cent of their entire receipts. Of the 25 Offices so named, three are in existence to-day in a thriving condition, one has recently passed away by amalgamation, others had amalgamated in a respectable manner previously, and only one can be said to have had a disreputable ending. The very bandying of the names of young Life Offices before the public in such a fashion was, in itself, enough to destroy them; and in several cases did effectually do so.

While these 25 Offices had to bear the brunt of the attack, the others were not allowed to escape "scot free." "My impression" (said the same writer in the same appeal), "nay, my entire " conviction as to others, notwithstanding the flaming accounts of " their prosperity, contained in reports and speeches at annual

" meetings, is *that they are rotten, and are in effect, though not " in design, fraudulent.*"

It may be convenient to state here (a little in advance of current dates) that the entire number of Insurance Associations registered under the Act of 1844 had, up to 30 April 1853, numbered 311—this was in 9½ years; but of these 140 only had become completely registered and commenced work, and of the latter only 96 were in existence at the date of the return. The total number of Life Assurance Offices (*i.e.*, Insurance Offices transacting Life business) in the United Kingdom old and new about the end of 1852 was 174, namely, entirely *Mutual* 42; proprietary, or having a guarantee fund independent of the premium fund, 132. The combined share capital of 177 of these latter was £72,391,740. The implied contention was that these existing offices were more than sufficient to transact all the Life Assurance the Kingdom needed; and hence, that no profit could result to new enterprizes: that, indeed, the supply of Life Offices was greater than any present or probable future demand.

At this time a Parliamentary Committee was appointed nominally to consider the Law relating to Friendly Societies—but in fact to solve the peculiar legal situation as bearing upon "Nomination" policies, still being issued by several of the new Life Offices. In the evidence given various suggestions were made by Actuaries and others towards a solution of some of the difficulties then dividing the Life Assurance world. The Committee recommended exemption from all Stamp Probate and Legacy duties, in common with Life policies not exceeding £500; but Parliament did not assent.

Perhaps that which did most to facilitate what immediately followed was the *Letter of J. Hooper Hartnill, Editor of the Post Magazine, to the Rt. Hon. E. Cardwell, M.P., President of the Board of Trade, on the inoperative character of the Joint-Stock Companies Registration Act as a means of preventing the formation of Bubble Assurance Companies, or of Regulating the action of those honourably and legitimately instituted.* In this letter was contained a most trenchant narration of certain insurance projects; of the character of the men; of their antecedents; of the amounts and nature of their depredations—time, place and circumstances all stated. It was impossible after such an exposure that enquiry could be withheld.

On 8 March 1853 a Select Parliamentary Committee was appointed *to take into consideration the subject of Assurance*

Associations. The Committee was fairly and impartially constituted. It called before it Actuaries, representatives of all classes of Assurance Companies, and others competent to give useful evidence on the subject under enquiry; and by the middle of August its Report, with the Minutes of Evidence taken, were presented to Parliament.

Early in the year 1853 several meetings of the Institute of Actuaries took place to consider the working of the Act of 1844; and at a meeting held on 19 April, at which the President, Mr. John Finlaison, and 66 other members were present, the following Resolutions were passed:

> "1. That the Act of 1844 has created an invidious distinction between the offices established prior to 1844 and those established since; and *that the Act in question ought to be forthwith rejected and provision made for putting all existing offices on an equal footing.*
>
> "2. That whilst this meeting recognizes the propriety of allowing the competition in business of Life Assurance to remain unrestrained, it cannot lose sight of the fact that a rate of expenditure much greater than was previously necessary is hereby occasioned; and the meeting is therefore of opinion that every company to be hereafter established should be required before commencing business to prove the possession of an investment in Government securities of a Capital of at least £10,000 (under conditions similar to those imposed in the case of Joint-Stock Banks) as some guarantee to the public of its ability, under present circumstances, to carry out the contracts which it may enter into.
>
> "3. That in the opinion of this meeting no legislation in Life Assurance Associations can be permanently effective that does not exact tests as to the respectability and acquirements of persons allowed to practice as Actuaries."

These resolutions were placed before the Parliamentary Committee. The Report presented by the Committee was in every respect a most fair one; but it is quite certain that the Committee did not regard the situation from the same stand-point as the Institute of Actuaries as a whole had reviewed it—and far less the "dissolution" point which some individual members of that body had assumed. The report as a whole should, perhaps, find a place here—but on the ground of space my present quotation shall consist of two paragraphs only:

> "6. With regard to the general condition of existing companies as far as any evidence has been laid before your Committee, they feel it their duty to report, that it is more satisfactory than they had been led to believe before they entered upon their enquiry. No doubt instances of great abuses and flagrant frauds have been disclosed by the witnesses examined, but in general these consisted of an open

violation of all law, more akin to swindling than to regular trade, and such as it would be difficult for any legislation to prevent, so long as private persons exercise so little precaution in the conduct of their own affairs. But while the Committee are enabled to speak in these satisfactory terms of existing offices, so far as the evidence has gone, their attention has been called to the great facilities which exist under the present state of the law, for insurance companies, in common with others, being brought into existence with no reasonable prospect of or guarantee for success, and not unfrequently without any *bonâ fide* intention of transacting business. It appears, by a return made to your Committee from the office of the Registrar, that since the passing of the Act in 1844, no fewer than 311 insurance companies of various kinds have been provisionally registered, of which only 140 were completely registered, and of which only 96 continue to exist at this time. And while your Committee have reason to believe that some of the companies which have ceased to exist during that period have been absorbed in other companies by whom their business has been taken over, yet at the same time they have no doubt that considerable traffic has been carried on in the mere creation of companies which never had any real prospect of a *bonâ fide* existence.

"7. So far, then, as regards the present state of the law, as established by the Act of 1844, it appears to be a very general, if not a universal, opinion, that it requires some amendment, and in which your Committee fully concur. They are deeply impressed with the opinion, that as the law now stands, it does either too much or too little; too much, inasmuch as any legislative enactments professing to protect the public in such matters, have a certain tendency to weaken and impair that individual vigilance which would be more surely exercised if no such attempt were made; and too little, inasmuch as the securities which the law provides are ill calculated to affect the object at which they aim. On the part of all the witnesses examined, your Committee found a laudable desire to improve the existing state of the law, differing no doubt widely in their views, as to the best mode of accomplishing that object. On the part of the old offices of extensive business, the witnesses who more particularly represented them, expressed their desire to be placed under one general rule, to make such annual reports, and to afford such other tests as could be given without an undue interference with their business, for the purpose of securing one safe and uniform system, by which a reasonable security may be afforded for the respectability and solvency of insurance companies."

In view of future protection, the Committee was of opinion: "That the business of Assurance Companies differs so much from "ordinary business, that it will be advisable to repeal all the "provisions of the Joint-Stock Act, so far as they relate to "Assurance Societies, and to deal with them in a separate Act."

The immediate effect of the Committee was not to lessen the number of new Insurance projects. In 1854 no less than 37 were completely registered, and 29 provisionally registered. Of the completely registered companies 24 were devoted to Life

Assurance, either alone or in conjunction with some other branch of Insurance. In 1855 and 1856 the number of new projects was large; in 1857 there was a falling off, which events to be narrated may account for.

Legislation.—It will be convenient now to review the course of legislation during the next few years. Parliament took no immediate action on the report of the Committee of 1853. In 1855 there was enacted a measure (18 & 19 Victoria, c. 133) for limiting the liability of members of certain Joint-Stock Companies—but Insurance Companies were excluded from its operation. In 1856 Mr. Wilson, the Secretary of the Treasury, introduced a Bill with a view to the improvement of the law regarding Insurance Companies; but partly from the measure being incomplete, and largely from the opposition of the older offices, the Bill was withdrawn at an early stage. One of its provisions was that registrations and publication of accounts should apply to *all* existing as well as all new Insurance Associations. By a blunder of legislation the following state of matters arose. An Act of this same Session (the 19 & 20 Vict., c. 47) for the Incorporation and Regulation of Joint-Stock Companies was passed; and by it was repealed the Acts of 1844 and 1845 respectively; but such repeal was not to take effect as to any Company registered under the Act of 1844 until such Company had obtained registration under this new Act—designated the Joint-Stock Companies Act, 1856. Yet the second section of this very Act declared that its provisions should not apply to persons associated together for Banking or Insurance.

The Insurance Offices were thus entirely left out in the cold as far as legislation was concerned. This state of things continued during the remainder of 1856 and into the next year. In 1857 there was enacted the 20 & 21 Victoria, c. 80, which declared that the Act of 1856 was not to be deemed to repeal the Act of 1844, so far as Insurance Companies were concerned.

Amalgamation.—The unsettled state of the law regarding Assurance Associations, and the disturbed state of public feeling regarding the security of the offices, combined in some degree with unfavourable commercial prospects, led to a considerable falling off of business with many of the Life Offices during part of 1855 and nearly the whole of 1856. As a consequence, no less than 26 of the Associations passed out of the list of going concerns during the last-named years; while only eight new ones were created. Now, therefore, commenced a new mode of disposing of business of Life Offices—this was by the process of amalgamating their business, capital and effects with older-established or more vigorously-conducted Companies. The *Eagle* Insurance Company, under the able direction of Mr. Jellicoe, took a prominent part in this movement, and absorbed several Life Offices of great respectability which did not find themselves strong enough to continue a separate existence; as also, it must be admitted, some others to which this description could hardly apply.

Nearly every one concurred in the expediency of lessening the number of Assurance Offices. But abuses soon crept in. While it was advantageous in every way to place weak Companies under the protection of strong ones—the terms of the arrangement being equitable on each side—it was useless and dangerous to place two struggling, or financially-impaired Offices together: they could afford each other no enduring support, and might readily become a source of combined weakness.

The numbers of new Insurance Offices during the years 1857–59 were much less than during the previous years from 1845 downwards; while the number of Amalgamations and other "departures" were very considerable. In 1857 no less than 30 Life Offices were so disposed of, while but two new ones came upon the scene. In 1858 about 22 Offices passed away, many of these considerable ones, and few came on to supply their places.

Days of Grace.—In 1858 an important decision occurred on a question of practice which had not been previously rendered clear by any legal decision, and wherein the authorities were conflicting, namely, concerning the question of death during "the days of grace", *i.e.*, the period allowed by custom in the United Kingdom within which a Life Policy may be renewed. The case which now arose was under a policy of reinsurance (but that did not affect the principle involved)—*Pritchard* v. *Merchant and Tradesmen's Mutual Life.* The original policy contained a proviso for 30 days' grace. The premium became due on 13 October, and days of grace therefore expired 12 November, on which day the life assured also expired. On 14 November the plaintiff sent the reinsuring Company a cheque for the premium. On the following day receipt was sent "for the premium for renewal of the policy to 13 October 1856, inclusive",—both parties being ignorant that the life assured was then dead. It was *held* that the payment *did not, under the circumstances, revive the policy.* This decision created a great deal of alarm amongst policyholders; the public press, hardly ever well-informed upon Insurance questions, fanned the flame of discontent; and nearly all the Life Offices in the United Kingdom declared that it was their practice to pay claims arising during the "Days of Grace."

The Scotch Offices.—Those offices, placed in the order of their foundation, were:—1. *Scottish Widows* (1815); 2. *North British* (1823—Fire 1809); 3. *Edinburgh* (1823); 4. *Scottish Union* (1824); 5. *Scottish Provincial* (1823); 6. *Standard* (1825); 7. *Scottish Amicable* (1826); 8. *Scottish Equitable* (1831); 9. *Caledonian* (1833 — Fire 1805); 10. *Northern* (1836); 11. *Scottish National* (1841); 12. *Scottish Provident* (1837); 13. *City of Glasgow* (1838); 14. *Life Association of Scotland* (1838); 15. *Colonial* (1846). The Managers of these offices formed an association amongst themselves, and all matters of practice were discussed and joint action usually taken. At the instance of Mr. Robert Christie they united in making a return of their position as at the end of 1861; these are the results obtained:

Annual Income from premiums and interest on accumulated funds	**£2,200,933**
Accumulated funds	**£12,807,057**
Sums assured, with bonuses thereon	**£54,692,877**

Limited Liability extended to Assurance Offices.—The year 1862 constitutes, in one important respect, a new era in the history of Life Assurance, for from this period henceforth Life Assurance Associations, whether mutual or proprietary, could obtain a form of incorporation, sufficient for all business purposes, and at the same time could secure a limitation of the liability of their members to such an amount (in proprietary companies at least) as should be mutually agreed upon. In mutual offices much care was still required to guard the interests of members against liabilities beyond their agreed contributions in the way of premiums upon their policies.

These advantages were secured by "The Companies Act, 1862" (25 & 26 Victoria, c. 89), which regulates not only the formation of but the winding-up of companies generally. Every association registered under this Act has to register a "Memorandum of Association," defining the nature of its proposed business; this can never be altered, and if the Directors in the conduct of the business exceed their powers, the act would be deemed *ultra vires*, and the shareholders would not be liable. The Directors might indeed incur personal responsibilities. But to guard against these dangers it is usual to prepare "Articles of Association", wherein the powers are defined in detail. The "Memorandum" and the "Articles" are filed at the Registry Office of Joint-Stock Companies, together with a list of shareholders, corrected annually; and these, all or any of them, might be inspected by *anyone* on payment of 1*s*. Under this Act it was not necessary to file any balance sheet annually or otherwise. (*See* 1870).

The real advantage of the Limited Liability Law is—apart from the technical advantages just enumerated—that responsible persons may become shareholders, with the full security that they know the utmost measure of their loss in the event of things going wrong. And the effect of this undoubtedly is to improve the status of proprietary bodies in Insurance Offices generally.

The effect of the Act upon the projection of Insurance Offices could not be realized until the following year (1863) when of 27 offices founded—there were others projected—only nine undertook the business of Life Assurance; and in the several following years the figures were about the same. It cannot therefore be said that this last legislation lent any especial inducement to the formation of Insurance Companies.

Post Office Assurance Scheme.—In 1864 the Government Scheme of Life Assurance and Annuities, to be worked through the Post Office, was brought into operation under the authority of a special Act of Parliament (27 & 28 Vict., c. 43). The minimum sum for which a Life policy could be granted was £20, the maximum £100. The portion of the scheme relating to Annuities is very elaborate. The rates of premium for Life Assurance were based upon the English Table, and were very reasonable. The scheme was received with a flourish of trumpets by the press—or by that portion of it not devoted to Assurance interests. The *Economist* said: "Mr. Gladstone has just presented to Parliament his Table "for selling small deferred annuities or assurances to the poor. "They are worth a little study. The day will certainly come—"may possibly come while Mr. Gladstone is alive—when the "greatest of undeveloped economic forces, the principle of insur-"ance backed by the State Guarantee—*i.e.*, of Insurance which "really insures,—may be applied upon a somewhat extensive "scale." The project has, nevertheless, proved to be an absolute and ridiculous failure. During the first ten years ending with 1874 there had been issued a total of 4,478 Life policies, assuring £343,797—those remaining then in force being 3,630, insuring £284,069. Of the Annuity contracts of various classes there had been issued in the like period 6,531 contracts, the purchase-money under which had been £80,374; and there remained in force 5,930, representing £70,600 of purchase-money.

The simple fact is, there is *genius* required in the conduct of Insurance—the genius of well-directed enterprize. All the Life Assurance that would result from voluntary action in the United Kingdom could easily be transacted by some half-a-dozen offices. It may be useful to state in this connection that the average age at which lives are insured in the United Kingdom is 35; the average age of Government Annuitants at the time of purchasing annuities is 63. The one class insures against premature death; the other against prolonged life.

When the data upon which the "Seventeen Offices' Mortality Table" was deduced in 1843 was under investigation, the fact became revealed that the duration of the *Irish* lives (*i.e.*, lives resident in Ireland) had been inferior to that of lives of the same age resident in England. The cause could not be entirely unravelled; but some of the offices transacting Life business then put up their scale of premiums as a precautionary measure. There was often a difficulty regarding proof of age, in consequence of the defective system of birth registration which prevailed in the South of Ireland, but that did not account for the variation. Subsequent events revealed the fact that systematic frauds had been carried on there, whereby intemperate and other under-average lives had become insured; and that this system was usually carried on by means of *personation*. A remedy had to be applied, and the simple one was devised, of extending the "Gambling Act" of 1774 to that portion of the United Kingdom. This was effected by a short Act of Parliament, 29 & 30 Victoria, chapter 42—"An Act to Amend the Law relating to Life Assurance in Ireland." No person could thenceforward lawfully insure a sum upon any life in which they had not a pecuniary interest. Subsequent frauds have been discovered on a somewhat extended scale, but this Act has stood in good stead in such cases.

This year constituted a somewhat remarkable one in the annals of Life Assurance. Then passed out of separate existence, by process of absorption, the *Amicable* Society, founded 1706, and also several other important offices.

Assignment of Life Policies.—Difficulties, chiefly of a legal or technical character, had long existed regarding the assignment of Life Assurance policies; and the process had been attended with some expense also. By the *Policies of Insurance Act* of 1867, the whole matter was simplified, and a simple form furnished to be used in such cases. The office might charge a fee for registration not to exceed 5*s*.

A plan of "Settlement Policies" was this year devised by Mr. Bunyon, who has done so much to expound the law of Life Assurance, alike in his special work on that subject and otherwise.

In 1868 an important return was made to the House of Commons, of Life Policy Stamps, issued during the six years 1863–68, from which the *maximum* new assurances were calculated with the following results:

Years	Industrial Policies		General Policies		Totals	
	No.	Amount Insurable	No.	Amount Insurable	No.	Amounts
		£		£		£
1863	110,115	2,752,875	70,453	26,596,825	180,568	29,349,700
1864	169,357	4,233,925	70,816	26,291,425	241,173	30,525,350
1865	211,577	5,289,425	73,808	26,627,750	285,385	31,917,175
1866	215,308	5,382,700	71,848	26,314,700	287,156	31,697,400
1867	206,186	5,154,650	64,655	24,248,350	270,841	29,403,100
1868	396,296	9,907,400	68,056	23,666,650	464,352	33,574,050

The effect of the commercial disaster which overtook the country in 1866 is here clearly seen in its effect upon the returns of the following year. The cause of the enormous growth of the business in this 1868 is not so easily apparent—the growth was entirely in the industrial department.

The year 1869 was distinguished by two important events:

1. *Institute of Actuaries' Mortality Table.*—This year was completed, under the superintendence of the Institute of Actuaries, a new Mortality Table, based upon the experience of Assured Life. It had been for some time felt that the rapid strides made in Life Assurance during the quarter-of-a-century which had intervened since the preparation of the "Seventeen Offices' Table" (1843), rendered it most desirable that the results should be tested in a scientific manner. Accordingly, a scheme was propounded for collecting the required data. Twenty of the more prominent British Life Offices contributed returns, extending to 147,000 lives, of persons who had been admitted to assurance, after passing medical examination as healthy lives; while of these 24,000 had died. These are the short results. The full history of the table is given in the volume published by the Institute. The table is generally known as the "*Institute of Actuaries' Table*", but is sometimes spoken of by American writers as the "Twenty

Offices' Experience." An elaborate series of monetary values based upon it has been published also by the Institute; and for all ordinary purposes of life contingency computations this table is superseding all others.

The mortality experience of the *Scotch* Offices which had contributed to the general results of the foregoing Table was afterwards analyzed separately by Mr. James Meikle, under the superintendence of a Committee of Actuaries of Scotch Offices, and furnishes some special results of great value.

2. *Failure of the Albert Life Office.*—In the month of August the entire Assurance world was convulsed by the sudden stoppage of the *Albert* Life Assurance Company which had been founded in 1838, under the designation of the *Freemasons and General Life*, &c. Early in its career it had absorbed several small Life Offices; but during the period from 1857 to 1862 it absorbed, by amalgamation and otherwise, a number of much larger Companies—several of which had themselves absorbed other Offices. Afterwards it took over the business of several Life Offices having important connections in India. Thus, by the end of 1865 it had, directly and indirectly, absorbed no less than 26 Life Offices—and, with its own business included, presented the feature of 27 Offices supposed to be combined in one—with a premium income of £330,000 per annum. Periodical (so-called actuarial) Valuations had been made, and a surplus of assets was reported. In 1868 the original manager died. The new manager took the commendable course of calling in skilled advice. It then transpired that a Valuation made in 1866 on the "net premium plan" had shown a deficiency of more than a quarter-of-a-million. Steps had been then taken to reduce and finally to stop surrender-values being paid, but it was too late. The credit of the Office had become so far affected that it was looked upon with suspicion. The Actuarial world had divined that there was something wrong, but its extent could not be known. When the real facts became patent, it was seen that (under the then state of the law) there was no course but to stop. The income had been decreasing since 1866; the claims were increasing, and with the payments for annuities and management expenses added, the outgo was greater than the revenue. The stoppage was followed by investigation into the causes of the deficiency, and it was then found that almost the entire deficiency in the funds was occasioned by the sums which had been paid or misapplied in acquiring the businesses of the various companies taken over. An effort was

made by a Committee of policyholders and others to reconstruct this Company, and for a time there seemed a fair prospect of success; but the legal difficulties arising, nominally from the complexities consequent upon the various amalgamations, finally broke down the project. Then it seemed likely that a life-time of litigation would ensue; and assuredly it would have done so if the ordinary processes of winding-up in the Court of Chancery had been pursued. Eventually, Parliament intervened, and under the authority of a special Act (1871) Lord Cairns was appointed arbitrator, with full powers to adjudicate upon and determine the rights of all parties. He entered upon the task with vigour, and although the difficulties at first seemed overwhelming, by the middle of 1875, or in little more than four years, at a cost of £70,233, the whole business was disposed of in a manner that gained marked approval. The costs during one year and nine months in the Court of Chancery had been £71,668. The awards made from time to time tell their own story of the vast interests involved in this first great failure of an Insurance Office.

Failure of the European Life.—There probably was not a well-informed man associated with Assurance interests who did not foresee that the break-down of the *Albert* would certainly involve the stoppage of the *European* Life, although it certainly took longer to bring about the accomplishment of this second downfall than could have been supposed. The Office did not actually succumb until 1872; but it is convenient to treat the two events together, the circumstances being so precisely analagous, and as a matter of fact the *European* was dead from the day the *Albert's* doors were closed. The *European* had been founded in 1819, as a proprietary Company—it would be more correct to say an *European* had been founded in that year. The *People's Provident* had been founded in 1853. In 1858 these two Companies amalgamated, and in the following year the joint concern assumed the original title of *European,* although it was in truth now a distinct Company. The *People's Provident* had itself taken over various Life Offices before its union with the *European.* The combined Companies soon entered upon a like crusade of acquiring the businesses of other offices, in the most indiscriminate and, as is now known, the most reckless manner. By means of these combinations the annual income had been largely increased, and in 1868 the gross revenue reached £363,503. The claims were increasing in a rapid rate. The assets were less than half-a-million, but there was uncalled capital of about the same amount.

A call was made: before it was paid a petition to wind-up the Company was presented. This was in September 1869. Other petitions followed. Independent Actuaries were called in; and a very large deficiency was declared. A conflict of actuarial opinions ensued. An effort was made to bring in new Directors and reconstruct the Company. Other petitions followed, and upon one of these an Order was made in July 1871, and provisional liquidators were appointed. An attempt was made to "reduce the contracts." This failing, negotiations were opened to transfer the entire business to another company; to this all parties seemed willing to assent, but an investigation of the risks induced the negotiating company to decline to proceed with the arrangement—the mortality experience had been 10 per-cent above the proper rate, and it was supposed many of the best lives had withdrawn from the Company. On 12 July 1872 the affairs of the Company passed into the Court of Chancery for a compulsory winding-up. An investigation showed that no less than 46 Life Offices had become merged into this one, and the conflict of interests which must result was appalling to contemplate. Bills were introduced into Parliament for compulsory enquiry into the affairs of the Company, and for the appointment of an arbitrator. The latter was passed (1872), and Lord Westbury was appointed arbitrator. He died, and was succeeded by Lord Romilly, who also died during the progress of the arbitration. Finally, in (1875), Mr. Reilly, who had acted as assessor to Lord Cairns under the *Albert* arbitration, was appointed to close the affair, with appeal to the Court of Chancery in certain cases. By 1879 the whole affair had become closed, and the total cost of the proceedings of the winding-up was found to have been £182,151.

By reason of the disaster which had fallen upon these Companies—very largely, no doubt, in consequence of ill-considered and even dishonestly-promoted *Amalgamations*—this mode of disposing of unsuccessful Life Offices fell much into disrepute; and the good which had resulted in more prudently-conducted transactions of this nature was altogether ignored. Returning reason has shown that Amalgamation is the one remedy which, in many cases may be resorted to with advantage to all parties.

Here ends what has been not quite logically termed the "prehistoric" period of Life Assurance Chronology.

7.—LIFE ASSURANCE—1870 TO 1883.

The year 1870 is a most memorable one in the history of Life Assurance in the United Kingdom, from several causes next to be enumerated.

1. *Life Assurance Companies Act.*—The Act of 1862 placed Insurance Companies upon precisely the same footing as other Joint Stock Associations regarding their legal constitution, powers of disposing of their business, non-registration of accounts, &c., &c. There was only one special regulation applying to Banking and Insurance Companies exclusively, namely, that they should prepare half-yearly a statement of their capital, assets and liabilities, and post the same in a conspicuous position in their usual place of business. There were Actuaries and others who had long contended that Insurance Associations, by reason of the peculiar nature of their business, demanded more special legislation. The events narrated at the close of the last section certainly lent force to their arguments, and in consequence of the panic which ensued, it became almost necessary that some legislative protection should be attempted. Equally able and far-seeing men deprecated panic legislation; but finally all parties agreed upon a common line of action, and a measure was passed through Parliament known as the Life Assurance Companies Act, 1870, namely, the 33 and 34 Vict., chapter 6, "*An Act to amend the law relating to Life Assurance Companies*", consisting of 25 clauses and various schedules, the substance of which was as follows: Every new Life Office was to deposit £20,000, in such securities as were usually permitted in Chancery Funds, before certificate of Registration of such Company could be granted. The interest resulting from the fund to be received by the Company, and the fund itself to be repaid when the accumulated Life premiums reached £40,000. Companies transacting other business than Life Assurance to keep Life Funds distinct. Accounts to be registered annually in the form prescribed by the Act. Actuarial reports in more detailed form to be deposited with the Board of Trade after each periodical Actuarial Investigation. Copies supplied to share and policyholders on demand. Deed of Settlement to be printed and supplied to share and policyholders on demand, on payment of 2*s*. 6*d*. or less. As to *Amalgamations*, petitions are to be made to the Court to sanction the same after advertizement in *Gazette*, and after statement of material facts relating to proposed arrangement has been sent to each policyholder of each of the amalgamating offices. If policyholders representing *one-tenth* or more of

entire business object, Court not to affirm arrangement. Statement of all arrangements and agreements between companies and parties concerned in any amalgamation of Life Offices to be deposited with Board of Trade. All documents filed under Act may be received in evidence. Heavy penalties for non-compliance. Life Companies may be wound up on application of one or more of the policy or shareholders, after security for costs shall have been given, and a *prima facie* case established. Then followed an altogether new and important feature: "22. The Court, in case "of a Company which has been proved to be insolvent, may, if it "thinks fit, *reduce the amount of the Contracts* of the Company, "upon such terms and subject to such conditions as the Court "thinks just, in place of making a winding-up order." Board of Trade to bring before Parliament annually all statements of accounts and reports deposited under this Act. The schedules relating to balance sheets and valuation returns form a not unimportant part of the Act, and are particularly explicit and practical.

The Act was amended in the following session, on a point regarding the deposit required to be made by new companies, and orders for return of the same. And again, and more materially in 1872, first as to the deposit, and then as to the separation of the life fund from the other funds of companies carrying on other branches of insurance business; again, as to powers of winding-up of Life Offices, there should be concurrent powers to wind-up the affairs of any office which had been incorporated with a company previously to such winding-up. The most material point in this Amending Act arises under section 5 regarding the basis of valuations of life policies in companies wound-up under the Act of 1870. The "Seventeen Offices' Experience Table", with interest at 4 per-cent, is to be employed.

2. *Married Women's Property Act.*—Another Legislative Act of 1870 which has a bearing upon Life Assurance is the Married Women's Property Act, section 10 of which provides that a married woman may effect a policy of insurance upon her own life or the life of her husband, for her separate use, and the same and all benefit, if expressed on the face of it to be so effected, should enure accordingly, and the contract in such a case should be as valid as if made with an unmarried woman. The Consolidating and Amending Act of 1882 gives power to a married woman to effect a policy on her own life for the benefit of her husband, or her husband and children. This is an important piece of legislation, but points of complexity continue to arise in practice in

carrying out their provisions. Where such policies have been effected by husbands in view of defeating the rights of their creditors, the latter may demand, out of the sum insured, an equivalent to the premiums so paid.

3. *American Life Offices in the United Kingdom.*—During the first half of the present century, several of the British Life Offices established agencies in the United States, and transacted a limited business there. The *Royal* commenced there in 1842. The *Eagle and Albion* (combined), and the *International* in 1844; the *British Commercial* in 1848; the *Liverpool and London* in 1853. The *Colonial* (1846) had also an agency there, I believe. The remarkable development of the Life Assurance Associations of the United States during the decade 1850–60 practically drove the British Offices from the field, so much so, that at the close of 1862 the first five offices named had only an aggregate of 2,608 policies existing there, assuring a little over £570,000. They had in effect retired from active competition. It had often been threatened that the American Offices would in their turn establish agencies in the United Kingdom. Experiments were made in a quiet manner, and considerable results were obtained, especially in the North of Ireland, and in some of the manufacturing districts of England. Finally, early in 1870, two powerful and progressive American Life Offices established themselves in London.

One of the first questions which arose under the Life Assurance Act 1870 was in respect to Life Assurance Offices commencing business in the United Kingdom after the passing of that Act. The two great American Offices, the *Equitable* and the *New York* Life, had each been carrying on business here before the Act came into force. As to the later comers, it was held by the Board of Trade that they were in the light of the provisions of that Act called upon to make the deposit. One of these companies paid in the deposit, and within forty-eight hours—its premium accumulations being over £40,000—made application for and promptly obtained an order for repayment of the deposit.

Statistics of Life Assurance.—There was a very general belief that the discredit brought upon Life Assurance by the events of 1869–72 (the Albert and European scandals) would cause a material falling off of business, at least in Europe, and there were causes in operation in the United States—a species of crusade to stamp out the weaker offices—which it might be expected would create a like effect there. Hence there was considerable curiosity to obtain statistical returns. When they were available it was seen that there had been a considerable falling off in new business, thus:

Countries	1869		1870		Decrease	
	No. of Policies Issued	Amount Insured	No. of Policies Issued	Amount Insured	No. of Policies Issued	Amount Insured
		£		£		£
Great Britain	50,086	24,158,687	45,328	20,252,467	4,758	3,906,220
Germany . .	95,696	12,278,431	66,516	8,991,689	29,180	3,286,742
United States	181,683	97,097,167	179,746	81,241,951	1,937	15,855,216

In the returns for Great Britain the industrial policies (under £25) have been excluded. Comparative returns could not be obtained from France and some other European countries; but Germany probably affords a fair illustration of the effect produced in Continental Europe generally.

It will be useful to reproduce here an estimate of the Life Assurance business of the world for the year 1870, as prepared in Germany, by Herr Karup, I believe.

Country	No. of Insurance Associations	Amount Insured
		£
United Kingdom (1870)	113	340,000,000
United States (1869)	69	406,400,000
France (1868)	16	62,200,000
Germany (1870)	30	24,000,000
Austria	11	13,000,000
Remainder of Europe . . .	25	30,000,000
Rest of World	30	37,400,000
Totals . .	294	913,000,000

The amount of Life Assurance per head of the population in each country on the basis of the above totals, spread over the estimated populations, produced the results following:

Country	Population	Insurance per head
		£
United Kingdom	30,200,000	11·23
United States	36,700,000	11·07
France	38,200,000	1·64
Germany	39,000,000	1·39
Austria	13,000,000	·99
Remainder of Europe	177,600,000	·17
Rest of World (including Australia)	953,300,000	·40
Totals . . .	1,288,000,000	·73

It thus appeared that the amount of Life Assurance upon every man, woman and child in the world averaged just under 15*s*. In the United Kingdom and the United States the average amount was within 1*s*. the same. But there is a prodigious amount of "Industrial" and Friendly Society Assurance in the United Kingdom not included in this return.

New Life Offices and Liquidation of Old ones.—The immediate effect of the Life Assurance Companies Act was most materially to repress the formation of Life Assurance Companies, and this still continues. As a matter of fact, I believe but six new Life Offices of the class contemplated by the Act have been founded in the 13 years the Act has been in operation, and about the same number of Life Offices, registered under the Friendly Societies Acts, have come into existence. These last are included in the returns of the present number of existing Life Offices. During the same period, various Life Offices have been amalgamated—still the best known mode of disposing of offices which cannot maintain an independent existence,—and there are others which might, with advantage to their policyholders, adopt this course. Some have passed directly into liquidation under the directions of the Court of Chancery. Here they became too often the prey of designing attorneys and unscrupulous liquidators. The oft-quoted evil deeds of "promoters" pale before the legalized iniquities of compulsory winding-up. As a result of these various changes, the number of Life Offices in the United Kingdom is a decreasing quantity. In 1872 there were 115 offices included in the annual returns; in 1877 there were 109; in the following year, 107; in 1883 there were but 105 Life Offices embraced in the Life Office returns, and of these 25 carried on their business in conjunction with Fire Insurance. Since 1861 very few companies of this compound character have been formed.

The limitation of the number of Life Offices has given to Life Assurance the advantage of a species of protective monopoly, whatever that advantage may be worth in these free-trade times. It is, indeed, very true that there are still in existence a sufficiency of Life Offices to transact the Life Assurance at the moment needed: but that argument does not quite cover the whole ground. Mr. Sprague said, in a letter addressed to the *Insurance Record*, under date 1 July 1873:

> "I believe I am right in saying, that all improvements in Life Assurance, of whatever character, have been originally introduced by new companies; and this must also be so from the nature of the

case. Speaking for myself, I say unreservedly, that I believe the system of Life Insurance, as at present practised in this country, admits of very substantial improvements in the direction of giving greater advantages to the assured; and I shall view with pleasure the foundation and growth of a new company . . . . . . which should aim at conferring on the assured such further advantages."

I need offer no comment on the enlightened views here expressed, further than to remark with satisfaction upon the character of the searching investigations which are being made into all points of practice by the rising members of the Institute of Actuaries, as also of the Actuarial Society of Edinburgh, and the Insurance and Actuarial Society of Glasgow.

The latest event of special note as to individual offices is the long-expected collapse of the *Great Britain* Life that occurred in 1881. The power under the Act of 1870 for reducing contracts was brought into play. A scheme was prepared under competent actuarial advice, sanctioned by the Court of Chancery, and adopted in the common interest of all parties; and it seems not unlikely that the former reckless mode of winding-up Life Offices of any magnitude may be regarded as entirely a thing of the past.

Life Assurance Accounts and Returns.—The annual accounts of Life Offices, as also the Reports of Actuarial Investigations required to be returned under the Life Assurance Act 1870, have been ordered by Parliament to be printed, and hence results annually a volume which is full of interest, while a series of such volumes constitutes a mine of valuable information, regarding which no information used to be volunteered. In this respect, too, all Life Offices, old and new, are placed upon precisely the same footing. It is true that the aggregate of the accounts here contained does not represent the full measure of Life Assurance in England. A large amount in sums below £200 per policy is transacted by offices registered under the Friendly Society Acts. Taking the returns, as annually condensed and published by Mr. Wm. White in his *Insurance Register*, we obtain many facts not generally collectively given. Thus the returns for 1872 showed that 115 Life Offices (omitting the American Offices) had an

Income from Premiums of	£10,256,057
" " Interest and Dividends . .	4,158,987
" " Consideration for Annuities .	273,427
" " other Sources	240,164
Total Income .	£14,928,635

While the same offices paid in

Claims and Bonuses	£8,156,216
Surrenders	498,938
Annuities	413,113
Commission and Expenses of Management	1,437,050
Dividends and Bonuses to Shareholders	403,240
Reduction of Premiums, Income-Tax, &c.	816,039
Total Expenditure	£11,724,596

Leaving a balance of £3,204,039 on the year's trading to be accumulated in augmentation of the Life and other funds.

The aggregate annual income of these offices is seen to be close upon 15 millions sterling; the aggregate Life and Annuity Funds to amount to over 92 millions; the share capitals and accumulations to about 10½ millions; the aggregate amount of Assurance in force to over 343 millions; the ratio of expenses of management to income 14 per cent.

I pass over a period of ten years and then institute a like comparison. The returns for 1882 from 107 offices give the following results:

Income from Premiums	£14,175,312
" " Interest and Dividends	5,569,158
" " Consideration for Annuities	603,292
" " Profits on Investments and other Sources	275,747
Total Income	£20,623,509

While the same offices paid in

Claims and Bonuses	£10,506,664
Surrenders	749,745
Annuities	500,330
Commission and Expenses of Management	2,743,096
Dividends, &c., to Shareholders	500,831
Cash Bonus and Reduction of Premiums	873,338
Income-Tax (£54,576) and other Payments, &c.	102,082
Total Expenditure	£15,976,086

Leaving a balance of £4,647,423 on the year's trading to be accumulated, &c. The aggregate Life Funds at the end of the year reached £134,011,957, covering sums assured under contracts 435 millions; the share capital and accumulations reached more than 11 millions; the ratio of expenses being under 14 per-cent. In consequence of an obvious defect in the schedules the amount of new assurances effected each year is not stated.

It is apparent from a perusal of these decennial returns *that Life Assurance is advancing in a far more rapid ratio than population.* It is most probable that it has done so all through the present century, but no certain proof can be adduced for the earlier periods. If the business results of the American Offices operating in the United Kingdom were included, the evidence would become much more striking.

Friendly Societies.—A Royal Commission on *Friendly Societies* had been appointed in 1870. The third Report of this Commission, published 1873, contains many details of interest upon Life Assurance generally, as also upon the Post Office scheme of Insurance, and Industrial Assurance. The various reports published under the authority of this Commission throw a flood of light upon the Provident Associations of this country. In 1875 new consolidating and amending legislation was applied to the Friendly Societies of the United Kingdom, of which there were believed then to be 21,659 registered Societies, having 1,787,291 members, with accumulated funds of £8,630,525. If the unregistered societies were added it was believed the members would be over four millions, with funds of about 11 millions. The funds are more particularly designed to meet sickness claims, and what precise proportion is available for death claims I am unable to state.

Add to these figures the amount of *Industrial* Assurances existing in various offices, with the returns of the ordinary Life Offices, as above given, *plus* the large business of the American Life Offices (not separately given), and the aggregate Provident Savings of the British people in the direction of Life and Health Assurance may be approximated.

Stamps on Life Insurance Policies.—The following will throw light upon various points in this enquiry. If the Legislature failed to reach the business of Life Assurance in 1692, they were not long to escape; but the new mode was moderate and just as compared with the former one. In 1694 a stamp duty of 6*d.* was imposed upon all policies of insurance. In 1698 this was increased to 1*s.*, and in 1711 an increase of 2*s.* 4*d.* was made upon all policies, to continue for a period of 32 years—making in all 3*s.* 4*d.* No unstamped policy could be used in proceedings in Law or Equity. In 1713 an additional 6*d.* was added, making 3*s.* 10*d.*, quite irrespective of the amount of the policy. In 1757 another 1*s.* was added, but as some of the former levies had expired, the sum per policy was therefore only 2*s.* 6*d.* In 1765 a distinction was made between policies issued in London and the country—in the former case, the stamp was 2*s.* 8*d.*; in the latter, 5*s.* 6*d.* In 1776 the stamps were again increased to 3*s*. 2*d*. and 5*s*. 6*d*. respectively. In 1777 an additional 5*s*. was imposed on policies of and over £1,000—hence, a policy of that amount or above required a stamp of 8*s*. 2*d*. in London, and 10*s*. 6*d*. in the country. There was a penalty of £10 for signing and sealing a policy not duly stamped. In 1797 all former Acts were repealed, and the stamps were declared to be 6*s*. and 11*s*. respectively. In 1801 Life Insurance policies are first specifically mentioned; but no change in stamps was made. *In* 1804 *Life Policies were exempted from all stamps;* but the exemption was of short duration. In 1808 life policies of under £500 were charged 15*s*.; over, 30*s*. In 1815 occurred a considerable advance: below £500, 20*s*.; up to £1,000, 40*s*.; £3,000, 60*s*.; under £5,000, 80*s*.; £5,000 and upwards, 100*s*. In 1835 a reduction as to policies not exceeding £100, namely, not exceeding £50, 2*s*. 6*d*.; £50 and not exceeding £100, 5*s*. In 1853 a reduction to 1*s*. per £100 was effected. In 1860 the stamp on policies not exceeding £25 was reduced to 3*d*. Under the Consolidating Act of 1870 the amounts were not altered, except that a policy not exceeding £10 requires a stamp of 1*d*. only; for £25, 3*d*.; for £50, 6*d*.; then 1*s*. per £100 up to £1,000, and all fractional parts of £1,000 additional 10*s*. For these duties no adequate advantages in the way of returns of the numbers of stamps used have been systematically obtained.

Conclusion.

Opinions are invited as to the general principles upon which legislation on Life Assurances should proceed. Beyond the indications conveyed in the preceding sketch, the writer would add hereon, that except in providing well-considered laws under which Life Offices in common with all other commercial associations may secure corporate privileges; may be protected from all vexatious interferences in the enjoyment of these; may be speedily reached through their responsible officers in the event of wrongdoing; and may be compelled to make their financial situation known to all concerned therein—beyond these things, the less legislative interference the better. Especially should there be no advantages to one class of company over another—of the old against the new; it is the weak which need the greater measure of protection. On the whole the present legislation affecting Life

Offices seems just, and the greatest care should be exercised in making further advances.

As regards the *Contract of Life Assurance* it is apparent that the tendency of late years has been in the direction of simplicity. In the early days of Life Assurance the contract was indeed simple enough. With the development of the large proprietary companies, in the first half of the present century, the conditions became more complicated, and in some cases increasingly severe. Stipulations, general rather than technical, should be provided against fraud; but for all legitimate business requirements, such as permission to travel and reside in foreign countries; permission to surrender annual premium policies for fully paid-up policies; facilities for admission of age, and rendering policies of certain standing absolutely indefeasible, are all steps in the right direction, and for the common good; while special advantages to any particular class are only to be gained at the expense of the whole. The modern practice of paying claims immediately upon proof of death as of right, appears to the writer of doubtful advantage, and will sometimes be attended with positive danger. Three months is a rational period, and very few estates are in a position to be wound up earlier than that. Solicitors are not always the most safe custodians of insurance monies, and the cases where real advantages would result are quite exceptional.

On the Value of the Option of Withdrawal in Deferred Annuity Contracts. *By* W. B. Paterson, F.F.A., A.I.A.

[Abstract of Paper read before the Actuarial Society of Edinburgh.]

THE subject of this paper was suggested by a perusal of the correspondence in the 12th and 13th volumes of the *Journal,* in which several members of the Institute engaged, and which left the matter in an unsatisfactory position.

Since that time no further notice of the subject appears to have been taken; but the publication of the last Government Annuity Tables renders further investigation of the subject possible.

Withdrawals may be divided into two classes, the one containing select and the other deteriorated lives. This distinction is essential to the solution of the problem, and it is to the want of it that the erroneous or incomplete results formerly arrived at may chiefly be attributed. The two classes are in this matter in entirely different positions. With the first the question of return is one of surrender; with the second the return must be provided for by an addition to the premium.

In the case of a select life, the value of a deferred annuity is, throughout the period of deferment, greater than the premium or premiums paid, net rates being assumed; and the return is more than provided for in the ordinary premium. A larger return might indeed be made without loss, and the option of withdrawal would not in any way affect such a return. The option or withdrawal does not reduce the value of the annuity, and therefore cannot limit the possible return. It fixes a minimum, not a maximum, return.

From these considerations it follows that the rate of withdrawal to be assumed must depend upon the rate of deterioration alone, and not upon any of the various other causes which may bring about the surrender of annuities.

Deteriorated lives are estimated in accordance with a theory explained by Mr. Sprague in his paper on the "Construction and Use of Select Mortality Tables" (*J.I.A.*, xxii, 421–441).

In this theory there appears to be a slight oversight. The theory takes no account of the possibility of deteriorated lives ultimately recovering. The number of deteriorated lives obtained by Mr. Sprague's method may be rightly employed as a measure of the deterioration in any body of lives, but cannot be held to be the actual number of deteriorated lives. In the present case the employment of the theory involves the assumption of an impossible amount of foreknowledge on the part of deteriorated lives. Those among them destined to die are assumed to withdraw, while those who are to regain health are assumed to retain their annuities. The tendency of the assumption is to increase the premiums, and the error is therefore of less importance than it would have been had it acted in an opposite direction.

Although the rate of withdrawal is measured by the rate of deterioration, these rates are not necessarily equal. The deteriorated lives may or may not withdraw immediately on deterioration; but the difference due to the distribution of withdrawals is exceedingly small. The assumption that the rates of deterioration and withdrawal are equal is the one most favourable to the granter of the annuity, and upon it the following demonstration proceeds.

Suppose that $l_{[x]}$ lives purchase annuities to be entered upon at age $x+n$ with return in event of death or withdrawal, these lives will be reduced by death in the first year to $l_{[x]+1}$ lives, of which $l_{[x]+1}-l_{[x+1]}$ are deteriorated, according to Mr. Sprague's theory already referred to. When these withdraw, $l_{[x+1]}$ lives remain and enter upon the second year. Similarly, the numbers entering upon the third, fourth, and nth years of deferment are $l_{[x+2]}$, $l_{[x+3]}$, and $l_{[x+n-1]}$ lives. The lives entering upon the annuities are $l_{[x+n]}$ lives, and the values of their annuities are of course select.

The first column of the table of l consisting of the above terms will therefore represent the numbers under observation at each age during the period of deferment. Let $\mathbb{N}'_{[x]}$, $\mathrm{M}'_{[x]}$, and $\mathrm{R}'_{[x]}$ be found from this column and treated as a separate mortality table. $\mathrm{D}'_{[x]}$ is equal to $\mathrm{D}_{[x]}$.

The formulas may then be derived in the same way as in the case of a deferred annuity with return only at death.

Let π be the single payment for an annuity deferred n years with return in event of death or withdrawal.

Then $\pi=$ (1) the value of the deferred annuity $=\dfrac{\mathbb{N}_{[x+n]+1}}{\mathrm{D}_{[x]}}$, and (2) the value of the return in event of death or withdrawal $=\dfrac{\pi(\mathrm{M}'_{[x]}-\mathrm{M}'_{[x+n]})}{\mathrm{D}_{[x]}}$.

$$\pi \text{ is therefore equal to } \frac{\mathbb{N}_{[x+n]+1}}{\mathrm{D}_{[x]}-\mathrm{M}'_{[x]}+\mathrm{M}'_{[x+n]}}.$$

The formula for the annual premium is

$$\frac{\mathbb{N}_{[x+n]+1}}{\mathbb{N}'_{[x]}-\mathbb{N}'_{[x+n]}-\mathrm{R}'_{[x]}+\mathrm{R}'_{[x+n]}+n\mathrm{M}'_{[x+n]}}.$$

This formula is based on the assumption that all deteriorated lives withdraw; but when the value of the annuity on any deteriorated life is greater than the sum to be returned on withdrawal, it will be in the interest of the annuitant to retain the annuity.

The following are specimens of the results given by the formulas:

Specimen Single Premiums for Deferred Annuities of £10, &c.

Age at Entry	Single Premiums (Males)				Percentage of Ordinary Premiums to provide for Option, as in Col. (4)		
	With Return on Death or Withdrawal (1)	With Return on Death only (2)	Addition for Option of Withdrawal (3)	Percentage of Ordinary Premium to provide for Option (4)	Annual Premiums		Single Premiums (Females)
					Males	Females	
	Age when Annuity entered upon 45						
40	119·259	117·880	1·379	1·17	1·23	1·15	1·21
	Age when Annuity entered upon 50						
40	89·712	88·675	1·037	1·17	1·25	1·27	1·37
45	109·20	107·92	1·280	1·18	1·24	1·31	1·38
	Age when Annuity entered upon 55						
40	64·725	64·040	·685	1·06	1·13	1·56	1·77
45	80·022	79·187	·835	1·05	1·11	1·67	1·83
50	97·746	96·745	1·001	1·04	1·08	1·76	1·85
	Age when Annuity entered upon 60						
40	44·107	43·673	·434	·99	1·01	1·67	1·97
50	68·905	68·284	·621	·91	·95	1·94	2·10
56	87·888	87·110	·778	·89	·92	2·03	2·12
59	98·245	97·656	·589	·60	·60	1·19	1·19
	Age when Annuity entered upon 65						
40	27·752	27·331	·421	1·54	2·39	1·99	2·68
50	45·230	44·239	·991	2·24	2·97	2·59	3·07
60	70·873	68·574	2·299	3·36	3·62	3·19	3·37

The only case in which an adjustment was necessary on account of the sum to be returned being less than the value of the annuity on some of the deteriorated lives was for age 40 at selection and 65 at entry upon the annuity in the table of single premiums for male lives, and in that case its effect was inappreciable—1 in the third decimal place.

The Government Annuity Tables are at present the only tables showing the effect of deterioration among annuitants. They must, therefore, in the meantime, be adopted as the exponent of the laws regulating it. The course which deterioration follows is entirely different in respect of male and female lives.

Column (4) in the tables of examples affords perhaps the best means of comparing the additions. The percentages there given make it possible to compare the additions for single and annual premiums as well as for males and females. For single and annual premiums the difference in the percentages is small at ages 60 and under. Above 60, however, it increases very considerably, the addition to the annual premium for age 40 at selection and 65 at entry upon the annuity being, for both male and female lives, about 1½ times the addition to the single premium. It is also useful for another purpose. From the fact that the figures in it depend almost entirely upon deterioration, and are therefore influenced little by other causes, they may with a fair degree of accuracy be applied to premiums derived from any table of mortality.

No figures are given in the Government Annuity Tables, 1883, for ages under 40, and there is accordingly no index of the rates of deterioration for early ages. The percentages in column (4) vary more with the age at entry upon the annuities than with the age at selection, and when a material variation does take place under any fixed age at entry upon the annuities, it is in the direction of a diminution for early ages at selection. No great risk of error would appear to attend the extension of the percentages in column (4) to earlier ages at selection; but great care would be required in extending them to earlier ages at entry upon the annuities, and it would be necessary to make liberal allowance for possible variations in the rates of deterioration.

To summarize, the following are the conclusions arrived at:

First.—The return of premium on the withdrawal of select lives is a mere question of surrender.

Second.—The return of premium on the withdrawal of deteriorated lives, except when the value of the annuity on a deteriorated life is greater than the return, must be provided for in the calculation of the premium.

Third.—In order to provide fully for the option of withdrawal, the annuitants must be assumed to exercise it to the best of their ability in their own interests, and therefore deteriorated lives upon which the value of the annuity is less than the return must be held to withdraw.

Fourth.—The percentages to be added to the premiums obtained from the Government Annuity Tables, 1883, in order to provide for the option of withdrawal, may, with a reasonable degree of accuracy, be applied to premiums derived from any table of mortality.

Fifth.—In view of the smallness of the additions, approximations which are more adapted to calculations than the percentages ascertained from the Government Annuity Tables, 1883, may safely be made use of when necessary on account of other mortality tables being employed. Additions of 1 per-cent for males entering upon annuities at ages 45 to 60 inclusive, 1½ for females at ages 55 and under, and 2 per-cent for females at ages 56 to 60 inclusive, appear to be sufficient.

The formula for an annual premium for a deferred annuity without return, which may be written $\frac{\mathbb{N}_{[x]+n+1}}{\mathbb{N}_{[x]}-\mathbb{N}_{[x]+n}}$, involves the assumption that the premiums are paid for n years or until death, should that event take place during the period of deferment.

In order to make the formula strictly applicable to the case of an office granting deferred annuities by annual premiums, where there is no obligation on the part of the holder to fulfil his part of the contract, provision must be made for the contingency of his exercising his option of withdrawal on the deterioration of the life upon which the annuity depends.

A close approximation to accuracy may be obtained by the use of a formula giving effect to withdrawal immediately on deterioration $\frac{\mathbb{N}_{[x+n-1]+2}}{\mathbb{N}'_{[x]}-\mathbb{N}'_{[x+n]}}$.

The demonstration of the formula is as follows:

The value of annual premiums for n years or until death or deterioration $=\frac{\pi(\mathbb{N}'_{[x]}-\mathbb{N}'_{[x+n]})}{D_{[x]}}=$ the value of a deferred annuity, on the assumption that in the event of a life deteriorating during $\overline{n-1}$ years, that is, before payment of the last premium, the contract is allowed to lapse $=\frac{\mathbb{N}_{[x+n-1]+2}}{D_{[x]}}$.

Therefore $$\pi=\frac{\mathbb{N}_{[x+n-1]+2}}{\mathbb{N}'_{[x]}-\mathbb{N}'_{[x+n]}}.$$

This formula does not pretend to absolute accuracy. In some cases of deterioration just before the date of the last premium it is for the advantage of the holder to maintain the contract; but the error involved in assuming withdrawal in every case of deterioration is very trifling, and, to obviate it, the formula required would be wholly unworkable.

The following examples of the results given by the two formulas show the differences to be very small.

Annual Premium to secure an Annuity of £10.

Age at Entry 40 — Deferred	By Ordinary Formula		By New Formula	
	Male	Female	Male	Female
20 years .	2·6055	3·1196	2·6337	3·1611
10 years .	9·5299	10·941	9·6250	11·047
5 years .	24·187	27·280	24·372	27·475

The annual premiums for contingent annuities are subject to the same chance of discontinuance.

NOTICES OF NEW BOOKS.

The Pole Star Life Insurance Company of Stockholm. Its Operations during the first 15 *Years of its Existence.*

This is a neatly-got-up volume prepared by the manager, Mr. Otto Samson, apparently for the purpose of showing to all persons interested in the company, what satisfactory progress it has made during the 15 years which have elapst since it was establisht in the year 1871. Table No. I gives particulars of the proposals considered in each year, the total being 14,173, of which 10,492 were accepted at the ordinary rate, and 2,015 at an increast rate. Table No. II gives the number of proposals declined in each year; and also of the policies terminated by death and other causes. The proportion of the proposals declined has decreast in recent years—probably, it is suggested, because the agents have gained experience, and are able to reject bad lives without having them examined by the medical officer. The total number of deaths during the 15 years was 440, and the number of persons who withdrew from the Society was 3,034. A very interesting table is given, showing how many of the 3,034 entered in each of the 15 years, and how many of those entering in any year withdrew in each subsequent year. Table III gives the number of lives insured, the number of policies existing, and the total sum assured thereby, at the end of each year. It states that the number of policies existing at the end of 1886 was 17,887, on the lives of 7,983 persons, and assuring a total sum of £3,037,246; the average amount of the policies being thus about £170. Adding the number of lives remaining insured, to the numbers of deaths and withdrawals, we get the total number of lives insured, 11,457. I have thus been enabled, by means of the above-mentioned table of withdrawals, to estimate roughly the probabilities that a policy will be discontinued in each of the 15 years following its issue. It is to be observed, however, that lapses at the end of the 1st year, are here entered as discontinuances in the 2nd year; and so in other cases.

Table 1.

Table showing the Probability that a Policy will be Discontinued in each of the 15 *Years after it has been effected.*

Year	Probability
1	·063
2	·083
3	·051
4	·038
5	·031
6	·026
7	·022
8	·019
9	·016
10	·014
11	·011
12	·009
13	·007
14	·005
15	·003
1–15	·398

From this it would seem that the probability of a policy being lapst or surrendered within 15 years after its issue, is about ·4.

Table III also gives information as to the reassurances, from which it appears that, with trifling exceptions, the lives reinsured have always numbered between 3 per-cent and 4 per-cent of the total number insured; while the amount reassured was about 4½ per-cent in the first year, 1872, and has gradually increast until it has become about 8½ per-cent in the last quinquennium. Subsequent tables give full information as to the occupations of the persons whose lives were proposed, the ages of the lives insured at the end of each year, and other matters which it is unnecessary to specify.

Tables XV–XX are concerned with the financial progress of the company, and give particulars of the income, outgo, and assets, in each of the 15 years. There are columns of percentages given, from which we learn that in the first quinquennium the premiums amounted on the average to about 3⅝ per-cent of the sum assured; and that the rate gradually diminisht, until, in the year 1886, it was about 3¼ per-cent. The average rate of interest yielded by the investments seems to have been a little over 5 per-cent in the first quinquennium; about £5. 2*s*. 6*d*. per-cent in the second quinquennium; and about £4. 19*s*. in the third quinquennium. The expenses, including commission, were 37½ per-cent of the premiums in the year 1872; 26·4 in 1873; and gradually diminisht to 13·7 per-cent in 1882. In 1883 the percentage increast to 15·4, and in the same year there was a great increase in the amount of new business; but by 1886 the rate of expense had fallen to 14·3 per-cent of the premiums. The book concludes with a diagram, which shows in a striking manner that the company has made much more rapid progress than the two older and the three younger Swedish offices; and that, as regards the total sum assured, it stands considerably in advance of them all.

The most interesting parts of the volume to an actuary are those that deal with the mortality experience. There is a good deal of information given regarding this, although not all the particulars that would be necessary for the purpose of deducing a mortality table from the experience. As far as can be judged from the particulars given, the mortality seems to have been extremely light; and it will be interesting if, at some future time, when the company has been longer establisht, the mortality experience is given in the usual form, as we shall then see whether the favorable promise of the first 15 years is realized.

Table X gives the actual and expected deaths and claims for each of the 15 years; but, unfortunately, this information loses much of its value in consequence of our not being informed by what table the expected deaths have been computed. It appears that during the first 5 years, 1872–76, the deaths were 44·4 per-cent, and the claims 39·3 per-cent of the expected; that in the second 5 years, 1877–81, the percentages were 62·1 and 60·1; and in the third 5 years, 1882–86, they were 78·2 and 75·7. This gradual increase is no doubt due to the wearing out of the benefit of selection. Taking the whole of the 15 years together, the actual deaths (440) were 69·5 per-cent of the expected (633); and the actual claims 66·8 per-cent of the expected. The expected claims have steadily increast, as compared with the total sum assured, from ·46 per-cent in 1872 to 1·21 per-cent in 1886. Other tables give particulars of the causes of death, the ages at death, and the occupations of those who died; but the most interesting table is No. XIII, which shows how many of the 440 deaths occurred in each insurance year, namely, 31 in the first insurance year, 36 in the second, and so on, as shown in the following table.

TABLE 2.

Length of Assurance	Number of Deaths
Under one year . . .	31
Between 1 and 2 years .	36
„ 2 „ 3 „ .	45
„ 3 „ 4 „ .	36
„ 4 „ 5 „ .	41
„ 5 „ 6 „ .	36
„ 6 „ 7 „ .	38
„ 7 „ 8 „ .	33
„ 8 „ 9 „ .	40
„ 9 „ 10 „ .	29
„ 10 „ 11 „ .	20
„ 11 „ 12 „ .	20
„ 12 „ 13 „ .	19
„ 13 „ 14 „ .	11
„ 14 „ 15 „ .	5
Total	440

Comparing the deaths in the first year with the total number of lives assured, 11,457, we see that the death rate was about ·3 per-cent, or 3 in 1,000, which I am inclined to think is about the lowest rate that life offices can ever hope to arrive at, whatever precautions they may take. It would, however, be useful to know what is the experience on this point, of the offices that transact a very select business; such, for instance, as the mutual offices which pay no commission.

The numbers of deaths in the second and subsequent years, do not increase so fast as I should have expected; and this has led me to compare them with the corresponding figures of other companies. For this purpose, I have referred to those reports as to the mortality of insured lives which I happen to have at hand; and have made use of such of them as contain the necessary information. In some of these reports, the example of the Institute has been followed as regards the "year 0" of insurance; while, in others, the greatly preferable course has been adopted of taking out the experience by policy-years. This diversity of practice is apt to lead to some confusion, if not carefully kept in mind; and more especially in those cases where, as in the experience of the *New York Mutual*, the example of the Institute has been followed in computing the years; but what the Institute calls the "year 0", is called the "year 1". In order to prevent any misunderstanding, I have thought it convenient to call the "year 0" of the Institute, the "year ½"; so that the Institute years 1, 2, 3, &c., are denoted by 1½, 2½, 3½, &c.

The statistics I have made use of are (1) the Institute observations, Healthy Males (H^M). (2) The experience of 23 German Offices, publisht in the year 1883 by the *Berlin Life Insurance Institute* (Healthy Males). (3) The experience of the *Mutual of New York* from 1843 to 1874; Table XIII. This experience includes all the lives insured by the company, no distinction being made between the healthy males and the females and other special risks. (4) The experience of the *Connecticut Mutual* Life Office from 1846 to 1878; Table II (*b*), which relates to all the male lives insured, and Table VIII (*b*) "Male Lives insured under premium-paying life and endowment policies"; the latter table having been formed, apparently, by excluding all lives insured under term policies, or under paid-up policies. (5) The experience of the *Provident Life and Trust Company*, of which I lately gave some account in this *Journal*.

The following table shows, for each of these, as well as for the *Pole Star*, the recorded numbers of deaths in each of the 10 years after entry, and the corresponding numbers of entrants:

Table 3.

Observations.	Institute H^M	German Offices H^M	Mutual of New York	Provident Life and Trust	Connecticut Mutual		Pole Star	
					Table II (*b*)	Table VIII (*b*)		
Number of Lives (or Policies)	130,243	341,744	Lives 101,967	Policies 25,000?	Males 97,790	Select Males 95,060	Lives 11,457	
Year	Number of Deaths (or Claims) in the Year							Year
½	295	1,062	247	43	730	640	31	1
1½	897	2,803	564	103	653	624	36	2
2½	1,032	3,044	570	101	579	558	45	3
3½	1,077	2,877	566	112	603	569	36	4
4½	1,111	2,667	541	98	570	528	41	5
5½	1,003	2,481	500	85	617	560	36	6
6½	999	2,379	408	67	560	487	38	7
7½	949	2,210	317	71	533	483	33	8
8½	885	2,025	200	40	534	467	40	9
9½	853	1,749	176	54	471	408	29	10
10½	850	1,538	153	46				

The figures in their present state enable us to make one interesting comparison, namely, to note in which year the greatest number of deaths occurred. In the H^M experience, this is the year 4½; in the *Provident Life and Trust* it is the year 3½; in the *Pole Star* it is the year 3; in the *New York Mutual* and *German Males*, it is the year 2½; while in the *Connecticut Mutual* it is the year 1.

The experience of the last-named company in this respect seems quite exceptional, and probably arises from the benefit of selection being less than is usually the case. As it appears that the office at one time experienced a heavy mortality among lives insured for terms of years at an extra premium, I have thought it desirable to include in my table the figures taken from the company's Table VIII (*b*), from which this class of lives is excluded; and it will be seen that these figures show a similar progression, but not so strongly markt; that is to say, the deaths for the first year are still more numerous than in any other year; but the excess of the deaths in the first over those in the second year, is very much less in proportion than in the figures taken from Table II (*b*).

In order to compare more easily the rates of mortality in the different offices, I have reduced all the numbers to a common radix of 100,000 entrants; and the following table exhibits the results so clearly that no further explanation seems necessary.

TABLE 4.

Calculated Numbers of Deaths during the first 10 *Insurance Years among* 100,000 *Entrants.*

Observations	Institute H^M	German Offices	Mutual of New York	Provident Life and Trust	Connecticut Mutual		Pole Star	
					Table II (*b*)	Table VIII (*b*)		
Year	Number of Deaths (or Claims) in the Year							Year
½	227	311	242	172	747	654	271	1
1½	689	820	553	412	668	638	314	2
2½	792	891	559	404	592	571	393	3
3½	827	842	555	448	617	582	314	4
4½	853	780	531	392	583	540	358	5
5½	770	726	490	340	631	573	314	6
6½	767	696	400	268	573	498	332	7
7½	729	647	311	284	545	494	288	8
8½	679	593	196	160	546	478	349	9
9½	655	512	173	216	482	417	253	10
10½	653	450	150	184				
Total	7,641	7,268	4,160	3,280	5,984	5,445	3,186	...
Deduct	327	225	75	92	...	...	...	...
Deaths in 10 years	7,314	7,043	4,085	3,188	5,984	5,445	3,186	...

The total number of deaths in the first 10 insurance years, will furnish a good approximate measure of the rate of mortality experienst in each of the groups of lives under consideration. In order to get this in the case of the first four in our table, we must deduct from the total deaths in the years ½, 1½, 2½, ... 10½, one-half of the deaths in the year 10½, as is done at the foot of the table. Comparing then the total deaths, it will be seen that the *Pole Star* and the *Provident Life and Trust*, exhibit results which are practically identical, and much more favorable than all the others. The *Mutual of New York* comes next, then the *Connecticut Mutual;* and, lastly, the *German* and *Institute* observations. These results agree pretty closely with the information given as to the relation between the actual and the expected claims. The actual claims of the *Pole Star* are stated by Mr. Samson to have been 69·5 per-cent of the expectation; and Mr. Wing says those of the *Provident Life and Trust* were 65·7 per-cent of the expected, and that the corresponding percentage in the case of the *New York Mutual* was 79·9 per-cent. In the Connecticut Mutual, the percentage was 85·3.

It must not be forgotten that this comparison will, for every insurance year after the first, tell unduly in favor of a young company, as compared with an older. Supposing a company to have been establisht for *n* years, the deaths in the first insurance year, may have occurred among any of the entrants of the whole *n* years; but the deaths in the second insurance year, can only have occurred among the entrants of the first ($n-1$) years; the deaths in the third insurance year, only among the entrants of the first ($n-2$) years; and so on; and the smaller *n* is, so much the more rapidly will the number of entrants decrease, from among whom the deaths in successive insurance years must have occurred. Making, however, every allowance for this, the mortality of the *Pole Star* seems to have been very favorable.

T. B. SPRAGUE.

The Life Assurance Companies of the Australasian Colonies.

THE following summary of the revenue accounts for the year 1886, of the twelve companies whose head offices are in the Australasian Colonies, has been compiled from a return printed in the *Australasian Insurance and Banking Record* of 15 January 1887:

				£	s.	d.		£	s.	d.
Amount of Funds at beginning of the Year				8,763,415	19	6	Claims and Surrenders*	749,930	9	10
Premiums—							Annuities	16,021	15	6
New	£282,016	1	11				Cash Bonuses and Dividends	45,727	15	3
Renewal	1,504,686	10	2							
				1,786,702	12	1	Expenses	422,852	16	2
Consideration for Annuities				17,165	16	6	Funds at end of Year	9,900,326	13	7
Interest				565,801	8	10				
Other Receipts				1,773	13	5				
				£11,134,859	10	4		£11,134,859	10	4

* Not separately stated.

The particulars of new policies issued by the twelve Australasian companies during the year are as follows:

	£	s.	d.	
35,469 Assurances and Endowments, for	9,865,351	0	0	
New Single Premiums thereon . .	22,478	18	9	
„ Annual „ „ . .	330,553	5	11	
35 Annuities granted for . . .	2,209	3	6	(per ann.)

CORRESPONDENCE.

THE GRAPHIC METHOD OF GRADUATION.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—If I make no reply to Mr. Higham's letter which you printed in your April number, it may probably be thought that I acquiesce in the views he expresses. I think it, therefore, desirable to state that I cannot accept the two propositions which he enunciates, as giving a fair account of the conclusions arrived at in my paper. I would rather describe my conclusions as follows: the series of numbers which represent, either the probabilities of dying at different ages, or the expectation of life at different ages, does not proceed by constant third differences; therefore a graduation by such formulas as Mr. Woolhouse's and Mr. Higham's, will not give theoretically correct results; but will have a tendency to distort, to some extent, the law of the facts. To what extent they do this, is a question I have not investigated. Judging from the example to which Mr. Higham refers us, namely, Mr. Woolhouse's adjustment (*J.I.A.*, xv, 396), the distortion is probably very small; but, even in that case, I think that clear indications may be found that it exists.

I agree with Mr. Higham, that the object of a good graduation, is to faithfully reproduce every well-pronounced characteristic in the original; but when a graduation has been made by the graphic method, we can best judge whether the desired object has been accomplisht, by comparing the graduated results with the original facts, rather than with another adjustment, made by Mr. Woolhouse's or Mr. Higham's formula. This is the course I pursued myself, in making the graduation which I lately submitted to the Institute; and I am not conscious of having obtained any assistance from the study of Mr. Higham's adjustment.

I am not sure that I correctly understand the concluding paragraph of Mr. Higham's letter. He speaks of the possibility of labor being bestowed on his untoucht results, which might possibly produce results better than anything hitherto produced; but he does not indicate in what way such labor is to be applied. I have felt an objection to Mr. Higham's method of procedure because, as hitherto explained by him, it is a mechanical procedure, which does not admit of the application of any judgment or skilled labor. In fact, I have looked upon the results given by his formula, as final results, which he intended should be left untoucht; and, if I have done him injustice in this respect, I trust that he will explain how he would proceed, in the event of his desiring to rectify and beautify the results given by his formula.

I am, Sir,
Your obedient servant,
T. B. SPRAGUE.

Edinbro',
1 *July* 1887.

CLAIM ACCELERATION RESERVE, &c.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—For some time past I have been intending, with your permission, to correct an oversight on *J.I.A.*, xxiv, 76, for it would seem that claim acceleration reserve should be based on the theoretical instead of on the actual date for payment, at any rate as long as the fraction combined with the annuity-value in capitalizing future premiums is dependent on the date of their falling due rather than of their being received. That is, that the interval necessary for proof of death and title ought not to be taken into account, unless the grace days allowed for renewals are considered on the other side; or, in other words, if claims are payable immediately, a full half-year's (not five months') interest must be reserved, unless the ½, or whatever it is, used with the a in valuing the premiums is not fixed by the average of their due-dates only, but regard is also had to any delay there may be in the cash reaching the office.

And, as I am writing, I would add that the formula on *J.I.A.*, xxvi, 54, looks less formidable if y be written for $\frac{x-1}{2}$; while, later on, $\frac{2x^2}{x^3}$ instead of $\frac{2}{x}$ is an ugly mishap.

I am, Sir,
Your obedient servant,
C. D. HIGHAM.

3, *Princes Street, Bank, London,*
26 *May* 1887.

FRIENDLY SOCIETY LEVIES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In the last number of the *Journal* (p. 389) Mr. King refers to the above subject, and gives very simple demonstrations of the formula for the value of the future death levies,

$$W=\frac{m(m-1)}{2}\bar{A}_{xx},$$

where m is the number of members, x the average age, and 1 the sum paid by each member at a levy. The proof he gives of the above, by the use of contingent assurances, I may say, was suggested to me some years ago by Mr. H. J. Rothery.

It frequently happens that levies are made not only at the deaths of the members but also at the deaths of their wives, and a similar method of dealing with these leads to an equally convenient formula by which to value them. If we assume that all the members are married, and that w_1, w_2, w_3, &c., represent the ages of the wives of the members x_1, x_2, x_3, &c., the value of the future levies on the deaths of wives, will be

$$\begin{array}{l}
\bar{A}^1_{w_1x_1x_2}+\bar{A}^1_{w_1x_1x_3}+\bar{A}^1_{w_1x_1x_4}+\ldots\ldots(m-1)\text{ terms}\\
+\bar{A}^1_{w_2x_2x_1}+\bar{A}^1_{w_2x_2x_3}+\bar{A}^1_{w_2x_2x_4}+\ldots\ldots\quad ,,\quad ,,\\
\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
+\bar{A}^1_{w_mx_mx_1}+\bar{A}^1_{w_mx_mx_2}+\bar{A}^1_{w_mx_mx_3}+\ldots\ldots\quad ,,\quad ,,
\end{array}$$

In all there will be $m.\overline{m-1}$ contingent assurances, and if we assume as before an average age x, for members and wives alike, we have

$$m(m-1)\bar{A}^1_{xxx}=\frac{m(m-1)}{3}\bar{A}_{xxx}.$$

Mr. King considers these formulas to be theoretical, not practical, solutions, since the number of members in a society is constantly subject to change by new entrants and by secessions; but I think this objection cannot be urged with greater force in the case of these contributions than in that of the fixed annual payments of the members, and that the above formulas represent the sums at which these particular assets should be taken in the valuation. On the one hand, it is clear that no account can be taken of their possible increase by the addition of new members, nor, on the other hand, is it necessary to take any account of their possible reduction by secessions, since (provided negative values have been properly excluded from the valuation) the net liability of the society will not be increased by such means. In this respect the levies are in the same position as the fixed contributions; their estimated value may be diminished by secessions, but since, at the same time, the society's liabilities are correspondingly diminished, this is of no moment.

In order to exclude negative values from a valuation in which future levies appear as an asset, the net liability for each member, after deducting the value of contributions, but *independent of the levies*, should be taken at not less than

$$(m-1)\bar{A}_{x_1x_2}=(m-1)\bar{A}_{x_1x}\text{ nearly}$$

(where x_1 is the age of the member and x_2 the mean age of the remaining $\overline{m-1}$ members, which may be taken as practically $=x$, the mean age of the whole society). For the assets, as regards the value of the levies, will be diminished in case of the secession of the member x_1 by the value of the assurances on x_1 jointly with the remaining $(m-1)$ members. At the younger ages, where alone negative values will appear, the above will be somewhat less in value than

$$(m-1)\bar{A}_{xx}$$

where x is the average age of all the members, as before; this latter quantity being constant, while the former varies with the ages x_1, x_2, it may be conveniently substituted.

As an example, if we suppose a society of 150 members making a levy of 1*s.* per member at each death, and of an average age giving $\bar{A}_{xx}=\cdot600$, the constant $(m-1)\bar{A}_{xx}=4\cdot47$; and provided the net liability in respect of each member, as above (independent of the levies) is taken at not less than this figure, negative values will be effectually excluded, and the values of the levies as given by the formula will be a perfectly good asset.

I am, Sir, your obedient servant,

5, Whitehall, S.W.; G. F. HARDY.
24 *August* 1887.

National Provident Insurance.

THE following is the full text of the Report of the Select Committee appointed by Parliament (originally in 1885, and renewed in subsequent Sessions) to enquire into the best system of National Provident Insurance:

Since the appointment of a Select Committee in 1885 to enquire into the proposals for a national scheme of provident insurance against pauperism, Parliament has been twice dissolved, and the enquiry resumed by a second Committee in 1886, and again interrupted, has now been brought to a conclusion. Your Committee, in making their Report, are conscious of the disadvantage inseparable from the absence from Parliament of some of those honourable Members who took part in the earlier part of the enquiry, and the fact that some members of your present Committee were not appointed until after the greater part of the evidence had been concluded.

Your Committee have to report, that although the reference was "to enquire into the *best* system of National Provident Insurance against Pauperism", the enquiry has practically narrowed itself into an examination of one particular scheme, namely, that formulated by the Reverend W. L. Blackley, Hon. Canon of Winchester, hereinafter referred to as Canon Blackley's scheme. Other proposals were submitted, by the Reverend John J. Stratton (366 *et seq.* [1885]), by Mr. C. P. Tebbutt (506 *et seq.* [1885]), by Mr. J. Broughton Edge (2445 [1886]), by Mr. Robert Davie (1485 [1886]), but the evidence always tended to revert to Canon Blackley's scheme, which had manifestly impressed itself, whether favourably or unfavourably, upon the minds of witnesses, to the exclusion of all other proposals.

CANON BLACKLEY'S SCHEME.

Canon Blackley's scheme may be briefly described as one for the compulsory insurance of all persons, of both sexes and of every class, by the prepayment between the ages of 18 and 21 years of the sum of £10 or thereabouts into a national friendly or provident society, thereby securing to the wage-earning classes 8*s.* per week sick pay, and 4*s.* per week superannuation pay after the age of 70 years. It proceeds on the assumption that, whereas it is a moral as well as a political duty for every individual to make provision for independence in sickness and old age, large numbers of the population of this country make no such provision, but ultimately come upon the rates, thus compelling the thrifty and the industrious to pay for the maintenance of the unthrifty and the idle. The evidence submitted to your Committee in favour of Canon Blackley's scheme was not, in the main, of a representative character, but from persons only responsible for their individual opinions.

This proposal was supported by a number of witnesses of various classes. Your Committee desire especially to draw attention to the evidence of some members of the working classes, as showing the desire felt by them for some means of securing their own independence, and for adequate security for any provision which they may make for themselves. Mr. Charles Symes, a cabdriver, who was put into a workhouse at three years of age, and remained there till he was 13 or 14 (676 [1885]), was asked (718 [1885]) why he had never joined a friendly society before he was over the age. "The thought", he answered, "never entered my mind that I should become old. Unhappily, I have it now clearly painted before me, turn which way I will; whenever I come out of the house in the morning I have got the workhouse before me; I never can get it at my back, and I ask you, gentlemen, now, whether my mind must not be in a bad state. If I was convinced that I was going, now, to have 4*s.* per week whenever I thought proper to apply for it, so that I should be quite convinced that I need not be compelled to go into the workhouse, then I should consider that the workhouse was at my back, and I might be quite serene in the matter; but I am in that unhappy state which I have been describing to you, and I know that there are many thousands more in the same state."

Mr. Richard Nesbitt, a Scotch gardener, stated that he had a family of seven children (1555 [1886]), who commenced, on hearing of Canon Blackley's scheme, to save their odd coppers, and put them into the bank; "and I am happy to say", he added, "that all these seven children will have their money ready to throw into the scheme as soon as it becomes law."

Mr. Charles John Goodwin, a compositor, was asked (2321 [1886]) his opinion upon making any general scheme compulsory. He replied that he thought it should be compulsory, "because I find there are a number of people who will never avail themselves of any of the methods open now to provide against these emergencies, but so soon as they become ill they readily fall upon their fellow-workmen for help in the shape of subscriptions."

The evidence of Sir Edward Walter, the founder and present commander of the Corps of Commissionaires, sixteen hundred strong, was given unhesitatingly in favour of Canon Blackley's complete scheme; and it is the more worthy of consideration from the fact that Sir Edward has established in his corps a system of compulsory provident insurance (1773, &c. [1886]), which is exceedingly beneficial to, and appreciated by, the men.

Mr. John Aird, of the firm of Messrs. Lucas & Aird, who employ at times from 12,000 to 14,000 men (300 [1887]), described (315, &c. [1887]) the creation and maintenance of a sick and accident fund (*see* Appendix No. 6 [1887]). "We created that fund, and from that time have deducted from every man 2*d.* per week, and from every boy 1*d.* per week; and although that has been going on for all this period (six or seven years), I do not remember any single case of complaint of such deduction being made." He states, in reply to Q. 514 [1887]: "I do not think the people would look upon it (a compulsory contribution) as compulsion; I think they would look upon it as a great privilege and a great safeguard."

On the other hand, your Committee received a considerable amount of evidence, from various sources, adverse to Canon Blackley's proposals, namely, from the official point of view, chiefly on administrative grounds; from the actuarial point of view, chiefly upon grounds of the insufficiency of the proposed initial sum for securing the required benefits; and from what may be called, without any sort of unfavourable reflection, the point of view of vested interests, namely, that of the representatives of the affiliated orders of friendly societies, who aver that to give the whole nation, by compulsory insurance, an independent provision against pauperism in sickness and old age, would limit the growth, in numbers, of their own voluntary organizations.

One of the most important objections to the scheme, and one which has been frequently urged, is, in the opinion of your Committee, the argument that in the proposed national assurance society there would be great difficulty in preventing malingering, and in keeping down the amount of sick pay that would have to be provided. This objection seriously affects all that part of the scheme which refers to the payment of benefit in sickness.

Your Committee believe that many of the poorest class would be unable to provide £10 between the ages of 18 and 21 without great difficulty; that many of those among them who might be able would be unwilling; and that the collection of the money from those whose work was fitful and uncertain would be almost impossible. Among the poorest class so many exceptions would have to be made in case of both women and men that the scheme would, to a great extent, fail to benefit many of those who at present add very largely to the pauperism of the country. It would not assist those who are out of work, or compel those who are idle and dissolute to go to work. And yet absence of employment for those who are willing to work, and the reluctance of the idle to work under any circumstances, are two of the most fruitful causes of poverty. It should be further noted that the very large proportion of paupers who are under 18 years of age would receive little or no benefit from the scheme.

The proposal that only wage-earners should receive the benefits of the scheme creates a distinction which your Committee fear would be quite unworkable. Many poor persons who are not wage-earners would be excluded, and many wage-earners who are perfectly able to provide for themselves would be admitted to the proposed benefits.

The upper and middle classes, none of whom would ever enjoy the benefits unless they fell to the level of wage-earners, would, your Committee think, strongly protest against this form of compulsory charity (a compulsory tax of a very different kind from the existing poor rate), part of which would be enjoyed by many wage-earners who are quite able to look after themselves. This scheme is not one for enabling working people to be insured with their own money alone, but is dependent for its success upon the enforced contributions of those members of the community that are not wage-earners, some of whom could ill afford such contributions, and would receive no benefit from them.

Your Committee think that the proposal for compulsion, which is an essential part of the scheme, is a proposal which is open to very strong objections. While the Committee are well aware that the promoters of this scheme have the most benevolent objects in view, and are actuated by a sincere desire to remove pauperism, they have grave doubts whether these objects would be obtained by the compulsory methods proposed. They believe that the great majority of the working classes would prefer to provide their own insurance in their own way, rather than be parties to any compulsory scheme. Anything which tended to undermine the self-taught habits of thrift and self-help which prevail among the working classes to a considerable extent, or to lead the wage-earners of the country to rely for support upon a national insurance society, practically guaranteed by the State, might be fraught with disastrous consequences. There is no proof that this scheme, according to which the employer would hand over a large part of the insurance-money without any personal effort on the part of the wage-earner, would teach habits of thrift. It might, it is to be feared, work in the opposite direction. It might lessen the feeling of responsibility which is shared by a constantly-increasing number of the working classes of providing their insurance for themselves. It might seriously impair that education in thrift and training in business which has been brought about by the building up and management of their own associations for self-help by the working classes, the value of which to the nation it is almost impossible to exaggerate.

An objection which your Committee think weighty enough to notice is that advanced by a representative of the Post Office (Cardin, 1581 *et seq.*, 1649 [1885]), who, though personally in favour of a scheme of national insurance (1579), foresees difficulty in the Post Office being made the means for the collection and distribution of the fund. This objection, even if maintained (and it appears to be one that presents itself against every successive proposal for increasing the usefulness of Post Office machinery), does not appear to be one fatal to Canon Blackley's scheme, supposing it to be approved of in other respects, and it might probably be met by an alteration in detail of the proposed system.

Another serious difficulty which presents itself against the adoption of Canon Blackley's scheme consists in the absence of any actuarial opinion of the adequacy of the proposed contribution (£10 at the age of 21) to secure the proposed benefits, especially as the tendency of the evidence was against the possibility of calculating for a rate of interest on accumulations higher than 3 per-cent. No actuary was found to certify that £10, prepaid at 21 and invested at 3 per-cent, would suffice for the purpose in view.

To this Canon Blackley replies that, although he named £10 as the probably adequate sum, he has never stated that it was the exact sum required, subject to no modifications by inquiry or experience; and that he has always associated such probable sum with a higher rate of accumulative interest. The actuary of the Manchester Unity having stated (941 [1885]) that the benefits proposed by Canon Blackley would cost (according to the experience of his society at a 3 per-cent accumulation) £18, the Reverend Canon has submitted that the same benefits would be reduced in cost as follows, according to the rate of interest realized:—

			£	*s.*	*d.*
Invested at	3¼	per-cent to	15	18	9
„	3½	„	14	3	4
„	3¾	„	12	17	7
„	4	„	11	3	10

Your Committee, however, are of opinion that it would not be prudent to embark on a scheme of such magnitude on the estimate of receiving a higher rate of interest than 3 per-cent; accordingly, if the initial payment of £10 is to be held adequate, it would be in prospect of the fund being largely enriched by the contributions of those classes who, not being wage-earners, would be debarred from claiming the benefits of insurance. Undoubtedly, these would amount to a very large figure, and would very materially reduce the initial payment required, but to what extent your Committee have not the means of ascertaining even approximately.

Mr. Ralph Hardy, an experienced actuary, being asked (1940 [1886]) whether he considered the prepayment of £10 sufficient to cover the demand on the fund, replied: "Not according to any basis with which I am acquainted, unless it could be shown satisfactorily that the number of abstentions from claims of those who became ineligible under the scheme would be such as to make the balance a sufficient payment." Your Committee were much assisted in arriving at this opinion by the evidence given, and documents submitted, by Mr. Sutton (App. 2, page 140 [1885]).

An idea appears to prevail that if national compulsory insurance were established it would cause the present poor laws to cease. That is not part of the proposal. The poor laws would remain, and it is contended that the population, as they grew into national contributors, would also grow out of reliance on the poor laws, and thus cause poor rates gradually to disappear.

Friendly Societies.

The admirable motives, labours, and results which form the history of the higher order of friendly societies naturally have caused your Committee to treat with great respect the strong objections brought against the scheme by the representatives of the affiliated orders.

These objections resolve themselves mainly into apprehension lest the operation of compulsory national insurance should interfere in the numerical increase of their own organization. So keenly apprehensive are some of the officials of these societies of this effect, that Mr. Reuben Watson, actuary to the Manchester Unity of Oddfellows, did not hesitate to reply to *Q.* 909 [1885], "If you could devise some scheme which would be for the welfare of all classes in this country, but which would be to the detriment of friendly societies, you would not object to it on that ground"? *A.* "Well, I think I should object to it. I consider that friendly societies have voluntarily done a very great deal of good in this country, and I think that they ought not to be interfered with by the establishment of any system which would be injurious to them."

No conclusive evidence was given to show that the establishment of a compulsory society on a national basis could endanger the funds or affect the soundness of any existing voluntary society conducted on a sound principle.

Little adverse evidence was tendered by or on behalf of any friendly societies, except the affiliated orders.

Your Committee, while deferring much for the present to the opposition of the affiliated orders, and acknowledging the energy and skill with which their organization is conducted, their laudable efforts to improve their financial position, and the degree of advantage to the classes from which their benefit members are drawn, do most earnestly desire to see further exertions made in the direction of securing the investments of their contributors.

It is well known that a considerable number of friendly societies show a deficiency in valuation.

It must, however, be frankly admitted that the Manchester Unity of Oddfellows (and the same remark applies to the Ancient Order of Foresters and some other societies) has made most praiseworthy efforts to collect trustworthy information for its guidance, and has framed graduated rates of contribution adequate to the benefits promised, which are in force in nearly all its lodges. There still remain, however, the bad bargains made with old members, and it is for this reason, in a very great measure, that many lodges still show deficiencies on valuation. It may be safely asserted that, speaking generally, the lodges of the Manchester Unity have, to the extent of nearly one-half of their number, succeeded in attaining actuarial solvency entirely by their own unaided efforts, and as regards the remainder, many of them are slowly but surely tending towards the same goal.

It is evident to your Committee, in spite of their acknowledgment of the increasing good work done by well-managed friendly societies, and of all the recently provided national aids to thrift, that their tendency is, while aiding the thrift of the thrifty, in no way to discourage or put an end to the waste of the improvident.

THE POOR LAWS.

On the 1st March 1886 (Lloyd, 186 [1886]) circulars were sent by the Council of the National Provident League to all the Boards of Guardians in England to the number of 649, and at the end of March to about 90 of the largest Parochial Boards in Scotland, requesting an expression of opinion as to the proposal for compulsory national provident insurance. By the 6th May following 86 resolutions in favour of the scheme had been received, and not one unfavourable.

The Barony (Glasgow) Parochial Board, representing the largest parish in Scotland, with 264,000 inhabitants, accompanied their resolution with an intimation that three of their officials, including the chairman, were ready to give evidence in favour of the scheme. The chairman and inspector afterwards appeared before your Committee (233 *et seq.* [1886]).

COMPULSORY INSURANCE IN GERMANY.

The attention of your Committee was naturally directed to the recent legislation in the German Empire, by which, one after the other, various industries have been progressively brought under compulsory insurance.

Dr. F. Aschrott, holding an office equivalent to an assistant-judge in the Court of First Instance, and who was commissioned to visit this country to inquire into the working of the poor law here, gave minute and valuable evidence (1690 *et seq.* [1885]) as to the establishment, growth, and popularity of compulsory insurance in Germany.

The principal difference between the system at work in Germany and that proposed by Canon Blackley appears to be fourfold:

1st. At present, at all events, compulsion is confined to the industrial classes, and, further, among them, to certain trades or occupations, although, eventually, it is intended to include them all. Thus the fact that every subscriber is entitled to the benefits of the fund has its advantage over a system in which a large number of contributors, including not only the landed and wealthy commercial classes, but the small traders and farmers, are debarred from sharing in the direct benefits; while, on the other hand, the objection may be urged that compulsion, applied to working men only, may be held to be invidious.

2nd. Payments to the fund are not made in an initial lump sum, but by weekly and lifelong deductions from wages. In this respect your Committee think that the German system shows an inferiority to Canon Blackley's proposal, inasmuch as prepayment enables a much smaller total payment to suffice as insurance, and, once got over, the workman is left free to dispose of his savings in whatever prudential investment he may select.

3rd. No provision is made in the German system for superannuation: the insurance is solely against accident or sickness, and thus what appears to your Committee to be the most desirable part of any scheme for national provident insurance is unprovided for.

4th. A choice is left to the insurer of three different clubs:

> (*a*) The *Betriebes Kranken Casse*, or Factory Club, restricted to any factory or workshop employing not less than 50 men;
>
> (*b*) The *Orts Kranken Casse*, or Local Club, restricted to the members of any separate trade in any town or district;
>
> And (*c*) the *Gemeinde Kranken Casse*, or Parochial Club, insurance in which is compulsory upon any workman who has not joined either of the other institutions. It is somewhat as if, supposing compulsory insurance to be the law in this country, the Legislature were to provide that every working man who was not a member of an approved friendly society should subscribe to a fund managed by the parochial authority.

Dr. Aschrott's evidence was clear (1716 *et seq.* [1885]) as to the favour with which this legislation has been received by the people, and the preference shown for the factory and local clubs over the parochial club, although the rate of contribution to the two former is higher than to the latter.

On the all-important question of malingering, he replied with confidence (1715 [1885]), and he appeared to be satisfied that the deferring of sick pay until after three days' sickness, and the large share confided to the workmen in the management and administration of the fund, were practically sufficient safeguards against any but exceptional imposture.

The actuarial and administrative difficulties in the way of universal insurance against sickness and accident are sufficient to prevent your Committee from recommending the adoption of a compulsory scheme of that nature; but they desire to urge on the Legislature the duty of encouraging and regulating the efforts of those, whether friendly societies or private employers of labour, who are at present endeavouring to facilitate provident insurance by working men; and, further, of watching with a careful eye the development of compulsory industrial assurance in Germany, and of special enquiry from time to time into the success of its administration, and its effect on the working classes in that country.

Your Committee are of opinion that it is highly desirable that the Legislature, which has made education compulsory, should cause instruction in sound principles of thrift and insurance to form part of that education. An elementary text-book in the principles of provident insurance, such as is recommended by Mr. Sutton (1628 [1885]), and in Mr. H. S. Tremenheere's paper (Appendix I. [1887], Section 10), might be comprised in a few pages, and if introduced by the Education Department into the routine of elementary schools, prove of infinite advantage to the whole mass of the population, and ultimately, by the diffusion of knowledge, sap the evils of improvidence and imprudent investment to which so many miseries have been ascribed in the evidence before your Committee.

The present system of registration of friendly societies cannot be considered satisfactory. Societies are allowed to register without any requirement on the part of the Registrar that their scales of contribution are adequate for the benefits promised. Your Committee would suggest that when the actuarial tables which are being prepared in the Registrar's office are complete, a minimum contribution for a given benefit should be fixed when the constitution of the society will admit of this, and no society in future should be registered with less than that minimum, and that it be provided by the rules that the contribution be raised, or the benefit reduced after valuation, if necessary, on pain of suspension of registry. Your Committee are aware that such a plan would be accompanied with considerable difficulties, but they believe these difficulties could be surmounted.

Were this plan carried out, and greater powers given to the office of the Registrar of Friendly Societies than it at present possesses in connection with the registration of societies' rules, and with a view to the securing an efficient audit, the proper investment of funds, and the protection of benefit funds from any inroads upon them for management expenses, it is believed that registration would be of far greater value to the members of these societies than under the present arrangements. It is desirable also that the facilities for the prosecution by a public authority of societies or officials, especially when fraudulent practices have taken place, should be enlarged. Your Committee think that the appointment of a Select Committee to inquire into the working of the Friendly Societies Acts might lead to very useful results.

Your Committee are of opinion that all persons hereafter appointed to the service of the Crown, whether civil or military, whose service at present counts towards pension, should contribute towards that pension by a percentage deducted from salaries or pay. The steady and rapid growth of the Pension List points to a proximate revision of the entire policy of burdening the public with the provision of pensions; the enterprise of private individuals and firms (such as has been witnessed to by Sir Edward Walter and Mr. Aird) indicate the advantage of self-help as a condition of employment (which it might be proper to supplement with State-help); and your Committee recommend that not only in service, counting under the present system towards pension, but also in the police and other unpensioned branches of the public service, contribution to a pension fund should be made obligatory.

The financial objections to be urged against Canon Blackley's proposals for national insurance do not press with such force against the deferred annuity part as against the sick-pay part of his scheme. As regards the principle of compulsion, there would possibly be quite as strong objections raised in this case as in the case of sick pay. But, bearing in mind that deferred annuities can be purchased early in life for comparatively small amounts, it would appear to be eminently desirable to give every reasonable opportunity and encouragement to young persons to purchase those annuities; thus, at all events, securing to them provision for their wants when past the age for labour. It was pointed out in evidence (1840, 1847 [1885]), that the present Government system of deferred annuities is capable of very considerable improvement, and in this direction the Committee are of opinion that much good might be done.

Another great objection from which this part of the proposal is entirely free, is that of interference or competition with any existing organizations whatever. No voluntary organization, even if providing the method, can attract the membership to such a fund.

Thus, although the Manchester Unity possesses calculated rates of payment, and actually has established a paid-up superannuation, one of its directors, Mr. Holmes, stated (767 [1885]), that two years after its establishment only four members out of 600,000 had joined it.

The provision of a pension, superannuation, or annuity, was most strongly urged by several witnesses, and, in fact, apart from general objections to compulsion, which were little pressed throughout, a large proportion of the objections brought against Canon Blackley's proposals were directed against the sick-pay portion of the scheme, and not against the provision of pensions.

Your Committee are, however, disposed to wait for the further development of public opinion, which they believe would ensue upon the carrying out of the recommendations contained above, before advising the adoption of a general obligatory system of superannuated pay.

Your Committee, although unable to recommend the adoption of Canon Blackley's scheme, feel that they cannot conclude their Report without recording their sense of the disinterested patience and energy with which he has laboured to remove the causes which tend to drive the poor into the workhouse. He has brought to light an immense deal of information on a subject which lies at the root of the happiness and welfare of large masses of the population, information which cannot fail to prove useful in any future legislation which may be undertaken, and his proposals, though in the opinion of your Committee they appear objectionable in some respects, and impracticable in others, contain more valuable suggestions, and seem to be based on more extended knowledge, than any of the other schemes which have been brought under their attention.

2 *August* 1887.

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